Ph.D. Preliminary Examination

MACROECONOMIC THEORY

Spring 2010

Majors and Minors: Answer ALL FOUR parts.

Please make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly.
Part I

Answer the question in Part I.
Question I.1: Dynamic Programming and Competitive Equilibrium

Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer’s utility function is

\[ \sum_{t=0}^{\infty} \beta^t u(c_t, x_t) \]

The consumer is endowed with 1 unit of labor in each period, some of which can be consumed as leisure, \( x_t \), and some of which is supplied as labor, \( \ell_t \). The consumer is also endowed with \( k_0 \) units of capital in period 0. Feasible allocations satisfy

\[ c_t + k_{t+1}(1 - \delta)k_t \leq f(k_t, \ell_t). \]

(a) Formulate the problem of maximizing the representative consumer’s utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman’s equation. Make appropriate assumptions on the parameters \( \beta \) and \( \delta \) and on the functions \( u \) and \( f \) that guarantee a solution to Bellman’s equation. Cite any results that you need, but do not prove anything.

(b) Define a sequential markets equilibrium (with borrowing and lending) for this economy. Suppose that you have solved the dynamic programming problem in part (a). Explain carefully how to calculate the sequential markets equilibrium. Make explicit any additional conditions that you need to impose on the parameters \( \beta \) and \( \delta \) and on the functions \( u \) and \( f \).

(c) Define an Arrow-Debreu equilibrium for this economy. Suppose that you have solved the dynamic programming problem in part (a) and that the parameters \( \beta \) and \( \delta \) and the functions \( u \) and \( f \) satisfy whatever additional conditions that you imposed in part (b). Explain carefully how to calculate the Arrow-Debreu equilibrium.
Part II

Answer both questions in Part II.
Question II.1: PIH

Consider a consumer that solves the following standard problem

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \text{ u quadratic}$$

s.t.

$$c_t + a_{t+1} = (1 + r) a_t + y_t$$
$$a_0 = 0, (1 + r)^2 = 1, \text{ No Ponzi}$$

and faces an income process given by

$$y_t = (1 - \rho) \bar{y} + \rho y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim N(0, \sigma), \bar{y} > 0, \rho \geq 0$$

(a) Solve for $\Delta c_t = c_t - c_{t-1}$ and $\Delta a_{t+1} = a_{t+1} - a_t$ as a function of $\varepsilon_t$ (Show your calculations)

(b) Find a range for the parameter $\rho$ for which the volatility of consumption changes ($\Delta c_t$) is higher than the volatility of income changes ($\Delta y_t$). Explain why this happens.

(c) Consider now $\rho = 0$ and consider the following realization of the shocks: $\{\varepsilon_t\}_{t=0}^{\infty} = \{0, -1, 0, 0, 0, \ldots\}$. Plot consumption and wealth for this realization. Argue that there is a period in which positive income growth does not lead to positive consumption growth and explain why this is the case.
Question II.2: Asset Pricing in Complete and Incomplete Markets

Consider a closed economy with a continuum of infinitely lived households. Each household has preferences represented by

$$E \sum_{t=0}^{\infty} \beta^t \frac{c_{1-t}^{1-\gamma}}{1-\gamma}$$

and is subject to idiosyncratic income shocks $y_{it}$ with

$$y_{it} = \exp(\varepsilon_{it}), \varepsilon_{it} \sim N\left(\frac{-\sigma^2}{2}, \sigma^2\right)$$

In period 0 each household is endowed with the same amount of shares $\lambda$ of a Lucas tree which in each period yields a constant fruit $d$. Assume also that in period 0 each agent starts with income $y_{i0} = 1$.

(a) Assume that agents can trade a full set of Arrow securities contingent on the realization of their idiosyncratic income shock. Define a stationary competitive equilibrium for this economy and solve for equilibrium consumption of each agent and for the (after dividend) price of the tree.

(b) Now assume that agents can only trade shares in the tree and a standard uncontingent bond. Write down the budget constraint of the agents and define a stationary competitive equilibrium for this economy.

(c) Show that the price of a share of the tree in the economy in point 2 is higher than the price of the tree in the economy in point 1 and give a brief intuition of why this is the case.
Part III

Answer the question in Part III.
Question III.1

Consider a cash credit good economy in which households have preferences of the form
\[ \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}) \]
where \( c_{1t} \) and \( c_{2t} \) denote consumption of cash and credit goods respectively, \( U \) is strictly concave, differentiable and satisfies the Inada conditions, and \( 0 < \beta < 1 \) is the discount factor. Household endowments are given by an endowment of \( y \) units of a composite good which can be converted into cash and credit goods according to the resource constraint
\[ c_{1t} + c_{2t} = y \]
The securities market meets at the beginning of the period The household’s securities market constraint is
\[ M_t + B_t = (M_{t-1} - p_{t-1}c_{1t-1}) - p_{t-1}c_{2t-1} + p_{t-1}y + R_{t-1}B_{t-1} + T_t \]
where \( M_t \) denotes cash balances, \( B_t \) denotes holdings of one-period debt, \( p_t \) denotes the price level, \( R_t \) denotes the (gross) interest rate on debt and \( T_t \) denotes lump sum transfers by the government. The cash in advance constraint is
\[ p_t c_{1t} \leq M_t. \]
Assume real debt holdings are bounded below by a large negative number.

(a) Define a competitive equilibrium.

(b) Assume government policy is characterized by a sequence of constant interest rates, \( R_t = R > 1 \) for all \( t \). Characterize competitive equilibria. What is the set of real allocations in such equilibria? Under what conditions on the utility function \( U \) is this set a singleton? Does the economy have a unique equilibrium? Prove your assertions.

Question III.1 continues on the next page.
Question III.1 continued:

(c) Now assume that monetary policy is described by a sequence of constant money supplies, $M_t = M$ for all $t$. Characterize the set of competitive equilibria when

$$U(c_1, c_2) = \log c_1 + \log c_2.$$  

(d) Define $F(c_1)$ as $c_1 U_2(c_1, y - c_1)$ and $G(c_1)$ by $c_1 U_1(c_1, y - c - 1)$. Let $c_1^*$ be defined $F(c_1^*) = \beta G(c_1^*)$. Show that if the absolute value of $F_1^*/G_1^*$ is less than $\beta$, the economy has a continuum of equilibria converging to a steady state.

(e) Now suppose that the money supply grows at a constant rate $\gamma$. Consider the same issue of equilibrium multiplicity in part (d). Under what conditions on $\gamma$ and $U$ do we obtain multiplicity?
Part IV: Stuff Related to the Fourth Mini

In the following there are 12 questions for 90 points. Answer questions for a total value of 60 points. Be as BRIEF as you can and good luck.
Recursive Equilibria (Production with land)

There is an economy with many identical consumers and infinite time. Consumers have preferences

$$E \left\{ \sum_{t=0}^{\infty} \beta^{2t+1} \left[ u^s(c_t, n_t, \ell_t) + u^w(c_t, n_t) \right] \right\}$$

where $c_t$ is own consumption at time $t$, $n_t$ is the fraction of time worked by the agent at time $t$ and $\ell_t$ is land that is used for the enjoyment of vacation but only in summer ($u^w$ is for winter and $u^s$ for summer). Utilities are increasing and strictly concave.

Output can be produced with labor and land according to a standard neoclassical production function

$$z_t F(L_t, N_t)$$

where $L_t$, is land used for production purposes. There are two Shocks to productivity $z$ have finite support and follow a Markov chain with transition matrix $\Gamma$. Total amount of land is 1.

1. (10 points) Imaging first that $u^s(c_t, n_t, \ell_t) = u^w(c_t, n_t)$ Define a steady state without seasons when $z_t$ is set to its unconditional mean. Make sure that you give a formula for the price of land.

2. (10 points) Define recursive competitive equilibrium of the stochastic economy. Explain the price of land, or alternatively the price of the firms who own the land.

3. (5 points) Imagine that in a recession (like this one), output is down by 10% half of which is due to $z$ and half by labor. Will the stock market go down in price? Discuss.

Now consider the version of the economy with seasons.

4. (10 points) Define a steady state. Clearly you have to define winter and summer values for consumption, leisure, land use, interest rates and land prices.

5. (5 points) Imagine the utility functions are non separable and that the government wants to smooth consumption with tax policies. Should it?
Search

Assume risk neutral works and firms in a one period world where there is a measure 1 of each. There is a Cobb-Douglas matching function. If matched output is 1, if not workers get .1 and firms get zero.

6. (5 points) Imagine that workers have to extent a given amount of effort $e$ to search. If the wage is set to .5 give a formula for this amount of effort.

7. (5 points) Imagine that there is wage posting. What happens?

8. (10 points) What happens under equal weight Nash bargaining?

9. (5 points) What happens under competitive search?

Monopolistic Competition

Imagine that preferences of a representative consumer in a static world are given by

$$u(\{c(i)\}_{i \in [0, A]}) = \left( \int_0^A c(i)^{\gamma} \, di \right)^{\theta/\gamma} (1 - n)^{1-\theta}$$

where $1 - n$ is leisure and $n$ is time spent working.

10. (15 points) Give an expression for the time worked as a function of the (identical) price of the goods, the wage and some initial wealth that the households may have.

Imagine now that each good $i \in [0, a]$ is produced by a firm that has a monopoly over its production. Output of these firms is produced with a linear technology $c_i = \gamma n_i$, where $\gamma$ is a positive parameter.

11. (5 points) What would be the price when representative consumers own all firms? This requires the computation of the consumer’s wealth.

12. (5 points) Is there a simple fiscal policy that improves on the welfare. Explain.