University of Minnesota

Department of Economics

Ph.D. Preliminary Examination

Mathematical Economics

Instructions: Answer ONE question from Part I and ONE question from Part II for a total of TWO (2) questions.
Answer ONE (1) question from Part I.
1. Secret versus Standard

Suppose that there are two agents, Ann and Bob, where Ann chooses a row and Bob chooses a column. There are two publicly verifiable signals. Consider the following monitoring technologies, i.e., conditional probability systems that give the signals’ conditional probabilities given each possible action profile.

(a) Show that for any profile of utility functions for Ann and Bob, $R$ is enforceable with recommendation-contingent contracts but not with just signal-contingent contracts.

\[
\begin{array}{ccc}
L & M & R \\
U & 1, 0 & 0, 1 & 1/2, 1/2 \\
D & 1, 0 & 0, 1 & 1/3, 2/3 \\
\end{array}
\]

(b) Show that for any profile of utility functions, $(U, L)$ is virtually enforceable.

\[
\begin{array}{ccc}
L & M & R \\
U & 1, 0 & 1, 0 & 1, 0 \\
D & 1, 0 & 0, 1 & 0, 1 \\
\end{array}
\]

2. Auctions with Interdependent Values

There are two agents. Agent 1’s possible types are $t_1 \in [0, 1]$, and agent 2’s types are $(t_2, s_2) \in [0, 1] \times [0, 1]$. There is an indivisible object to be allocated to the agents. Every agent gets 0 from not obtaining the object. Agent $i$’s payoff from the object equals $v_i(t_1, t_2, s_2) = t_i + s_2$.

(a) Characterize the ex post efficient allocations.

(b) There is money available to provide incentives. Prove that no efficient allocation is implementable.

(c) Find an allocation that maximizes welfare subject to being implementable.

(c) Find necessary and sufficient conditions on agents’ utility functions for an ex post efficient allocation to be implementable in this setting.
Answer ONE (1) question from Part II.
QUESTION 1

Let $W$ be a finite state space. Let $\pi$ be a prior probability distribution on $W$ such that $\pi(w) > 0$ for every state $w$.

Let $N = \{1, 2, \ldots, n\}$ be the set of agents. For each agent $i$, let $P_i : W \to 2^W \setminus \emptyset$ be his possibility correspondence. $P_i(w)$ is the set of states that agent $i$ considers possible when the true state is $w$. For each agent $i$ and event $E \subseteq W$, define $K_i(E) = \{w | P_i(w) \subseteq E\}$. $K_i(E)$ is the event that agent $i$ knows event $E$.

Let $t : W \to \mathbb{R}^N$ be a function that satisfies $\sum_{i \in N} t_i(w) = 0$ for every state $w$. The function $t$ can be thought of as a trade contract that specifies the net monetary transfer to each agent in each state. Let $F^t_i$ denote the event that agent $i$ finds the trade contract $t$ strictly profitable:

$$F^t_i = \left\{ w \left| \frac{\sum_{w' \in P_i(w)} \pi(w') t_i(w')}{\sum_{w' \in P_i(w)} \pi(w')} > 0 \right. \right\}.$$

Let $F^t$ be the conjunction of the $F^t_i$'s for every $i$ (i.e., $F^t = \bigcap_i F^t_i$), so that $F^t$ is the event that every agent has strict willingness to trade. Let $K^n F^t$ be recursively defined as $\bigcap_i K_i K^{n-1} F^t$, with $K^0 F^t = F^t$. Finally, define

$$CKF^t := \bigcap_{n \geq 1} K^n F^t.$$

$CKF^t$ is the event that it is common knowledge that every agent has strict willingness to trade. We say that the no-trade result obtains if $CKF^t = \emptyset$ for every trade contract $t$.

For any subset $W' \subseteq W$, we say that $W'$ is a partitional subspace if $\forall i, \forall w \in W'$, (1) $P_i(w) \subseteq W'$, and (2) $\forall w' \in P_i(w), P_i(w') = P_i(w)$.

For any subset $W' \subseteq W$, define

$$D(W') = \{w \in W | P_i(w) \subseteq W' \text{ for some agent } i\}.$$

$D(W')$ is the collection of states at which at least one agent considers only states in $W'$ to be possible. We say that $(W, \{P_i\}_{i \in N})$ satisfies terminal partitionality if there is a non-empty partitional subspace $W' \subseteq W$ such that $\cup_{n \geq 0} D^n(W') = W$, where $D^n(W')$ is defined recursively as $D(D^{n-1}(W'))$, and $D^0(W') = W'$.

Prove that if $(W, \{P_i\}_{i \in N})$ is terminally partitional, the no-trade result obtains.
QUESTION 2

Let $W$ be a finite state space. Let $\Delta(W)$ be the set of all probability distributions on $W$. An act is a function $a : W \rightarrow \mathbb{R}$. Let $A$ be the set of all acts.

A maxmin decision maker is a decision maker whose preference over $A$ can be summarized by a compact and convex subset $C \subseteq \Delta(W)$ such that he prefers act $a$ to act $b$ (i.e., $a \succsim b$) if and only if

$$ \min_{p \in C} \sum_{w \in W} p(w)a(w) \geq \min_{p \in C} \sum_{w \in W} p(w)b(w). $$

A Choquet expected-utility maximizer is a decision maker whose preference over $A$ can be summarized by a capacity $v$ on $W$ such that he prefers act $a$ to act $b$ if and only if

$$ \int a(v) \geq \int b(v), $$

where the integrals are Choquet integrals.

Question 2(a): Construct an example of a maxmin decision maker who is also a Choquet expected-utility maximizer.

Question 2(b): Construct an example of a maxmin decision maker who is not a Choquet expected-utility maximizer.