Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MINORS

Fall 2013

The time limit for this exam is $3\frac{1}{4}$ hours.

**Answer one question from each part, for a total of four questions.**

You may use calculators to make calculations during the examination. However: in answering any question that requires you to justify your answer, if you do use a calculator, you must mention at what point in your answer you obtained results using a calculator, and what it was you were seeking to calculate.

Note: This examination should have 14 pages including this one (Check to make sure!)
Part I

Answer one question from Part I.
Question I.1

Consider a production function $f : \mathbb{R}_+^n \to \mathbb{R}_+$ with $n$ inputs and one output. Assume that $f(0) = 0$.

(a) State a definition of $f$ having (strictly) increasing returns to scale.

(b) Prove that if $f$ exhibits increasing returns to scale, then, for any strictly positive input prices $w_i$ (where $i = 1, \ldots, n$) and strictly positive output price $p$, either the firm’s output at the profit-maximizing production plan is zero or otherwise the profit-maximizing production plan is not well defined (i.e. it does not exist).

(c) Consider the following example of production function with two inputs:

$$f(x_1, x_2) = \left(\min\{x_1, x_2\}\right)^2$$

Does this $f$ exhibit increasing returns to scale?

(d) Does the cost-minimization problem for production function $f$ of (c) have a solution for arbitrary prices $w_1 > 0$, $w_2 > 0$ and output level $y > 0$? Justify your answer.
Question I.2

Describe the Ellsberg paradox. Show that the pattern of preferences in the Ellsberg paradox is inconsistent with any expected utility function. Give an example a preference relation (or a utility function) defined on the set of bets (or gambles) of arbitrary amounts of money on a ball of any color that is consistent with the pattern in the Ellsberg paradox. Justify your answer.
Part II

Answer one question from Part II
Question II.1

In pure exchange economies in which consumers may have nonconvex preferences, the potential presence of such nonconvexities can cause one of the welfare theorems not to hold.

(a) State both welfare theorems clearly. Be sure to begin by defining notation for a pure exchange economy, defining its competitive equilibria and its Pareto optimal allocations, and specifying what assumptions are needed for your definitions to make sense.

(b) Prove the welfare theorem that does not require convexity.

(c) For the welfare theorem that does require convexity, indicate where convexity of preferences is needed in the proof. Explain.
Question II.2

Consider a pure exchange economy under uncertainty with asymmetric information where expected utility maximizers have state-dependent utility functions and prices may depend on states of the world. In particular, consider an example with two commodities (x and y), two traders, and three equally-probable states of the world denoted L, M, and H. [Think of low, medium and high temperature where x is ice cream and y is hot coffee.] Each agent (denoted by subscripts 1 and 2) has an initial endowment vector which is independent of the state, in particular, $e_1(L) = e_1(M) = e_1(H) = (1, 1)$ and $e_2(L) = e_2(M) = e_2(H) = (1, 1)$. Suppose that agent 1 knows whether the state is L or is “M or H”, and has state-dependent cardinal utilities given by the following:

$$u_1(x, y; L) = \frac{1}{3} \log x + \frac{2}{3} \log y$$
$$u_2(x, y; M) = u_2(x, y; H) = \frac{2}{3} \log x + \frac{1}{3} \log y.$$ 

Agent 2 knows whether the state belongs to \{L, M\} or is H. Utilities for agent 2 are as follows:

$$u_2(x, y; L) = u_2(x, y; M) = \frac{1}{3} \log x + \frac{2}{3} \log y$$
$$u_2(x, y; H) = \frac{2}{3} \log x + \frac{1}{3} \log y.$$ 

In each state of the world, normalize prices to sum to 1.

(a) Calculate state-dependent demands for each agent.

(b) Calculate the competitive equilibrium under uncertainty.

(c) Is it Pareto optimal? Define and explain what Pareto optimality means in this context.

(d) Are your equilibrium prices state dependent? Can either agent 1 or agent 2 or both learn from prices? Can they learn payoff relevant information that they did not have initially?

(e) Is the equilibrium you found in part (b) a rational expectation equilibrium? Explain.
Part III

Answer one question from Part III
Question III.1

This question has two steps.

Step 1

(a) Find all the sets that survive iterated elimination of weakly dominated strategies in the following game:

<table>
<thead>
<tr>
<th></th>
<th>l</th>
<th>m</th>
<th>r</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>0,0</td>
<td>2,2</td>
<td>0,0</td>
<td>-2,4</td>
</tr>
<tr>
<td>M</td>
<td>6,2</td>
<td>4,8</td>
<td>-8,2</td>
<td>2,6</td>
</tr>
<tr>
<td>B</td>
<td>-2,-4</td>
<td>4,-6</td>
<td>-2,2</td>
<td>-2,8</td>
</tr>
</tbody>
</table>

Be sure that you consider all possible sequences of elimination.

(b) Find all the sets that survive iterated elimination of strictly dominated strategies in the same game

Step 2 Below you will find several normal form games; these are exactly those for which you found the Nash equilibria in the past. For each of these games find:

(a) The set of Nash equilibria and Nash equilibrium payoffs for the stage game

(b) The set $co(F^0)$

(c) The vector of minimax values $(v^i)_{i \in I}$

(d) The set of feasible and individually rational payoffs

The games:

(a) 

<table>
<thead>
<tr>
<th></th>
<th>l</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>-2,2</td>
<td>-3,3</td>
</tr>
<tr>
<td>B</td>
<td>4,-4</td>
<td>-1,1</td>
</tr>
</tbody>
</table>

Question III.1 continues on the next page.
Question III.1 continued:

(b)

\[
\begin{array}{c|cc}
 & l & r \\
\hline
T & 6,1 & 1,1 \\
B & -1,-1 & 1,6 \\
\end{array}
\]

(c)

\[
\begin{array}{c|cc}
 & l & r \\
\hline
T & 1,0 & 4,4 \\
B & 2,2 & 0,1 \\
\end{array}
\]

(d)

\[
\begin{array}{c|cc}
 & l & r \\
\hline
T & 4,3 & 1,2 \\
B & 2,1 & 1,7 \\
\end{array}
\]
Question III.2

(a) Find all the transformations of the utility functions of a player that leave the Best Response correspondence of that player unchanged.

(b) Consider all $2 \times 2 \times 2$ normal form games, that is games with two players, each player with two actions. Illustrate the result in the first part (a) above in these games.

(c) Introduce a distance among all $2 \times 2 \times 2$ normal form games, and consider all games with the property that all other games at a distance less than some $\epsilon > 0$ have the same number and type (mixed or pure) of equilibria. You can now partition these games into subsets, each with the same number and type of equilibria. Characterize these subsets.
Part IV

Answer one question from Part IV
Question IV.1

(a) Write the extensive form for the repeated game which is repeated for two periods, with $\delta = 1$, and has stage game

<table>
<thead>
<tr>
<th></th>
<th>$l$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>4,1</td>
<td>5,6</td>
</tr>
<tr>
<td>$B$</td>
<td>2,0</td>
<td>-1,4</td>
</tr>
</tbody>
</table>

(b) Find the subgame perfect equilibria of the repeated game. Prove your answer in detail.
Consider the infinitely repeated game with stage game given by the PD game, and consider the following tentative agreement:

Play \((C, C)\) in every period as long as both players have played \((C, C)\) in the past. After any deviation, play \((D, D)\) for two periods, independently of the past history, then go back to play \((C, C)\), as long as both players have played \((C, C)\) in all periods following the first deviation.

For which, if any \(\delta\) is this agreement a subgame perfect equilibrium?