Preliminary Examination

Growth and Development

Fall 2014

To the best of your ability, answer all questions.
Question 1

Consider an economy with a representative consumer whose preferences over flows of agricultural products $A_t$, manufactured products $M_t$, and services $S_t$ are given by

$$\int_0^\infty e^{-\rho t} \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \right) dt,$$

where $\rho$ and $\sigma$ are positive parameters and composite consumption $C_t$ is given by

$$C_t = (A_t - A)^{\beta_A} M^{\beta_M} (S_t + S)^{\beta_S}.$$

The parameters $A$, $S$ and $(\beta_A, \beta_M, \beta_S)$ are positive, and $\beta_A + \beta_M + \beta_S = 1$. Intermediate output $Y_t$ is produced using a constant returns to scale technology with capital and labor as inputs,

$$Y_t \leq F(K_t, z_t N),$$

where $N > 0$ and $z_t = ze^{\delta t} > 0$ grows at a positive rate. Capital can be accumulated according to

$$DK_t \leq -\delta K_t + X_t,$$

where $\delta$ is a positive. The consumption and investment goods can be produced from intermediate output, subject to the time-t resource constraint

$$A_t + M_t + S_t + X_t \leq E + Y_t,$$

where $E$ is a flow of endowments of the intermediate good. The initial capital stock is positive.

a. Determine the cost-minimizing choices of $A_t$, $M_t$ and $S_t$ for a given level of composite consumption $C_t$. Verify that the cost function in units of intermediate good is $A - S + PC_t$, for some positive constant $P$.

b. Suppose it so happens that $E = A - S$. Give the equations that determine the balanced growth path, suitably defined.

c. What happens to consumer expenditure shares over time, assuming that $E = A - S$ is positive? If intermediate output is produced in the sector in which it is used, what can you say about sectoral labor shares when $E = A - S = 0$?
Question 2

Consider an economy with a representative consumer endowed with $H$ units of labor whose preferences over consumption flows $C_t$ are determined by

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\int_0^\infty e^{-\rho t} \ln(C_t) dt,
$$

where $\rho$ is positive and

$$
\ln(C_t) = \int_0^1 \ln(C_{j,t}) dj.
$$

Given the right blueprint, good $j$ can be produced using a linear labor-only technology with productivity $z_{j,t}$. The most productive blueprint is in the possession of the agent who created it, and becomes available to everyone when someone creates a more productive blueprint for good $j$. There are two technologies for creating a blueprint for good $j$ with productivity $\lambda z_{j,t}$, where $\lambda > 1$. On the one hand, anyone can hire $m_t \geq 0$ units of labor to create such an improvement at the Poisson rate $\gamma m_t$. On the other hand, the owner of the most advanced blueprint for any other good $j' \neq j$ can hire $x_t \geq 0$ units of labor to create such an improvement for good $j$ at the Poisson rate $f(x_t)$. The parameter $\gamma$ is positive, and the function $f$ is strictly increasing and strictly concave, and $f(0) = 0$.

Let $r_t$ be the interest rate and $w_t$ the wage. In the following, take for granted that the equilibrium will be symmetric and write $v_t$ for the profits from producing good $j$ and $q_t$ for the price of the most advanced blueprint for good $j$. The average rate at which the most productive blueprint for a particular good can be surpassed is denoted by $d_t$.

**a.** Explain why profits are given by $v_t = (1 - 1/\lambda)C_t$ and why $C_t = \lambda w_t L_t$ when $L_t$ is the aggregate amount of labor used to produce the goods $j \in [0, 1]$. At any point in time, what determines $w_t$?

**b.** What is the Bellman equation for $q_t$? Explain. Use the assumption that utility is logarithmic to derive a Bellman equation for $q_t/C_t$, and verify that it does not depend on the interest rate $r_t$.

**c.** State the first-order conditions for the amounts of labor that new entrants and the owners of frontier blueprints use to try to create better blueprints.
d. Conjecture that there a balanced growth path along which $M > 0$ units of labor are used by new entrants trying to create new blueprints, and $X > 0$ units of labor by the owners of frontier blueprints. Determine the equilibrium values for $M$, $X$, and $q_t/C_t$. What is the resulting growth rate of aggregate consumption?

e. Suppose blueprints created from existing blueprints (rather than from scratch by new entrants) stay within the same firm. Explain intuitively why this model cannot generate an empirically plausible distribution of employment across firms.
Consider an economy with a representative consumer endowed with one unit of labor whose preferences over consumption flows $C_t$ are determined by

$$\int_0^\infty e^{-\rho t} u(C_t) dt,$$

where $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$. There is a positive initial capital stock $K_0$. Capital can be used either to produce consumption goods or to produce more capital. If $X_t \in [0, K_t]$ units of capital are used together with one unit of labor to produce a flow $Y_t$ of consumption goods,

$$Y_t = F(X_t, 1),$$

then the capital stock grows according to

$$DK_t = A \cdot (K_t - X_t).$$

The production function $f(x) = F(x, 1)$ is given by

$$f(x) = \begin{cases} \alpha \xi^{1-\alpha} + (1 - \alpha)\xi^{-\alpha}x, & x \in [0, \xi] \\ x^{1-\alpha}, & x \in [\xi, \infty) \end{cases}$$

All parameters are positive, $\alpha \in (0, 1)$, and $A > \rho > (1 - \alpha)(1 - \sigma)A$. Note that $f(0)$ is positive and that $f$ is continuously differentiable on $[0, \infty)$.

Let $q_t$ be the price of capital, and write $v_t$ for the rental price of capital, both in units of consumption. The interest rate is $r_t$, also in units of consumption.

**a.** A production function of the form $f$ is implied by cost minimization given (i) a Cobb-Douglas technology that uses capital and labor and (ii) a linear technology that uses only labor. Show this using a diagram that displays the isoquants for these two technologies. No need for algebra.

**b.** Explain why $v_t \geq Df(X_t)$ and $v_t \geq Aq_t$. When do these inequalities have to hold with equality? Explain why $r_tq_t = v_t + Dq_t$. 

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c. Conjecture that the equilibrium is described by a constant ratio $X_t/K_t \in (0,1)$. Show that in such an equilibrium $DC_t/C_t = g$ and $r_t = r$, where

$$g = \frac{(1 - \alpha)(A - \rho)}{\alpha + (1 - \alpha)\sigma}, \quad r = \frac{\alpha \rho + (1 - \alpha)\sigma A}{\alpha + (1 - \alpha)\sigma}. \quad (1)$$

Verify that $X_t/K_t = (r - g)/A$ and note that $X_t \geq \xi$ corresponds to $K_t \geq K_* = A\xi/(r - g)$.

d. Describe the competitive equilibrium starting from an initial capital stock $K_0 \in (0, K_*)$. Provide details as time permits.