Answer all questions.
Show your work to receive credit.
Each section receives equal weight.
True, partly true, or false. Prove all claims.

1. It is possible to run an F-test for whether at least one of a set of additional variables has a non-zero coefficient in the true regression model by knowing only the $r^2$ from two regressions, one with the additional set of variables and one without them.

2. If the true errors in a regression model are AR(1) but one uses the OLS assumptions to construct standard errors then the magnitude of the standard errors will tend to be overstated.

3. Let $y$ be the dependent variable and $x_1$ and $x_2$ be explanatory variables. Suppose we project $x_1$ off of $x_2$ to get $\tilde{x}_1$. Frisch-Waugh-Lovell says that if we regress $y$ on $\tilde{x}_1$ we will get an estimate for $\beta_1$ and a vector of regression residuals that is exactly the same as what we would get if we regressed $y$ directly on $x_1$ and $x_2$.

4. The instrumental variable estimator is consistent even when one has weak instruments.

5. Consider two consistent estimators with one efficient and the other not. Then the asymptotic covariance between these two estimators is zero.
SECTION 2

Consider a model that is non-linear in parameters:

\[ y_i = e^{x_i \beta_0} + \epsilon_i \]

with \( \epsilon_i \) i.i.d. Suppose we construct estimates of the residuals for any possible candidate value of \( \beta \):

\[ e_i(\beta) = y_i - e^{x_i \beta} \]

and then use as an estimate of \( \beta_0 \)

\[ \hat{\beta} = \arg\min \sum e_i^2 \]

1. Write down the generalized method of moments representation of this estimator.

2. Is GMM consistent?

3. Is this estimator exactly identified, overidentified, or underidentified?
Answer all questions. Carefully write down all assumptions you need to make while answering questions.

Consider the following binary choice model:

\[ y_i = \begin{cases} 
1 & \text{if } X'_i \beta + \varepsilon_i > 0 \\
0 & \text{otherwise} 
\end{cases} \]

where \( X_i \) is a \((k \times 1)\) random vector that contains a constant such that \( X_i = (1, X'_{2i})' \) and the error term \( \varepsilon_i \) follows a standard normal distribution. Then suppose you have \( n \) i.i.d. observations on \((y_i, X_i)\) from the model.

Do the followings:

1. Describe the following estimators of \( \beta \) and derive their asymptotic distributions
   
   (a) The non-linear least squares estimator
   
   (b) The optimal weighted non-linear least squares estimator
   
   (c) The maximum likelihood estimator
   
   (d) The efficient GMM estimator that uses all moment conditions implied by the model

2. Consider a semiparametric version of the model as \( y_i = \textbf{1}(h(X_{2i}) + \varepsilon_i > 0) \) where \( \varepsilon_i \) follows a standard normal distribution and \( h(\cdot) \) is an unknown function of interest
   
   (a) Describe a sieve ML estimator of \( h(\cdot) \)
   
   (b) Show the consistency of the sieve ML estimator under suitable conditions
1. Suppose \( x_t \) and \( y_t \) satisfy

\[
x_t = x_{t-1} + u_t, \quad y_t = x_t + v_t,
\]

where \((u_t, v_t)\) is a two-dimensional vector of white noise. The variances of both \( u_t \) and \( v_t \) are positive. Find the parameters of the univariate Wold representation for \( \Delta y_t \) and determine the Beveridge-Nelson decomposition for \( y_t \). Under what circumstances (if any) does \( x_t \) correspond to the stochastic trend implied by the Beveridge-Nelson decomposition for \( y_t \)?

2. The process \( \{u_t, v_t\}_{t=1}^\infty \) is white noise with a covariance matrix

\[
E \left[ \begin{pmatrix} u_t \\ v_t \end{pmatrix} \begin{pmatrix} u'_t \\ v'_t \end{pmatrix} \right] = \begin{bmatrix} \Sigma_{uu} & \Sigma_{uv} \\ \Sigma_{vu} & \Sigma_{vv} \end{bmatrix},
\]

where \( \Sigma_{vv} \) is non-singular. The process \( \{x_t, y_t\}_{t=1}^\infty \) is determined by

\[
x_t = Ax_{t-1} + u_t, \\
y_t = Cx_{t-1} + v_t,
\]

starting from a mean-zero random variable \( x_0 \) that has a well-defined covariance matrix and that is uncorrelated with the white noise \( \{u_t, v_t\}_{t=1}^\infty \). Suppose \( p_0 \) is a mean-zero random variable that is also uncorrelated with \( \{u_t, v_t\}_{t=1}^\infty \) and satisfies \( E[(x_0 - p_0)p'_0] = 0 \).

Write \( H_t = \{p_0, y_1, \ldots, y_t\} \) and for any random variable \( z \) let \( P[z|H_t] \) denote the linear combination of random variables in \( H_t \) that minimizes \( E[(z - P[z|H_t])^2] \). Define

\[
p_t = P[x_t|H_t], \quad V_t = E \left[ (x_t - p_t)(x_t - p_t)' \right].
\]

Construct a recursion for \( p_t \) and \( V_t \).