Preliminary Examination

Growth and Development

Spring 2009

Four questions of equal weight. Try to answer as many as you can. Some questions may take more time than others.
Question 1

Consider the impact of a cartel on the location of industry. Let there be three periods, \( t = 1, 2, 3 \), corresponding to before, during, and after the cartel.

Suppose the industry is sugar. There is an economy-wide market for sugar and its price is denoted by \( p_t \). Sugar can be produced with land and the time of an entrepreneur, in fixed proportions. One entrepreneur in region \( j \) can produce at most \( y_j \) units of sugar.

In each region, there is a potential supply of entrepreneurs to the industry. Potential entrepreneurs are indexed by real numbers. In region \( j \) and period \( t \), entrepreneur \( n \) can earn \( w_{j,t}(n) \) in his or her next best occupation. The function \( w_{j,t}(\cdot) \) is strictly increasing. The rental price of land in region \( j \) and period \( t \) is \( r_{j,t} \).

a. Solve for the “number” of entrepreneurs that enter a given region \( j \) in period \( t = 1 \).

In period 2, the government and the industry form a cartel. It has two parts. First, the price of sugar is increased (say through an increase in a tariff), so that \( p_2 > p_1 \). Second, the entry of new entrepreneurs is forbidden. Between periods 1 and 2, the rental price of land also increases, to \( r_{j,2} > r_{j,1} \).

b. What happens to the number of acres of land devoted to sugar from period 1 to period 2 in region \( j \)? Explain.

In period 3, the cartel is ruled illegal. The price of sugar falls to \( p_3 = p_1 \), and there are no longer entry restrictions. Nothing else changes from period 2 to 3.

c. Characterize how the acres of land devoted to sugar change from period 2 to period 3 in region \( j \).

d. Use this simple model to discuss how the location of the sugar industry changed in the United States after the New Deal sugar cartel was abolished. What, in fact, happened to industry location? How can the model explain that?
A major innovation in the late 19th century was barbed wire. Previous to barbed wire, fencing in the United States was primarily wood fencing. In areas where wood was "scarce," fencing was expensive and little done. Fencing was valuable for many reasons. For example, it increased the yield of a unit of land since fewer crops were destroyed (from roving animals) with a fence. Here we explore estimating the "surplus" (to be defined shortly) generated by the innovation.

Consider a county in the United States whose only product is wheat. Let each farmer in the county have one unit of land and be indexed by \( i \in [0, 1] \). Farmer \( i \) produces \( Y_{N,i} \) units of wheat with no fence and \( Y_{F,i} > Y_{N,i} \) with a fence of any type. Before the innovation of barbed wire fence, wheat prices were \( p_W \) and the cost of building a wood fence was \( c_W \), in units of some outside good. Both were stable over time and expected to remain so. After the unanticipated innovation of barbed wire fence, it became possible to build a fence at a cost \( c_B < c_W \), and the innovation caused the price of wheat to fall, permanently, to \( p_B < p_W \). Fences last forever.

Farmers maximize expected profits in units of the outside good, discounted at a continuously compounded rate \( \rho \). Define the surplus \( s_i \) generated by the innovation of barbed wire fence to be the increase in expected discounted profits of farmer \( i \) brought about by the innovation.

Throughout the following, assume you have data on \( \rho \), \( (p_W, c_W) \), and \( (p_B, c_B) \).

**a.** Under what conditions would farmers in the county choose to build a fence or not, before and after the innovation?

Suppose you know farmer \( i \) built a barbed wire fence.

**b.** Construct an interval for \( \Delta_i = Y_{F,i} - Y_{N,i} \) that is as small as possible given the data.

**c.** What is the least upper bound you can find for the surplus \( s_i \) of this farmer?

Suppose nobody in the county built a wood fence and a fraction \( \phi \in (0, 1) \) built a barbed wire fence. Assume you have data on \( \phi \).

**d.** What is the least upper bound you can find for the aggregate surplus generated by the innovation in this county?
Question 3

Consider the following model of structural change. There is a representative consumer whose preferences over output from $I+1$ different sectors are given by

$$
\int_0^\infty e^{-\rho t} \ln(C_t) dt, \quad C_t = \left[ \sum_{i=0}^{I} C_{0,t}^{1-1/e} \right]^{1/(1-1/e)},
$$

where $\rho > 0$, $e > 0$ and $I \in \mathbb{N}$. The aggregate labor supply $L$ is exogenous and there is a positive initial capital stock $K_0$. The technology is given by

$$
C_{0,t} = z_{0,t} F(K_{0,t}, L_{0,t}) - (DK_t + \delta K_t)
$$

and

$$
C_{i,t} = z_{i,t} F(K_{i,t}, L_{i,t}), \quad i \in \{1, \ldots, I\},
$$

where

$$
\sum_{i=0}^{I} L_{i,t} \leq L, \quad \sum_{i=0}^{I} K_{i,t} \leq K_t,
$$

and all capital and labor inputs must be non-negative. The production function $F$ exhibits constant returns to scale.

a. Determine the relative prices $p_{i,t}/p_{j,t}$ and $s_t = p_{0,t}/P_t$, where $P_t = \left[ \sum_{i=0}^{I} p_{i,t}^{1-1/e} \right]^{1/(1-1/e)}$ is the price index for the composite good $C_t$.

b. Let $X_t$ be total consumption expenditures on goods 0 through $I$, measured in units of good 0. Show that $X_t = z_{0,t} F(K_t, L) - (DK_t + \delta K_t)$ and $C_t = X_t/s_t$.

Now suppose the production function is $F(k, l) = k^\alpha l^{1-\alpha}$ for some $\alpha \in (0, 1)$. Suppose further that $z_{i,t} = z_i e^{\theta_i t}$, for some non-negative $\theta_i$.

c. Determine the growth rate of total consumption expenditures, measured in units of good 0, along the aggregate balanced growth path. Does consumption of the composite good also grow at a constant rate?

d. Determine $L_{i,t}/L_{j,t}$ for all $i$ and $j$ in $\{1, \ldots, I\}$. Baumol argues that labor moves away from sectors with rapid technological progress and towards sectors with low rates of technological progress. Is he right?
Question 4

There is a unit continuum of infinitely lived agents whose preferences are

$$\int_0^\infty e^{-\rho t} \ln(C_t) dt,$$

where $\rho > 0$. Everyone is endowed with a flow of one unit of effort per unit of time that can be used to supply labor, manage a firm, or be an entrepreneur. Labor earns a wage of $w_t$ units of consumption. A unit flow of entrepreneurial effort generates a new firm at a Poisson rate $\Gamma > 0$. Once the firm is created, the entrepreneur automatically becomes its owner-manager. The firm ceases to exist if this manager quits, or if the manager loses his or her ability to manage the firm. The latter happens at a Poisson rate $\delta > 0$. Markets are complete and the risk-free interest rate is $r_t$.

A firm with productivity $z_t$ can produce $F(z_t, m_t, l_t)$ units of output with $m_t \in [0, 1]$ units of managerial effort and $l_t$ units of labor. The remaining managerial effort improves firm productivity according to $Dz_t = Bz_t(1 - m_t)$, where $B > \rho + \delta$. The manager cannot hire someone else to perform these tasks. For simplicity, assume $F(z, m, l) = (zm)^{\alpha l^{1-\alpha}}$ for some $\alpha \in (0, 1)$.

a. Show that a manager who sets $z_t m_t = x$ will hire $((1 - \alpha)/w_t)^{1/\alpha} x$ units of labor and generate a flow $A w_t^{1-1/\alpha} x$ of output minus labor costs, where $A = [\alpha^\alpha (1 - \alpha)^{1-\alpha}]^{1/\alpha}$.

Suppose there is a balanced growth path with $[C_t, w_t] = [C, w] e^{\kappa t}$ for some $\kappa > 0$. Conjecture further that owner-managers never want to quit.

b. Show that the value $q_t[z]$ of a firm with productivity $z$ at time $t$ is

$$\frac{q_t[z]}{w_t^{1-1/\alpha}} = \max_{m_{t+a} \in [0, 1]} \int_0^\infty e^{-\beta a} Az_{t+a} m_{t+a} da,$$

where $z_t = z$ and $Dz_{t+a} = Bz_{t+a}(1 - m_{t+a})$ for all $a \geq 0$. Determine $\beta$.

c. Conjecture that the optimal allocation of managerial effort is $m_t = m \in (0, 1)$. Use this to calculate $q_t[z]$. Show the conjecture can only be true if $\beta = B$. Determine $\kappa$.

Entrepreneurs who succeed in creating a firm can copy and start with the average productivity of the incumbent firms.

d. Suppose all incumbents have the same productivity in equilibrium. Proceeding under the conjectures made so far, determine $m$, $w$ and the number of managers along the balanced growth path.