Preliminary Examination

Econometrics

University of Minnesota

Fall 2014

Answer all questions.
Show your work to receive credit.
Each section receives equal weight.
SECTION 1

Consider the following model:

\[ y_i = X_i \beta + \epsilon_i \]

for \( i = 1, \ldots, n \) satisfying the ideal assumptions (e.g. \( X'X \) invertible, \( E[\epsilon|X] = 0 \), \( E[\epsilon \epsilon'|X] = \sigma^2 I \)), with \( X_i \) 1 X k.

Define another 1 X k vector \( Z_i \) by \( Z_i = (\phi_1(X_i), \phi_2(X_i), \ldots, \phi_k(X_i)) \) for some functions \( \phi_1, \phi_2, \ldots, \phi_k \) each from \( R^k \) to \( R^1 \). Let \( Z \) and \( X \) be the matrices where the \( n \) vectors \( Z_i \) and \( X_i \) are stacked in the usual way. Assume \( Z'X \) is non-singular.

Define a new estimator

\[ \tilde{\beta} = (Z'X)^{-1}Z'y \]

a) Is \( \tilde{\beta} \) unbiased?
b) Compute \( V(\tilde{\beta}) \).
c) Define fitted values \( \tilde{y} = X\tilde{\beta} \) and residuals \( \tilde{\epsilon} = y - \tilde{y} \). Are \( Z \) and \( \tilde{\epsilon} \) uncorrelated? How about \( \tilde{y} \) and \( \tilde{\epsilon} \).
d) Show that the OLS estimator \( \hat{\beta} \) has a smaller variance than \( \tilde{\beta} \) (compare their variances directly).
The following table reports results from two regressions of log(Quantity) on log(Price) for 51 cable markets of approximately the same size (data is centered). The explanatory variables are some of the characteristics of the cable system in the market, including monthly expanded basic price (monthly price), the channel capacity of the system and whether pay-per-view is available (the data are from 2002). The OLS column reports the results from regressing log(Q) on log(P) and the characteristics. The second column uses the tax on the cable company as an instrument. The tax is the percent of cable company revenues that goes back to the local community, also known as the franchise fee.

a) What is in the demand error in this setup?

b) Write down the model of monopoly pricing in this setting. What is the argument that makes the tax a valid instrument?

c) Suppose the instrument is valid. Can you suggest a reason for the movement why the price coefficient changes in the way it does when we move from OLS to IV.

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS Coefficient (Std. Error)</th>
<th>Instrumental Variables Coefficient (Std. Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>-0.04 (0.01)</td>
<td>-0.21 (0.18)</td>
</tr>
<tr>
<td>channel capacity</td>
<td>0.03 (0.01)</td>
<td>0.08 (0.06)</td>
</tr>
<tr>
<td>pay-per-view avail.</td>
<td>0.85 (0.18)</td>
<td>2.5 (0.25)</td>
</tr>
<tr>
<td>Observations</td>
<td>51</td>
<td>51</td>
</tr>
</tbody>
</table>
Consider the model
\[ y_i = \beta_0 + \beta_1 x_i^2 + \epsilon_i, \]
where (conditional on \( x_i \)) \( \epsilon_i \) is independent and identically distributed with mean zero and variance \( \sigma^2 \). For \( N \) observations on \((y_i, x_i)\), write down a criterion function for obtaining parameter estimates of this model. Describe the moment conditions that characterize the parameter estimate.
SECTION 4

Suppose you have a sample of i.i.d data \( \{X_1, \ldots, X_n\} \) where each \( X_i \) follows a uniform distribution such that the density function is given by

\[
f(X_i) = \frac{1}{\theta_0}, \quad 0 \leq X_i \leq \theta_0.
\]

Do the followings

1. Obtain an ML(Maximum Likelihood) estimator of \( \theta_0 \)
   
   (a) Show that the ML-estimator in part (a) is consistent or inconsistent
   (b) Show that the ML-estimator in part (a) is unbiased or biased
   (c) Obtain \( E[X] \) as a function of \( \theta_0 \)
   (d) Derive an M(Method of Moment) estimator of \( \theta_0 \) based on part (d)
   (e) Show that the M-estimator in part (e) is consistent or inconsistent
   (f) Show that the M-estimator in part (e) is unbiased or biased
   (g) Derive the asymptotic variance of the M-estimator in part (e) and propose a consistent estimator of the asymptotic variance
1. Consider the two-dimensional process \( (x_{1,t}, x_{2,t}) \) that satisfies
\[
\begin{bmatrix}
A_{1,1}(L) & A_{1,2}(L) \\
A_{2,1}(L) & A_{2,2}(L)
\end{bmatrix}
\begin{bmatrix}
x_{1,t} \\
x_{2,t}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{bmatrix}
\]
for some two-dimensional vector \( \varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}) \) of white noise. Suppose \( \det[A(z)] = 0 \) implies \( |z| > 1 \). The order of the lag polynomial \( A_{n,k}(L) \) is \( s_{n,k} \in \mathbb{N} \). Show that \( x_{1,t} \) is an ARMA\((p,q)\) process. Determine \( p \) and \( q \) and show that the autoregressive lag polynomial for \( x_{1,t} \) has an inverse with non-negative powers of \( L \). **Hint:** use a \( 2 \times 2 \) matrix of the type \( \det(B)B^{-1} \).

2. Suppose \( \{Y_t\}_{t=0}^\infty \) is defined by \( Y_t = X_t + \varepsilon_t \), where \( X_0 = 0 \) and \( X_t = X_{t-1} + Z_t \) for all \( t \in \mathbb{N} \). The processes \( \{\varepsilon_t\}_{t=0}^\infty \) is i.i.d. with mean zero and positive variance \( \sigma^2 \). The process \( \{Z_t\}_{t=-\infty}^\infty \) is a Markov chain that is independent of \( \{\varepsilon_t\}_{t=0}^\infty \). It can take on values \( \zeta(1) = x \) and \( \zeta(2) = y \neq x \). The transition probabilities \( \Pr[Z_{t+1} = \zeta(j) | Z_t = \zeta(i)] = P_{i,j} \) are given by the matrix
\[
P = \begin{bmatrix}
1 - a & a \\
b & 1 - b
\end{bmatrix},
\]
where \( a \in (0, 1) \) and \( b \in (0, 1) \). Let \( \mathcal{H}_t \) be the collection of random variables \( \{Y_s, Z_s\}_{s=0}^t \). Answer the following questions for any \( t \in \mathbb{N} \).

a. Show that \( \mathbb{E}[Z_{t+1} - \mu | Z_t] = \rho(Z_t - \mu) \) and give expressions for \( \mu \) and \( \rho \).

b. Determine \( \mathbb{E}[Y_{t+N} - Y_t - \mu N | \mathcal{H}_t] \) as well as \( g_t \), its mean-square limit as \( N \to \infty \).

c. Define \( M_t \) to be the mean-square limit \( M_t = \lim_{T \to \infty} \mathbb{E}[Y_T - Y_0 - \mu T | \mathcal{H}_t] \). Determine \( M_t \) and verify that \( \mathbb{E}[M_{t+1} | \mathcal{H}_t] = M_t \).