Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MAJORS

Fall 2013

The time limit for this exam is $3\frac{1}{4}$ hours.

Answer one question from each part, for a total of four questions.

You may use calculators to make calculations during the examination. However: in answering any question that requires you to justify your answer, if you do use a calculator, you must mention at what point in your answer you obtained results using a calculator, and what it was you were seeking to calculate.

Note: This examination should have 15 pages including this one (Check to make sure!)
Part I

Answer one question from Part I.
Question I.1

Consider preference relation \( \succeq \) on the consumption set \( \mathbb{R}_+^L \). Suppose that \( \succeq \) is reflexive and complete.

(a) State a definition of \( \succeq \) having a utility representation. Is utility representation, if it exists, unique?

(b) State a theorem providing sufficient conditions on \( \succeq \) to have a utility representation. Be as general as you can and clearly define any extra properties of \( \succeq \) that you use.

(c) Prove your theorem assuming additionally that preference relation \( \succeq \) is strictly increasing (i.e., strongly monotone).
Question I.2

(a) In what sense is it true that risk compensation identifies a von
Neumann-Morgenstern (or Bernulli) utility function? State the result clearly.

(b) Prove the result from part (a). If your proof relies on the Theorem of Pratt, you may
use it without proving the theorem, but you need to state it clearly.

(c) Apply your result from part (a) to risk compensations that do not depend on (initial)
wealth. Provide a characterization of utility functions with wealth-independent risk
compensation.
Part II

Answer one question from Part II
Consider a pure exchange economy with \( \ell \) goods and \( n \) traders, each having consumption set \( \mathbb{R}_+^\ell \), initial endowment \( e_i \in \mathbb{R}_+^\ell \), and preferences \( \succeq_i \) which are assumed to be complete continuous preorders on \( \mathbb{R}_+^\ell \).

(a) Define competitive equilibrium in this economy using this notation.

Define a budget based competitive equilibrium or BBCE as follows:

A BBCE is an \( n \)-tuple \( ((x_1^*, B_1^*), (x_2^*, B_2^*), \ldots, (x_n^*, B_n^*)) \) with \( x_i^* \in B_i^* \subseteq \mathbb{R}_+^\ell \) for all \( i = 1, 2, \ldots, n \) such that

1. \( x_i^* \in \mathbb{R}_+^\ell \) for all \( i \) (automatic from the notation above defining a BBCE)
2. \( x_i^* \) maximizes \( \succeq_i \) on \( B_i^* \).
3. \( \sum_{i=1}^n x_i^* = \sum_{i=1}^n e_i \) (so no free disposal).

To avoid confusion below, a competitive equilibrium will now be called a price based competitive equilibrium, abbreviated PBCE.

(b) Prove that every such economy has a BBCE. (Hint: This follows trivially from a simple observation.)

(c) What does your proof in part (b) tell us about the potential usefulness for economics of this statement about the existence of BBCE or, indeed, the definition of BBCE? In particular, does your result guarantee that there are equilibria of economic interest?

(d) Prove that every PBCE gives rise to a BBCE in a natural way so that we could say that any PBCE “is” a BBCE.

(e) Is it true that every BBCE allocation is Pareto optimal? Explain why or why not.
Question II.1 continued:

(f) Is it true that every Pareto optimal allocation in the economy is a BBCE allocation for some BBCE? Explain why or why not. For your answer, you may make additional standard “textbook” assumptions that appear in the second welfare theorem, but if you do so, you must state the additional assumptions that you are making and briefly indicate why each additional assumption is needed.

(g) Does your answer to part (f) change if we are allowed to reallocate individual initial endowment vectors in the economy (of course keeping the total endowment vector the same)? Explain your reasoning.
Question II.2

This question concerns nonconvexities in pure exchange economies. Suppose we have an economy with \( \ell \) commodities and \( n \) traders, \( i = 1, 2, \ldots, n \), each having initial endowment vector \( e_i \in R^+_{+} \) and preferences \( \precsim_i \) which are assumed throughout to be continuous complete preorders on \( R^+_{+} \).

(a) Define weak convexity of \( \precsim_i \).

(b) State and define an assumption that guarantees that a weakly convex preference \( \precsim_i \) is necessarily convex. Prove your assertion.

(c) Using this notation, define competitive equilibrium.

(d) If preferences are not assumed to even be weakly convex, what goes wrong in the standard fixed point argument that is used to prove existence of competitive equilibrium? Be very precise in stating which of the (sufficient) conditions on (which?) correspondence for the application of a (which one? state it!) fixed point theorem are violated.

(e) Suppose that \( \precsim_i \) are convex for all \( i \) but each consumer has a feasible consumption set \( X_i \subseteq R^\ell_{+} \), where \( e_i \in X_i \) but \( X_i \) is not necessarily convex, although it is closed. What goes wrong in the standard fixed point argument that is used to prove existence of competitive equilibrium? As in part (d), identify a correspondence and state which of the (sufficient) conditions for your fixed point theorem would be violated.

(f) In part (e), why did we assume at the beginning that the \( \precsim_i \) were defined on \( R^\ell_{+} \) rather than \( X_i \)?

(g) What happens if \( \precsim_i \) are not necessarily convex and consumers have feasible consumption sets \( X_i \subseteq R^\ell_{+} \) (with \( e_i \in X_i \)) where the \( X_i \) are closed but not necessarily convex?

Question II.2 continues on the next page.
Question II.2 continued:

(h) What could happen if the $X_i$ were not necessarily closed (but they are convex and preferences are all convex), but as before, $e_i \in X_i \subseteq \mathbb{R}_+^l$ for all $i$? You may assume that preferences are strictly convex if you think it helps, but this probably will not help you to answer this question.

(i) Give a few examples of economic phenomena that could be described by the situations analyzed here.
Part III

Answer one question from Part III
Question III.1

(a) Define (i) the Best Response Correspondence for a Normal Form Game, (ii) a perturbation $\eta$ of the mixed strategy set in a normal form game, as introduced in the definition of perfect equilibria.

(b) For a perturbation $\eta$, let $u_\eta$ be the utility function induced by the perturbation. Let $BR^i_{T^i}(s, u^i)$ be the best response to $s$ of player $i$ with utility $u^i$ and strategy set $T^i \subseteq S^i$. What is the connection between $BR^i_{S^i_\eta}(s, u^i)$ and $BR^i_{S^i}(s, u^i)$?

(c) Prove the existence of perfect equilibria of normal form games.
Question III.2

(a) Find all the transformations of the utility functions of a player that leave the Best Response correspondence of that player unchanged.

(b) Consider all $2 \times 2 \times 2$ normal form games, that is games with two players, each player with two actions. Illustrate the result in the first part (a) above in these games.

(c) Introduce a distance among all $2 \times 2 \times 2$ normal form games, and consider all games with the property that all other games at a distance less than some $\epsilon > 0$ have the same number and type (mixed or pure) of equilibria. You can now partition these games into subsets, each with the same number and type of equilibria. Characterize these subsets.
Part IV

Answer one question from Part IV
Question IV.1

In answering this question you may assume that action sets and public signals set are finite.

(a) Define repeated game with imperfect public monitoring and public behavioral strategies. Prove that every repeated game with imperfect public monitoring has an equilibrium in public behavioral strategies.

(b) A Markov strategy in a repeated game with imperfect public monitoring is a behavioral strategy that depends only on the value of the public signal in the past period. Prove that there is an equilibrium in Markov strategies.

(c) Give an example of a game and an equilibrium is in public behavioral strategies which is not Markov.
Question IV.2

(a) Consider a pure strategy Nash equilibrium $\hat{s}$ of an extensive form game. Prove that the strategy profile induces an equilibrium in every subgame that is reached by $\hat{s}$.

(b) Define an extension of the concept of backward induction for a game of perfect information with a countable set of nodes, not necessarily finite.

(c) Does this procedure define an equilibrium in pure strategies? Prove your claim in detail.