Preliminary Examination

International Trade

Spring 2009

Answer a total of THREE (3) questions. Each question must come from a DIFFERENT section. (There are three sections, and some sections have more than one choice)
Part I

Answer ONE (1) question from Part I
In I.1, part b, there is an equation that reads

\[ F(x) = 1 - x^\gamma. \]

It should read

\[ F(x) = 1 - x^{-\gamma}. \]
Question I.1

Monopolistic competition with heterogeneous firms and trade

Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

\[
\begin{align*}
\max (1 - \alpha) \log c_0 + \frac{\alpha}{\rho} \log \int_0^\mu c(z)^\rho \, dz \\
\text{s.t. } p_0 c_0 + \int_0^\mu p(z)c(z) \, dz = w\bar{\ell} \\
c(z) \geq 0.
\end{align*}
\]

Here \( 1 > \alpha > 0 \) and \( 1 > \rho > 0 \). Furthermore, \( \mu > 0 \) is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function \( y_0 = \ell_0 \).

(a) Suppose that the producer of good \( z \) takes the prices \( p(z') \), for \( z' \neq z \), as given. Suppose too that this producer has the production function

\[
y(z) = \max \left[ x(z) (\ell(z) - f), 0 \right].
\]

where \( x(z) > 0 \) is the firm's productivity level and \( f > 0 \). Solve the firm's profit maximization problem to derive an optimal pricing rule.

(b) Suppose that good 0 is produced with the constant-returns production function \( y_0 = \ell_0 \). Suppose that firm productivities are distributed on the interval \( x \geq 1 \) according to the Pareto distribution with distribution function

\[
F(x) = 1 - x^\gamma,
\]

where \( \gamma > 2 \) and \( \gamma > \rho/(1 - \rho) \). Potential firms pay a cost \( \phi > 0 \) to sample from this distribution. Define an equilibrium for this economy.

(c) Find an expression for the productivity of the least productive firm that produces. That is, find an \( \bar{x} > 1 \) such that no firm with \( x(z) < \bar{x} \) produces and all firms with \( x(z) \geq \bar{x} \) produce.
(d) Suppose now that there are two symmetric countries that engage in free trade. Each country $i, i = 1, 2,$ has a population of $\ell$. Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country $i$ faces a fixed cost of exporting to country $j, j \neq i,$ of $f_e$ where $f_e > f_d = f$ and an iceberg transportation cost of $\tau - 1 \geq 0$. Define an equilibrium for this world economy.

(e) Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in part (c). [You should explain carefully how to calculate any cutoff values, but you do not actually need to calculate it.] Compare this value, or these values, with that in part (c).

(f) Briefly discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.
Question 1.2

Dynamic Hecksher-Ohlin Model

Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a1} c_{2t}^{a2})$$.

The investment good is produced according to

$$k_{t+1} = d x_{1t}^{a1} x_{2t}^{a2}$$.

Feasible consumption/investment plans satisfy the feasibility constraints

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$
$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}$$.

where

$$k_{1t} + k_{2t} = k_t$$
$$\ell_{1t} + \ell_{2t} = 1$$

The initial value of $k_t$ is $\bar{k}_0$. All of the variables specified above are in per capita terms. There is a measure $L$ of consumer-workers.

(a) Define an equilibrium for this economy.

(b) Write out a social planner's problem for this economy. Explain how the solution to this social planner's problem is related to that of the one-sector social planner's problem

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

s.t. $c_t + k_{t+1} = d k_t^{a1}$
$c_t, k_t \geq 0$
$k_0 = \bar{k}_0$.

[You can write down a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]
(c) Solve the one-sector social planner's problem in part (b) [Recall that the policy function for investment has the form \( k_{t+1}(k) = Adk^{a_1} \), where \( A \) is a constant that you remember or can determine with a bit of algebra and calculus.]

(d) Suppose now that there is a world made up of \( n \) different countries, all with the same technologies and preferences, but with different constant populations, \( L^j \), and with different initial capital-labor ratios \( k^i_0 \). Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

(e) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

(f) Let \( s_t = c_t/y_t \) where \( y_t = p_{1t}k_t + p_{2t} = dk^{a_1} \) is world GDP per capita. Transform the first-order conditions for the one-sector social planner's problem in part (b) into two difference equations in \( k_t \) and \( s_t \). Use the first-order conditions for the consumers problem of the equilibrium in part (d) to show that

\[
\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left( \frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left( \frac{y_0^i - y_0}{y_0} \right).
\]

(g) Use the solution to the one-sector social planners problem in part (c) to solve for \( s_t \). Discuss the economic significance of the result that you obtain.
Part II

Answer the question from Part II
Question II.1

You have just been hired by the IMF and your first assignment is to assess whether there is good international risk sharing between countries 1 and 2.

(a) Briefly explain what economists mean by "international risk sharing".

(b) Briefly explain under what assumptions you can test for international risk sharing using only aggregate country specific data (i.e. aggregate consumption, output etc in countries 1 and 2).

(c) Now consider the standard 2 countries, 1 good model of international business cycles:

(i) Show that if utility of the representative agent in each country is

\[ u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + V(1-l) \]

where \( c \) is consumption, \( l \) is labor, \( V \) is a concave function and \( \sigma > 0 \), then perfect international risk sharing implies that the growth rate of consumption (i.e. \( g_{tc} \equiv \log(c_{lt}) - \log(c_{lt-1}) \)) should be equalized across the 2 countries in each date. Give an example of a market structure in which equilibrium allocation display perfect risk sharing.

(ii) Suppose that period utility of the representative agent in each country is given by

\[ u(c, l) = \frac{(c^\theta (1-l)^{1-\theta})^{1-\sigma}}{1-\sigma} \]

where \( \sigma \neq 1 \), and \( 0 < \theta < 1 \). Argue that in this case allocations which display consumption growth not equalized at every date can be consistent with perfect risk sharing. In particular consider, as an example, a period in which you observe \( g_{1c} > g_{2c} \). Show that if \( \sigma > 1 \) and in that period leisure \((1-l)\) grows less in country 1 than in country 2, the observation is consistent with perfect risk sharing. Is the fact that when \( g_{1c} > g_{2c}(g_{1c} < g_{2c}) \) then \( g_{1(1-l)} < g_{2(1-l)} \) \((g_{1(1-l)} > g_{2(1-l)})\) roughly consistent with macroeconomic data?
(d) Consider now a model economy with 2 countries and 2 goods in which country 1 receives a stochastic endowment of apples $A_t$ and country 2 a stochastic endowment of bananas $B_t$. Preference of representative households in both countries are

$$E \sum \beta^t \frac{c_{1\cdot t}^{1-\sigma}}{1-\sigma}$$

and consumption in the 2 countries is produced using the following technologies

$$c_1 = \left( \omega a_1^{\gamma-1} + (1-\omega)b_1^{\gamma-1} \right)^{\frac{1}{\gamma-1}}$$

$$c_2 = \left( \omega b_2^{\gamma-1} + (1-\omega)a_2^{\gamma-1} \right)^{\frac{1}{\gamma-1}}$$

$$\gamma > 0, 0 < \omega < 1$$

and in each period market clearing implies

$$A = a_1 + a_2$$
$$B = b_1 + b_2$$

(i) Solve for the real exchange rate (price of $c_2$ in terms of $c_1$) and the terms of trade (price of bananas in terms of apples) as a function of $a_1, a_2, b_1$ and $b_2$. Give conditions on $\omega$ under which the real exchange rate is constant.

(ii) Write down conditions that characterize perfect risk sharing in this economy and explain how you would test for those in the data.

(iii) Assume now that $c_i = a_i^{5}b_i^{5}, i = 1, 2$. Show that in this case a market structure without any intertemporal markets (i.e. financial autarky) yields equilibrium allocations consistent with perfect risk sharing. Explain why it is the case.
Part III

Answer ONE (1) question from Part III
Question III.1

Supporting Positive Debt

Bulow and Rogoff considered an environment in which if agents defaulted on their debts they were prohibited from borrowing again but they still could save in a state-contingent fashion at the existing rates. They claimed that in such an environment it is not possible to support positive debt. Hellwig and Lorenzoni took a similar environment and claimed to show that it is possible to support positive debt.

(a) Sketch out a model and develop a version of the Bulow Rogoff argument that it is not possible to support positive debt.

(b) Sketch out an alternative version of that model in which it is possible to support positive debt.

(c) Discuss in detail the differences in assumptions.

(d) Are the assumptions needed for the Hellwig Lorenzoni argument to apply supported in the data? Explain.
Question III.2

Government Intervention and Borrowing

(a) Describe an environment in which, absent government intervention, the private sector borrows an inefficient amount. Prove your result as carefully as you can.

(b) Can a similar result occur if the only friction in the economy is enforcement constraints? Discuss as precisely and formally as you can.
Question III.3
Sudden Stops

We described in class some economies in which a sudden tightening a country’s borrowing constraint could lead to an increase in output. Can you carefully describe an economy in which a sudden tightening in a country’s borrowing constraint leads to a fall in output?

(a) Describe the economy.

(b) Establish your result as precisely as you can.

(c) Carefully explain the intuition behind why your result is different from the one we discussed in class.