Answer any three of the following four questions. The answers will be equally weighted in the examination grade. Your copy of the exam questions should have 7 pages.

Analytical solutions can be derived for some of the problems this examination contains. If the algebra involved proves too cumbersome, however, essentially full credit will be given for careful explanations of the steps that could be followed to derive analytical solutions. If you feel that additional assumptions are required before a unique solution to the problem can be found, specify what they are and why you need them.

Please indicate clearly which questions and which part of the question you are answering. Note also that irrelevant material, even if correct, will receive no credit.
QUESTION 1

In this problem, we will consider econometric models of demand estimation in a differentiated products market. Suppose that there are \( i = 1, \ldots, I \) agents observed in \( t = 1, \ldots, T \) markets. Each agent makes a choice between \( j = 1, \ldots, J \) mutually exclusive alternatives. Let \( x_{jt} = (x_{jt,1}, \ldots, x_{jt,K})' \) be a \( K \times 1 \) vector of characteristics for product \( j \) that is observed by both the econometrician and the consumers in the market. Note that the product characteristics are time dependent, reflecting the possibility that product mix or other product attributes may change over time. Let \( p_{j,t} \) denote the price of \( j \) at time \( t \).

Specify the random utility as:

\[
u_{ijt} = x'_{j,t} \beta - \alpha p_{j,t} + \varepsilon_{ij}\]

Assume that the error term is iid extreme value so that the market share for \( j \) at time \( t \) is:

\[
s_{jt} = \frac{\exp(x'_{j,t} \beta - \alpha p_{j,t})}{\sum_{j'=1}^J \exp(x'_{j',t} \beta - \alpha p_{j',t})}
\]

1. Show that the own and cross price elasticities satisfy:

\[
\eta_{jk} = \left. \frac{\partial \Pr(\text{i chooses } j)}{\partial p_k} \right|_{p_k = p_{j,t}} \frac{p_k}{\Pr(\text{i chooses } j)}
\]

\[
= \begin{cases} -\alpha p_j (1 - s_j) & \text{if } j = k \\ -\alpha p_k s_k & \text{if } j \neq k \end{cases}
\]

2. Describe three possible drawbacks implied by this formula that may be encountered in applied work on demand analysis.

3. In Berry (1994) and BLP (1995), the authors extend this model by adding in a product \( j \) time \( t \) specific unobservable. That is, we specify utility as:

\[
u_{ijt} = x'_{j,t} \beta - \alpha p_{j,t} + \xi_{j,t} + \varepsilon_{ij}\]
which generates market shares:

\[
    s_{jt} = \frac{\exp(x'_{jt}\beta + \xi_{j,t} - \alpha p_{j,t})}{\sum_{j'=1}^{J} \exp(x'_{j',t}\beta + \xi_{j',t} - \alpha p_{j',t})}
\]

Please describe three reasons why we may wish to include \( \xi_{j,t} \) in our model.

4. The product specific unobservable \( \xi_{j,t} \) will be co-linear with \( x_{j,t} \). Describe three strategies that can be used to identify \( \xi_{j,t} \) separately from \( \alpha \) and \( \beta \). What are the advantages and limitations of each strategy.

5. It is common to further extend the model by including random coefficients. What are the potential advantages of using this specification?
QUESTION 2

Consider a production function given by

\[ q_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \eta_{it} \]

with inputs labor and capital, productivity shock \( \omega_{it} \), and i.i.d. shock \( \eta_{it} \).

1. Suppose capital and labor are correlated with \( \omega \). Suggest two possible approaches to addressing this simultaneity problem (only one may be a "control function" approach). Provide exact conditions under which each estimator will be consistent.

2. Suppose instead of \( q_{it} \) we observe \( p_{it} q_{it} \) and we deflate with an industry deflator \( P_t \). What condition must hold for the estimators proposed above to be consistent?

3. If firms face downward sloping demand curves, describe the possible bias of the capital and labor coefficients. Be specific.

4. Suppose firms face common input and output prices and treat \( \eta_{it} = 0 \) when choosing input levels. Assume there are no adjustment costs for labor or capital. Comment on "If firm 1 is twice as productive as firm 2, then we should shift as many resources as possible to firm 1 to increase aggregate productivity growth."

5. Suppose unobserved managerial ability increases the output that a firm gets out of both labor and capital inputs. Write down a production function specification consistent with this setting. Describe in detail how you would estimate this model?
QUESTION 3

Suppose the inverse demand in an industry is $P(Q_t) = A - Q_t$. There are two firms. Let $q^t_j$ denote the output of firm $j$ in time $t$ so that $Q^t = q^t_1 + q^t_2$. Marginal cost is zero. **However, there is a fixed cost $\phi > 0$ that must be incurred when output is positive.** When a firm moves and picks output, the output choice is fixed for two periods. Assume the fixed cost $\phi$ is paid in first of the two periods when the firm sets a positive output level. Firms alternate in moves, with firm 1 picking in odd periods and firm 2 picking in even periods.

1. Suppose there are two periods, $t = 1, 2$. In period 1, firm 1 sets $q^1_1 = q^2_1$ and firm 2’s output is initially fixed at $q^1_2 = 0$. In period 2, firm 2 picks $q^2_2$. Let $\beta$ denote the discount factor. Characterize the subgame perfect equilibrium of this game. Determine how the equilibrium sequence of outputs varies with the parameter $\beta$ and the fixed cost $\phi$. Explain the intuition for why the equilibrium sequence of outputs varies with $\beta$ and $\phi$.

2. Suppose there is an infinite horizon. The two firms alternate moves, firm 1 picking its location in odd periods, firm 2 in even periods, so the decisions are fixed for two periods. Suppose the fixed cost $\phi$ for a positive output choice that must be paid by the firm whose turn it is to move is a random variable drawn i.i.d. each period from a continuous distribution $F(\phi)$ with support $[0, \infty)$. Define a Markov-perfect equilibrium for this game.

3. Completely work out the solution for the problem of part 2 under the assumption that $\beta = 0$. Next discuss how allowing for $\beta > 0$ changes the nature of the solution. In particular, when $\beta > 0$, what additional things need to be taken account in the firm’s decision problem?
QUESTION 4

Consider the following two-sided matching game between venture capitalists and startup companies. Let \( i \in \{1, \ldots, I\} \) index venture capitalists and \( j \in \{1, \ldots, J\} \) index startup companies. A venture capitalist can match with only one company, and a startup company can match with only one venture capitalist. Hence, we have a one-to-one, two-sided matching game. For simplicity, assume that \( I = J \) so that there are equal numbers of venture capitalists and startup companies in the matching market. The outside option of non-matching yields the level of utility low enough to ensure that everyone will match in equilibrium.

Venture capitalists have preferences over investments in companies, and startup companies have preferences over matches with individual venture capitalists. To state these preferences, let each potential match have a surplus, and let the surplus of the match between \( i \) and \( j \) be denoted \( S_{ij} \). The surplus represents the expected net present value, at the time of the investment decision. In an empirical application, one will typically assume that \( S_{ij} \) is a function of \( i \)'s characteristics \( (X_i) \), \( j \)'s characteristics \( (X_j) \), and their interactions \( (X_{ij}) \) to capture match-specific synergies. I.e., \( S_{ij} = f(X_i, X_j, X_{ij}) \). These surpluses are assumed to be distinct (i.e., no tie), and the same surpluses determine the preferences of both the VCs and the companies. The surplus is divided between the VC and the company according to a fixed sharing rule determined by \( \lambda \in (0, 1) \), where the VC receives the fraction \( \lambda \), and the company receives \( (1 - \lambda) \). Note that \( \lambda \) is NOT subscripted by \( i \) or \( j \), meaning that the uniform sharing rule applies to all potential matches. This model, thus, features non-transferable utility (NTU).

Then VC \( i \)'s utility from matching with company \( j \), \( U_{ij} \), is given by:

\[
U_{ij} = \lambda S_{ij}
\]
Similarly, company $j$’s utility from matching with VC $i$, $V_{ij}$, is given by:

$$V_{ij} = (1 - \lambda)S_{ij}$$

According to these preferences, VC $i$ prefers an investment in company $j$ to an investment in company $j'$ when $S_{ij} > S_{ij'}$, and company $j$ prefers a match with VC $i$ to a match with VC $i'$ when $S_{ij} > S_{i'j}$.

Let $\mu$ denote matching. If $i$ is matched with $j$ under matching $\mu$, we write $\mu(i) = j$. Using this notation, a match between VC $i$ and company $j$ can be stated in two equivalent ways: as $\mu(i) = j$, or as $\mu(j) = i$.

(1) Define stable matching of this game. Write down a system of inequalities that matching $\mu$ needs to satisfy for it to be stable.

(2) Is $\lambda$ identified? Explain.

(3) Suppose that the setup is slightly different from above: Each venture capitalist chooses its most preferred company and matches with it. If more than one venture capitalists choose the same company as their most preferred one, assume that multiple copies of the company will be created so that every venture capitalist’s demand is satisfied. Does this model make sense at all? Explain. Write down the set of inequalities that should be satisfied for matching $\mu$ to be an equilibrium of this alternative model. Compare the set of inequalities to those in (1).

(4) The comparison of these different setups illustrates how a typical demand model differs from a matching model. Explain.

(5) Given your discussions in (3) and (4), explain why estimating the matching model (the original model we consider) using a usual discrete choice framework (probit or logit) is not a good idea.