Labor Prelim

In the following there are various blocks of questions. You have to answer only one of those blocks

Be BRIEF as you can and good luck. You have 4 hours.

Block C

This block has 4 questions for 100 points.

Target answering 70 points out of the following (a form of choice)

1. (25 points) Briefly discuss 3 theories of the shape of wages over the life cycle and put forth very simply 3 models that implement these ideas without uncertainty.

2. (20 points) Pose two theories of inequality of earnings at age 40 and discuss how to use theory and data to sort the contribution of the mechanism described by each theory.

3. (35 points) Imagine couples that live two periods. Their consumption is all joint public but there are no gains from living together. This is that if a couple spends $2 is as if 2 people living separately spent $1 each. In the second period some men die (with probability \( \gamma \)) and they make zero income. There are two groups of couples, the first one both members have income of $2 each. In the second group of couples, the men make $3 and the women $1. The interest rate is zero. We see that the first group of couples saves $2 and that the second group saves $1.5. If the decision of how much to save were given to a counseling advisor and if we knew that there are no insurance markets, the per period utility function is log, and the discount rate \( \beta \) is 1,
   A) Could you back up the Pareto weighs that the counselor uses to arrive to its recommendation?
   B) Alternatively, if you knew that these people could live alone but choose not to, could you say something about what is the bargaining power of males and females?
   C) Could you imagine saying something about which one of the two is a better theory based on this evidence? Explain.

4. (20 points) Imagine we measure the stochastic process of earnings for people and is iid around a deterministic life cycle profile with constant variance \( \sigma^2 \) at each age. Imagine that \( x \) is the mean of wealth at 40 and \( y \) is the variance of wealth at age 40. Imagine that preferences are CRRA, with risk aversion of 2. Describe somerely how would you use the computer to answer
   A) What discount rate accounts for mean wealth held?
   B) How much of the wealth variance at 40 could be accounted for by differences in the luck of earnings draws.
1. (30 points) The evolution of female labor force participation over the last century in US.

- Describe the evolution of aggregate female labor force participation (LFP) over the period.
- Using cohort analysis, describe the evolution of female LFP by age, marital status and presence of children. What different information does cohort analysis convey?
- How has LFP changed over the life cycle for married women of different cohorts? Has the evolution been geographically homogeneous?

2. (20 points) What are the main theories about the evolution of female LFP that you know? Briefly describe their main ingredients.

3. (30 points) Consider the following static problem of a married couple:

\[
\max \lambda_f \left[ \mu \log(c^1_f) + \nu \log(c^2_f) + (1 - \mu - \nu) \log(l_f) \right] \\
+ \lambda_m \left[ \mu \log(c^1_m) + \nu \log(c^2_m) + (1 - \mu - \nu) \log(l_m) \right] \\
\text{subject to} \quad c^1_f + c^1_m + qk \leq (1 - \tau_f)wl^1_f + (1 - \tau_m)wl^1_m \\
\quad c^2_f + c^2_m = Ak^\theta (l^2_f)^{1-\theta} \\
\quad l_f + l^1_f + l^2_f = 1 \\
\quad l_m + l^1_m + l^2_m = 1 \\
\quad l^1_m \geq 0, \quad l^2_m \geq 0, \quad l^1_f \geq 0, \quad l^2_f \geq 0
\]

where \(c^1_f\) and \(c^1_m\) are the consumption of the market good by the women and the man, \(c^2_f\) and \(c^2_m\) are the consumption levels of home good, \(l^1_f\) and \(l^1_m\) are the hours they work in the market and \(l^2_f\) and \(l^2_m\) are the hours they work in the home.

- Derive under which conditions on the parameters the solution implies \(l^2_m = 0\). Given these conditions, show that hours worked by women in both market and home are independent of \(A\) (technological progress in the household).
- How would you answer change if the utility functions were not logarithmic?
- What assumptions in Greenwood, Seshadri and Yorukoglu guarantee that female work hours increase in response to an improvement in home technology?

4. (20 points) If you were to write down a theory on the evolution of female LFP, which would be your main ingredients and why? Any other fact you would like to look up?
1. [20 points] In the United States, the college premium—the average wage of college graduates relative to the high-school graduates—fell persistently during the 1970s and then increased significantly in the 1980s and 1990s.

(a) Suppose that, in addition, you are told that the relative supply of college graduates (i.e., the number of workers with a college degree relative to the number of workers with a high school degree) has increased very fast in late 1960s and 1970s, but this trend has slowed down after that. Assume that the latter trend is exogenous and write a simple model that generates the non-monotonic behavior of the college premium observed in the data. Be as specific as possible: State your assumptions—the necessary functional forms, etc., and make any necessary derivations to support your arguments. [12 points]

(b) Based on the empirical evidence you know, how can you justify taking the relative supply of college workers as exogenous? What would be the problem in your model if the relative supply cannot be taken exogenous? [8 points]

2. [35 points] This question considers the life cycle permanent income model and asks you to think about how you can identify some properties of a stochastic income process by observing the joint dynamics of consumption and income. To this end, consider individuals whose income can be written as:

\[
\begin{align*}
y_t &= y_t^P + \eta_t \\
y_t^P &= y_{t-1}^P + \epsilon_t
\end{align*}
\]

where \((\eta_t, \epsilon_t)\) are i.i.d over time and across individuals. Preferences are quadratic and are defined over consumption only. Individuals can borrow and save at a constant interest rate \(r\) and also discount future utility at the same rate \(r\). Therefore, the maximization problem of
the individual can be written as:

$$\max E_t \left[ -\frac{1}{2} \sum_{\tau=0}^{T-t} (1 + r)^{-\tau} (c^* - c_{t+\tau})^2 \right]$$

s.t

$$\sum_{\tau=0}^{T-t} (1 + r)^{-\tau} (y_{t+\tau} - c_{t+\tau}) + A_t = 0$$

where $A_t$ is financial wealth at the beginning of period $t$.

(a) [12 points] Write down an expression that directly links the transitory and permanent income shocks $(\eta_t, \epsilon_t)$ to consumption change, $\Delta c_t$. According to this equation, how does consumption respond to permanent income shocks? How does it respond to transitory innovations? [Hint: This question will require you to fully solve the maximization problem above. If you get stuck, you can simplify the problem by setting $r = 0$ and $T = 2$.]

(b) [8 points] Suppose that you are interested in measuring the size (ie., variance) of permanent and transitory income shocks. You have panel data on consumption and income collected from $N$ individuals for $T^*$ years. Write down a set of data moments (covariances)—that use both consumption and income data—that will allow you to recover the two variances you are interested in. What is the minimum time span of the panel that will allow you to identify these two parameters?

(c) [7 points] You realize that consumption in your data set is measured with error. That is, your consumption data is $c^{**}_t = c_t + \nu_t$, where the measurement error $\nu$ is i.i.d over time and across individuals and also independent of true income innovations. Which variation in the data you can look at to measure the variance of measurement error (read: which moment you should compute that will allow you to calculate the variance of $\nu$)?

(d) [8 points] Finally, you are told that in reality when individuals experience a permanent change in their income (ie, an $\eta$ shock) their consumption goes up by less than one for one. Is your model consistent with this fact? If not, how can you modify your model so that the optimal consumption choice of the agents in the model is consistent with this empirical observation?

3. [20 points] Suppose that you are shown two empirical trends from the US data. The first trend is that male wage inequality (measured, for example, as the cross-sectional standard deviation of log hourly wages) has gone up substantially from 1980 to 2000. The second trend: household-level consumption inequality (measured as cross-sectional standard deviation of household non-durable consumption expenditures over time), has increased by much less during the same period. Describe three different hypotheses that could reconcile the
different behaviors of these two trends. Be as specific as you can. You do not need to write explicit equations for each hypothesis; however, you need to clearly explain why your proposed hypotheses can generate the two trends described above.

4. [25 points] Consider the following version of the Ben-Porath model of human capital accumulation. An individual is born with human capital level $h_0$, and can accumulate more human capital according to:

$$h_{t+1} = h_t + A(h_t i_t)$$

where $A$ and $\alpha$ are known parameters, and $i_t \in [0, 1]$ is the fraction of time the individual spends accumulating human capital. Individuals spend the remaining $(1 - i_t)$ fraction of their time working in the market. Each unit of human capital, $h_t$, earns a wage rate of $W$ in the labor market. So the wage income of an individual with human capital $h_t$ who invests $i_t$ is equal to $Wh_t (1 - i_t)$ in that period. Finally, individuals have a stochastic life-time: Every period an individual faces a constant probability, $1 - \delta$, of death, in which case his utility from that period on is equal to zero. Finally, assume that the individual consumes his/her income every period, discount future at rate $\beta$ and his utility is linear in consumption: $u(c_t) = \beta^t c_t$

[Hint: the fact that lifetime is stochastic will not require you to use anything we haven’t learned in class. In fact, it will make the solution of the problem simpler in certain ways compared to the case when individuals face a finite fixed horizon]

(a) Write down the individual’s optimization problem as a dynamic programming problem. What is (are) the state variable(s)? [6 points]

(b) Do you need to make any assumptions not given above to make this problem well-defined? [5 points]

(c) Derive the first order conditions for optimal investment choice. Obtain a closed-form solution for investment. What can you say about the optimal investment path (is it increasing, decreasing, constant, etc)? How about the net change in human capital per period $(h_{t+1} - h_t)$? [10 points]

(d) Suppose that the individual solves the problem above assuming that the wage rate will be $W_1$ for ever. At some time period $t^*$ (before the individual makes his investment decision for period $t^*$) it is announced that the wage rate will jump to $W_2 > W_1$ and stay there for all $t \geq t^* + 1$. How does the path of investment and $h_{t+1} - h_t$ change starting at $t^*$? [4 points]