University of Minnesota
Department of Economics
Ph.D. Preliminary Examination

Public Economics
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Instructions:

There are two questions. You must answer both of them.

Make your answers as neat and concise as possible. If you need to make additional assumptions when answering the questions then make sure you note them.
1 Laibson and Social Security

There are three periods and one agent with time inconsistent preferences. Technically, this is modelled as a game with three different agents, each operating in one period. This is supposed to capture the idea that you’d like to commit today to eat less in the future (or save more, or drink less, or exercise more) and continue to pig out today, but you know that you can’t commit to actions by future versions of yourself.

There is an endowment of \(w\) units of the one single consumption good in period 1, but none in period 2 or period 3. Denote consumption in period \(t\) by \(c_t\).

Utility of the period 1 agent is given by: \(u(c_1) + \delta \left[ \beta u(c_2) + \beta^2 u(c_3) \right]\)

Utility of the period 2 agent is given by: \(u(c_2) + \delta \left[ \beta u(c_3) \right]\)

Utility of the period 3 agent is given by: \(u(c_3)\)

Assume that there is a savings technology available so that storing one unit of the good until the next period gives \(1 + R\) units of the good the next period.

To make matters simple, assume that \(R = 0\), and that \(u(c) = \log(c)\).

a) What would consumption and savings be if there was commitment? I.e., if the time 1 agent could choose everything.

b) What is the equilibrium (subgame perfect) level of consumption and savings in each period without commitment? I.e., when each agent chooses his own consumption and savings for the future period(s) from the remaining amount of the endowment.

c) Compare the solutions in a) and b). Under what conditions (on \(\delta\)) are the solutions the same?

d) Show that the commitment solution can be implemented with a system in which income is taxed and rebated in the future. What periods is the tax needed and when should it be rebated?
Enforcement Constraints, Externalities and Capital Accumulation

Consider an economy with a measure 1 of agents. Agents can be of two types, $A$ or $B$, half of the population is of each type. The time horizon is infinite and there is a single consumption good per period. Each type of agent can operate a production technology which uses capital and is given by

$$y_{it} = A_i^t k_{it}^\alpha l_{it}^{1-\alpha}$$

where $y_{it}$ denotes output using $i$'s production technology, $k_{it}$ and $l_{it}$ denote the amount of capital and labor allocated to agent $i$, $i = A, B$ and the productivity $A_i^t$ follows a cyclic pattern,

$$A_i^t = \begin{cases} \bar{A} & t \text{ even} \\ \underline{A} & t \text{ odd} \end{cases}$$

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where $\bar{A} > \underline{A}$. Each type of agent has preferences given by $\Sigma \beta^t U(c_t)$. Each agent supplies one unit of labor inelastically. Capital depreciates at rate $\delta$.

1. What is an allocation? What is the appropriate notion of resource-feasibility?

2. Define a competitive equilibrium and prove the first and second welfare theorems for this environment.

Now suppose that at any date $t$ agents can take the capital allocated to them and operate the capital in autarky and hire labor at a competitive wage. That is, they can be excluded from markets for intertemporal trade but can use the capital allocated to them in period $t$ and consume and save that capital as they see fit using their technology.

3. What participation constraints are appropriate in this environment?

4. Define a competitive equilibrium. Does the first welfare theorem hold in this environment?

5. If so, provide a proof. If not, sketch out the reasons the first welfare theorem may not hold. What externalities are there in this environment?