Ph.D. Preliminary Examination

Macroeconomic Theory

Fall 2014

Answer ALL FOUR parts

Please read any instructions before each part and make your answers neat and concise. Read sections carefully and answer all questions. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly.

You have 5 hours to complete the exam.
Infinitely lived consumers and dynamic programming

Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer’s utility function is

$$
\sum_{t=0}^{\infty} \beta^t \left( \gamma \log c_t + (1-\gamma) \log x_t \right)
$$

Here $0 < \beta < 1$ and $0 < \gamma < 1$. The consumer is endowed with 1 unit of labor in each period, some of which can be consumed as leisure, $x_t$, and some of which is supplied as labor, $\ell_t$. The consumer is also endowed with $\bar{k}_0$ units of capital in period 0. Feasible allocations satisfy

$$
c_t + k_{t+1} \leq A k_t^a \ell_t^{1-a}
$$

Here $A > 0$ and $0 < \alpha < 1$.

a) Formulate the problem of maximizing the representative consumer’s utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman’s equation.

b) Suppose that you guess that, in the solution to the dynamic programming problem, labor, $\ell$, and leisure $x$, are constants. Suppose further that you guess that the value function has the form $a_0 + a_1 \log k$. Solve for the policy functions $c(k)$, $\ell(k)$, $x(k)$, $k'(k)$.

c) Define an Arrow-Debreu equilibrium for this economy. Use the solution to part a to calculate the Arrow-Debreu equilibrium.

d) Define a sequential markets equilibrium for this economy. Use the solution to part a to calculate the sequential markets equilibrium.

e) Suppose now that there are equal amounts of two types of consumers in the economy. The two types of consumers have the same discount factor $\beta$. They have different utility functions in each period, $\gamma^1 \log c + (1-\gamma^1) \log x$ and $\gamma^2 \log c + (1-\gamma^2) \log x$, $\gamma^1 \neq \gamma^2$; different endowments of labor in each period, $\bar{\ell}^1$ and $\bar{\ell}^2$, and different initial endowments of capital, $\bar{k}^1_0$ and $\bar{k}^2_0$. Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

f) Does the equilibrium allocation for the economy in part e solve a dynamic programming problem like that in part a? Carefully explain why or why not. If it does solve such a problem, write down the appropriate Bellman’s equation.
Question 2

Consider an infinite horizon growth model with a representative consumer, with log utility and inelastic labor supply:

$$\sum_{t=0}^{\infty} \beta^t \log(c_t).$$

Assume that there is a representative firm with a Cobb-Douglas production function \( y = Ak^\alpha n^{1-\alpha} \) and full depreciation \( \delta = 1 \).

a) Find the policy functions for the associated planning problem and use these to give an expression for \( y_t \), output in period \( t \). Show your work.

Now assume that there is also a government that taxes all income at a uniform rate, \( \tau \), in every period, and uses the revenue generated to purchase output, \( g_t \), in each period. Assume that the government balances the budget in every period.

Note that \( \tau \) corresponds to the economy in a) with no government.

b) Set up and define a TDCE (Tax Distorted Competitive Equilibrium) for this economy.

c) Show that the TDCE allocation solves a planner’s problem and give this problem.

d) Find expressions for \( y_t \) for this economy.

e) Which is larger, output when \( \tau = 0 \), or output when \( \tau > 0 \)?
A Cash-Credit Goods Model with Private Money

Consider a cash credit goods in which households have preferences of the form \( \sum_{t=0}^{\infty} \beta^t (\log c_{1t} + \phi \log c_{2t}) \), where \( c_{1t} \) and \( c_{2t} \) denote consumption of cash and credit goods respectively, \( \phi \) is a parameter, and \( 0 < \beta < 1 \) is the discount factor. Households supply labor inelastically. The resource constraint is given by

\[ c_{1t} + c_{2t} + k_{t+1} = k_t^S + (1 - \delta)k_t \]

where \( k_t \) denotes the capital stock, the capital share parameter \( \alpha \) and the depreciation rate \( \delta \) are both positive and less than 1.

It is convenient to let \( q_t \) denote the price of money (or the inverse of the price level). The representative household’s budget constraint is given by

\[ q_t M_{t+1} + c_{1t} + c_{2t} + k_{t+1} \leq q_t M_t + R_{kt} k_t + w_t l_t + q_t T_t \]

where \( M_t \) denotes cash balances, \( R_{kt} \) denotes the return to capital and \( T_t \) denotes lump sum transfers of cash by the government. The household can use fraction \( \theta \) of its capital stock to purchase cash goods. The cash in advance constraint is

\[ c_{1t} \leq q_t M_t + \theta k_t. \]

(a) Define a competitive equilibrium.

(b) Assume that the aggregate stock of money grows at a constant rate, \( \pi \). Characterize the steady state.

(c) Show that if \( \pi \) is above a critical threshold, the economy has a nonmonetary steady state in which the price of money is zero.

(d) Show that below the critical threshold, comparing steady states, per capita output is higher if the growth rate of money is higher.

(e) Do your results in part (d) imply that the Friedman Rule is not optimal in this economy?
Stuff related to the Fourth Mini

In the following there are 9 questions for 100 points. Be as BRIEF as you can and good luck.

Growth Models

Consider an economy with two equal sized regions indexed by $i \in \{M, B\}$ (Madrid and Barcelona). Each region is populated by a continuum of infinitely lived identical agents. Agents cannot change regions.

In Region $M$ there is a technology given by $f^M(z, k^M, n^M)$, where shocks follow a Markov chain with transition $\Gamma^M_{z, z'}$. This transition is such that there is a strong positive autocorrelation. All states of nature occur eventually and the stationary distribution is symmetric. Region $B$ has a technology given by $f(k^B, n^B)$ (no shocks). The wealth owned by people of region $M$, ($M$-people) is $\bar{A}^M$ while that of $B$-people is $\bar{A}^B$. The only form of wealth is real capital.

Preferences are given by the expected discounted value (both regions agents have the same discount factor, $\beta$) of region specific strictly concave current utility functions, $u^M(c, \ell)$ and $u^B(c)$ (while region agents in region $M$ like leisure, those in region $B$ do not).

Assume for the first two questions that output cannot be transferred across regions. Try to be as simple as possible in your answers by exploiting the specific properties that are described.

1. (10 points) Set the value of the shock to its unconditional mean $\bar{z}^M$, and then define what the steady state of this economy is, and state the conditions that have to be satisfied. Is any restriction on the initial wealth distribution meaningful?

2. (10 points) Define Recursive Competitive Equilibrium. Make sure that you not only define the required objects but you also state the conditions that such objects must satisfy. Would this equilibrium allocation be the same than that that would entail if we posed a separate equilibrium for each region?

Now assume that output can be moved between regions, but that it takes one period to ship output to the other region, i.e. output produced in one region in period $t$ can become either consumption or investment in the other region only in $t+1$.

3. (10 points) Define Recursive Competitive Equilibrium. Make sure to point to what is different now compared to the previous question. Also, make sure that the market structure that you are imposing implies that equilibria are Pareto Optimal. (You do not have to prove it).

4. (10 points) Write the first order conditions faced by agents of each country and argue what is the nature of the market arrangements that occur (when regions have a deficit and why).

Now, assume that the production function of the $B$-region is given by $f^B(k^B, n^B, N^M)$, with $f^B_{N^M}(k^B, n^B, N^M) < 0$. This is a negative externality: productivity of $B$-people goes up when $B$-people work harder.
5. (10 points) Discuss whether a tax on labor income together with a lump sum subsidy in region $M$ would improve things in a Pareto sense. What about if we add a lump sum tax in region $B$ with the proceeds going to agents in region $M$?

Search meets Aiyagari

Imagine a household that cannot borrow or issue state contingent assets but can save at rate $r$. It can either work or not work. Work yields disutility $\hat{u} < 0$. The household cares about consumption according to per period utility $u(c)$, is infinitely lived, and discounts the future at rate $\beta$.

Jobs are of finitely many types $s \in S$, and if they do not get destroyed, they evolve with transition $\Gamma_{s,s'}$ that satisfies stochastic dominance ($s_A > s_B$ implies $E[s'|s_A] > E[s'|s_B]$), and it also satisfies $E[s'|s] > s$. The probability of a job being exogenously destroyed is job specific and is given by $\Lambda^s$. Jobs with higher efficiency units also have a lower probability of getting destroyed. In addition to exogenous destruction, the household can quit the job if it chooses to do so and then the household will not work for a period.

A household that does not work, it can by incurring in $\bar{u} < 0$ utils, search for a job. Searching is nicer than working, i.e., $\hat{u} < \bar{u}$. If the household searches, it gets the second worst possible job with probability $\gamma$ and finds no job with probability $1 - \gamma$. If the household does not search it will not find a job for sure.

6. (15 points) Write down the problem of the households including state variables.

7. (15 points) Is it possible that the household quits under some circumstances all the possible types of jobs? How do those circumstances differ between two different jobs?

8. (5 points) Is it possible that it is more likely that the household quits the best than the worst job? Elaborate a bit.

9. (15 points) Construct the updating operator of the distribution of types.