Ph.D. Preliminary Examination

MACROECONOMIC THEORY
Fall 2013

Majors and Minors: Answer ALL FOUR parts.

Please read the instructions before each part and make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly.

You have 5 hours to complete the exam.
Part I. Please answer the following question.

Infinitely Lived Consumers and Dynamic Programming

Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer’s utility function is

\[ \sum_{t=0}^{\infty} \beta^t \left( \log c_t + \gamma \log (\ell_t - \ell_t) \right) \]

Here \( 0 < \beta < 1, \gamma > 0 \). The consumer is endowed with \( \ell \) unit of labor in each period and with \( k_0 \) units of capital in period 0. Feasible allocations satisfy

\[ c_t + k_{t+1} \leq \theta k_t^{\alpha} \ell_t^{1-\alpha} \]

Here \( \theta > 0 \) and \( 0 < \alpha < 1 \).

a) Write down the Euler equations and the transversality condition for the problem of maximizing the representative consumer’s utility subject to feasibility conditions.

b) Formulate the problem of maximizing the representative consumer’s utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman’s equation.

c) Guess that the value function has the form \( a_0 + a_1 \log k \). Guess that in the solution to the dynamic programming problem the optimal labor supply \( \ell(k) \) is constant. Solve for this constant \( \ell \). Solve the dynamic programming problem.

d) Show that the policy functions from the solution to the dynamic programming problem in parts b and c satisfy the Euler equations and transversality condition in part a.

e) Define a sequential markets equilibrium for this economy. Explain carefully how to use the solution to the dynamic programming problem in part c to calculate the sequential markets equilibrium.

f) Define an Arrow-Debreu equilibrium for this economy. Explain carefully how to use the solution to the dynamic programming problem in part c to calculate the Arrow-Debreu equilibrium.
Part 2. Please answer the following question.

Consider an infinite horizon, representative consumer, economy in which preferences are given by:

\[
U = \frac{1}{1-\beta} \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

where \(c_t\) is consumption in period \(t\). Each household has an initial stock of capital given by \(k_0\) and is subject to the law of motion for capital:

\[
k_{t+1} \leq x_t
\]

where \(x_t\) is investment by the household in period \(t\). That is, assume that there is complete depreciation.

Assume that labor is inelastically supplied and that this supply is given by one unit in each period. Output is given by

\[
c_t + x_t \leq F(k_t, l_t),
\]

where \(F\) satisfies all of the normal conditions.

Assume that the government taxes income earned on capital at rate \(\tau\) in every period and that all revenue generated in this way is lump sum rebated to the households. There are no other taxes, and no other government spending.

(a) Carefully define a TDCE for this environment/fiscal policy.

(b) Show that the allocation that results from the TDCE you defined in part (a) can also be obtained by solving a Planning Problem with a representative consumer. Clearly and carefully lay out what this Planning Problem is, and show that its solution is the TDCE allocation from part (a).
Part 3. Please answer the following question.

Default and Government Debt

Consider a simple production economy populated by a large number of identical infinitely lived individuals. In each period $t$, there are two goods: labor $l_t$ and a consumption good, $c_t$. The period utility function is $U(c_t, l_t)$ and individuals discount the future at rate $\beta$. A constant returns-to-scale technology is available to transform one unit of labor into one unit of output. The output can be used for private consumption or for government consumption. The per capita level of government consumption in each period denoted $g_t$, is exogenously specified. This government consumption can take on a finite number of values and is independently drawn over time. The government can finance this debt with proportional taxes on labor income and by issuing one-period debt. Government debt must be nonnegative in all periods. The initial stock of government debt is given and is positive.

(a) Define a competitive equilibrium for exogenously specified policy.

(b) Suppose now that a benevolent government wishes to implement the best competitive equilibrium. Set up the Ramsey problem.

(c) Suppose now the government lacks commitment. Assume policies and allocations can depend on the entire history of policies. Define a Sustainable Equilibrium.

(d) Without commitment, what is the best sustainable equilibrium if $g_0$ is positive and $g_t = 0$ for all $t \geq 1$?

(e) Without commitment, what is the best sustainable equilibrium if $g_t = g > 0$ for $t = 1, ..., T$ and $g_t = 0$ for all $t \geq T + 1$.

(f) Conjecture conditions on the stochastic process for government consumption, and on the discount factor so that the Ramsey outcome is sustainable. Try to prove your conjecture for extra credit!
Stuff related to the Fourth Mini

In the following there are 8 questions for 100 points. Be as BRIEF as you can and good luck.

Monopolistic Competition

Imagine that preferences of a representative consumer in a static closed economy are given by

\[ u(\{c(i)\}_{i \in [0, A]}, n) = \left( \int_0^A c(i)^\gamma \, di \right)^{\theta/\gamma} - \chi n^2 \]

Where \( 1 - n \) is leisure and \( n \) is time spent working. Output is produced with one unit of labor that is taken to be the numeraire. The households are the workers and own the firms.

1. (10 points) Give an expression for the price that each firm charges.

2. (10 points) What is the value of the firm? Verify that total income earned is equal to total expenditures.

3. (10 points) What is the Pareto Optimal allocation? What can you say about the case \( \chi = 0 \).

Lucas Shopping trees and idiosyncratic endowments

Imagine an economy with a continuum of infinitely lived households that discount the future at rate \( \beta \), and cannot borrow or issue state contingent assets but can own shares of a mutual fund. The mutual fund owns a measure 1 of apricot trees each one of them delivering a kg (unit) of apricots that we denote \( c^a \) each period and it also owns a measure one of blueberry bushes, each one of them also delivering one kg of blueberries, denoted \( c^b \), per period. Households have a random endowment of a good that can only be consumed by the household itself, this good cannot be traded or sold. Denote it \( c^h \), it follows a Markov process with transition matrix \( \Gamma^h \). The household has utility \( u(c^a, c^b, c^h, d) \) where \( d \) is search effort.

Both apricots and blueberries are traded in decentralized markets, with identical search frictions. Clearly in order to purchase apricots and blueberries they have to be found. Let \( M(T^i, D^i) \) be the identical matching function for both markets, \( i \in \{a, b\} \). Total search effort is the sum of the search effort posed in each one of the two markets. Let \( p_i \) be the price of fruit \( i \) for \( i \in \{a, b\} \), where the numeraire is shares of the mutual fund.

4. (20 points) Write down the problem of the household including state variables.

5. (10 points) Write down a formula for the value of both trees in terms of the numeraire.

6. (10 points) Imagine that the utility function can be written \( u(c^a + c^b, c^h, d) \). What can you now say about the prices of the trees?

7. (15 points) With this utility function and with the fruit being complementary to the good \( c^h \), will productivity go up in good times? Explain.
8. (15 points) Imagine that the utility function can be written \( u(c^a) + v(c^b + c^h, d) \). Is the price of apricots going up when there is a large endowment of the \( h \) good? What about the price of blueberries? Can you say something about the relative price of apricots and blueberries?