Preliminary Examination

Growth and Development

Spring 2010

Answer four questions, at least one from each of the three sections.

All questions are of equal weight.
Section I: Question 1 of 2

Consider an economy with a representative consumer whose preferences over consumption flows \( c_t \) are determined by

\[
\int_0^\infty e^{-\rho t} \ln(c_t) dt.
\]

The consumer is endowed with one unit of labor. A flow of \( \Lambda \) units of labor can be used to produce a unit flow of machines. A type-\( z \) machine can be combined with \( l \) units of labor to produce a flow of \( z \min\{1, l\} \) units of consumption.

At time \( t \), the type of every newly produced machine is \( z_{t,0} = \Gamma x_t \), where \( x_t \) is the lowest type of machine being used to produce consumption at time \( t \). The type of a machine produced at time \( t \) is \( z_{t,a} = z_{t,0} e^{\theta a} \) at time \( t + a > t \).

The parameters \( \rho, \Lambda \) and \( \theta \) are positive, and \( \Gamma > 1 + \rho \Lambda \). Conjecture that this economy has a balanced growth path with wages growing at a rate \( \kappa \) and machines becoming obsolete at an age \( \tau \).

a. Find the equilibrium conditions for a balanced growth path.

b. What is the effect on \( \kappa \) of an increase in \( \theta \)?

c. Use the balanced growth conditions to show that \( x = (\kappa - \theta)/\rho \) satisfies

\[
(1 + \rho \Lambda)(1 + x) = \Gamma + \frac{x}{\Gamma^{1/x}}.
\]

Use a diagram that shows the left- and right-hand sides of this condition as a function of \( x \) to determine the effect on \( \kappa \) of an increase in \( \Gamma \). You may use without verification that the right-hand side is a convex function of \( x \).

d. Along a balanced growth path, how do you have to de-trend machine types to make the distribution of de-trended types stationary? What is that stationary distribution?
Section I: Question 2 of 2

Consider the following model of structural change. There is a representative consumer whose preferences over output from $I + 1$ different sectors are given by

$$
\int_0^\infty e^{-\rho t} \ln(C_t) dt, \quad C_t = \left[ \sum_{i=0}^I C_{i,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)},
$$

where $\rho > 0$, $\varepsilon > 0$ and $I \in \mathbb{N}$. The aggregate labor supply $L$ is exogenous and there is a positive initial capital stock $K_0$. The technology is given by

$$
C_{0,t} = z_{0,t} F(K_{0,t}, L_{0,t}) - (DK_t + \delta K_t),
$$

and

$$
C_{i,t} = z_{i,t} F(K_{i,t}, L_{i,t}), \quad i \in \{1, \ldots, I\},
$$

where

$$
\sum_{i=0}^I L_{i,t} \leq L, \quad \sum_{i=0}^I K_{i,t} \leq K_t,
$$

and all capital and labor inputs must be non-negative. The production function $F$ exhibits constant returns to scale.

a. Determine the relative prices $p_{i,t}/p_{j,t}$ and $s_t = p_{0,t}/P_t$, where $P_t = \left[ \sum_{i=0}^I p_{i,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)}$ is the price index for the composite good $C_t$.

b. Let $X_t$ be total consumption expenditures on goods 0 through $I$, measured in units of good 0. Show that $X_t = z_{0,t} F(K_t, L) - (DK_t + \delta K_t)$ and $C_t = s_t X_t$.

Now suppose the production function is $F(k, l) = k^\alpha l^{1-\alpha}$ for some $\alpha \in (0, 1)$. Suppose further that $z_{i,t} = z_i e^{\theta_{i,t}}$, for some non-negative $\theta_i$.

c. Determine the growth rate of total consumption expenditures, measured in units of good 0, along the aggregate balanced growth path. Does consumption of the composite good also grow at a constant rate?

d. Determine $L_{i,t}/L_{j,t}$ for all $i$ and $j$ in $\{1, \ldots, I\}$. Baumol argues that labor moves away from sectors with rapid technological progress and towards sectors with low rates of technological progress. Is he right?
Section II: Question 1 of 2

1. Consider the following dynamic programming problem, which is one of the subproblems in a search model of marriage,

\[ V(b) = \max_{c, \ell} \left[ U(c, \ell) + b + \beta \int \max(V(b'), V^s) \, dF(b'|b) \right] \]

Here, \( b \) is the level of love in marriage and follows an AR(1) process; \( V(b) \) is the value function during marriage, \( V^s \) is the value function while single (which does not depend on the love shock); \( U(c, \ell) \) is a period utility function that depends on consumption and leisure, and \( \bar{\ell} \) is the total time endowment in a period (we ignore the complication of the husband and wife choosing separate consumption and leisure).

a. Assume that \( V^s \) is known. Write an algorithm that solves for \( V(b) \). Be very specific. You must specify how you will evaluate the integral (what method, why?), how you will pick the grid (the limits, the grid point locations if applicable), the algorithm must be such that a computer scientist who doesn’t know any economics (no dynamic programming knowledge) should be able to read your algorithm on her own and implement it exactly using Fortran, C, or Matlab, and obtain the right answer. You should also explain your motivation for choosing the particular methods you choose at each step.
This question asks you to solve the baseline model in Aiyagari (1994, QJE). This model embeds a standard income fluctuations problem in general equilibrium by assuming a Cobb-Douglas production function with capital and labor as inputs. The capital is supplied by households (obtained from the consumers consumption-savings problem). Therefore, you need to clear the capital market by finding the equilibrium interest rate. As in Aiyagari assume that the idiosyncratic income process for a typical consumer follows an AR(1) process.

a. Describe Tauchen’s (1986) method for discretizing an AR(1) process. There is more than one implementation of it. Feel free to discuss just one. Be very specific.

b. Describe Rouwenhorst’s (1995) method for discretizing an AR(1) process. Be very specific. If you have a process with high persistence, would you prefer Rouwenhorst’s or Tauchen’s method? Justify your answer.

c. Suppose that you want to solve the Aiyagari model with a risk aversion of 20. Without taking an enormous number of grid points, what computational tricks can you employ to solve the dynamic program within the Aiyagari model accurately. Be specific with the description of the method.

d. Describe precisely how you would solve for the general equilibrium in the Aiyagari model. What does it involve? Describe the specific approach you would take to clear the capital markets. Be very specific. Is the algorithm you are describing guaranteed to converge? Why, or why not? [You can assume that you know how to solve the dynamic programming problem for a given interest rate, so you do not need to describe that part.]
Consider an industry that produces a good $y$ that sells for $(1+\tau)p_y$, where $p_y$ is the world price, and $\tau$ is the tariff rate faced by foreign firms selling in the domestic market. This good is produced with two inputs, an entrepreneur’s time and a machine. Entrepreneurs are indexed by $\gamma$. An entrepreneur of type $\gamma$ has access to the following production function

$$y = \gamma \min(n, I),$$

where $n$ is the entrepreneur’s time and $I$ is an indicator function, where $I = 1$ if the entrepreneur has the machine, $I = 0$ otherwise.

Entrepreneurs decide to produce this good or to work in their next best alternative occupation. In particular, the entrepreneurs have a unit time endowment which they devote to producing good $y$ or to the alternative occupation. Suppose all entrepreneurs have the same next best alternative, the value of which is $w$. Finally, the price of the machine is $q$.

**a.** Let the parameter $\gamma$ be distributed continuously with density $f(\cdot)$ on $[0, \bar{\gamma}]$. Under what conditions will entrepreneurs enter into the industry producing $y$?

**b.** Under these conditions, what happens to the set of entrepreneurs that produce $y$ as the tariff is increased?

**c.** What happens to the dispersion of labor productivity in the industry as the tariff is increased, in particular, what happens to the range of labor productivities?

**d.** What happens to the range of capital productivities as the tariff is increased?

**e.** We studied a number of papers in class where the authors proposed that one country, say country A, had more “distortions” on businesses than another country B if the distribution of productivities in country A was more “dispersed” than that in country B. If we interpret higher tariffs as meaning greater distortions, does this “idea” hold in the simple model above? Explain.