Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MAJORS

Spring 2010

The time limit for this exam is 3\(\frac{1}{4}\) hours.

**Answer one question from each part, for a total of four questions.** Please note that there are three questions you can choose from in Part IV.

You may use calculators to make calculations during the examination. However: in answering any question that requires you to justify your answer, if you do use a calculator, you must mention at what point in your answer you obtained results using a calculator, and what it was you were seeking to calculate.

Be sure you clearly define all **boldfaced/underlined** terms. Also, please be sure to define precisely any notation that you introduce.

Note: This examination should have 15 pages including this one (Check to make sure!)
Part I

Answer one question from Part I.
Question I.1

Let $Y \subset \mathbb{R}^L$ be a finite production set, i.e., a set consisting of a finite number of production plans, $Y = \{y^1, y^2, \ldots, y^n\}$. Let $\hat{Y}$ be the convex hull of $Y$, that is the set of all convex combinations of production plans $y^1, \ldots, y^n$. Further, let $\pi^*_Y, s^*_Y, \pi^*_\hat{Y}$ and $s^*_\hat{Y}$ be the (maximum) profit functions and the supply correspondences for $Y$ and $\hat{Y}$, respectively.

(a) Show that $\pi^*_Y(p) = \pi^*_\hat{Y}(p)$ for every $p \in \mathbb{R}^L$.

(b) Show that $s^*_Y(p) \subset s^*_\hat{Y}(p)$ for every $p \in \mathbb{R}^L$. Show that the inclusion is strict for some price vectors if $n \geq 2$. 
Question I.2

Suppose that uncertainty is described by $S$ states of nature with $S \geq 3$. Consider the following preference relation $\succeq$ on the set of state contingent consumption plans (or acts) $\mathbb{R}_+^S$.

$$c \succeq c' \text{ if and only if } \min_s c_s \geq \min_s c'_s.$$ 

(a) Show that $\succeq$ does not have a state-separable utility representation.

(b) Consider a probability vector $\pi = (\pi_1, \ldots, \pi_S)$ such that $\pi_s > 0$ for all $s$. Show that $\succeq$ is strictly risk averse with respect to $\pi$. Derive risk compensation for risky gamble $z \in \mathbb{R}^S$ at deterministic initial wealth $w$ where $E_{\pi}(z) = 0$. 

Part II

Answer one question from Part II
Consider a two-person two-commodity pure exchange economy with no free disposal in which each consumer has consumption set $\mathbb{R}^2_+$ and initial endowment $(1, 1)$. Assume that the preferences of each consumer are arbitrary and can be different for the two consumers, but can be represented by utility functions

$$ u_1 : \mathbb{R}^2_+ \rightarrow \mathbb{R} \quad \text{and} \quad u_2 : \mathbb{R}^2_+ \rightarrow \mathbb{R} $$

respectively.

(a) What conditions must be satisfied by the preference relations $\preceq_1$ and $\preceq_2$ of each consumer?

(b) For this economy, define **competitive equilibrium** using the notation specified above.

(c) For this economy and using this notation, define the sets of weakly and strongly Pareto optimal allocations.

(d) Is it possible that all Pareto optimal allocations give each consumer a utility exactly equal to one while all other feasible allocations give each a utility of exactly zero? Provide an example or explain why there can be no such example.

(e) Is it possible to have such an economy in which, at any Pareto optimal allocation, consumer 1’s utility is strictly greater than consumer 2’s utility? Explain why or why not.

**Question II.1 continues on the next page**
Question II.1 continued:

(f) Now let the initial endowment vectors be \( e_1 \in \mathbb{R}_+^2 \) and \( e_2 \in \mathbb{R}_+^2 \) but \( e_1 + e_2 = (2, 2) \).

Can there be an economy in which \( u_1(x^*_1, y^*_1) > u_1(x_1, y_1) \) where \((x^*_1, y^*_1)\) is any competitive equilibrium allocation for consumer 1 and \((x_1, y_1)\) is any feasible allocation for 1 which is not a competitive equilibrium allocation. Explain. Can the analogous statement hold for consumer 2’s competitive equilibrium and feasible allocations with the inequality reversed (i.e., \( u_2(x^*_2, y^*_2) < u_2(x_2, y_2) \))? Explain.

(g) What additional assumptions on preferences are needed for the Second Welfare Theorem?
Question II.2

This question asks you to compare and contrast results regarding the existence and characterization of competitive equilibria in pure exchange economies in which all (finitely many) traders, \( i = 1, \ldots, n \) have complete continuous preorders \( \preceq_i \) as preference versus twice continuously differentiable utility functions \( u_i \) representing smooth preferences. Both economies have \( \ell \) (finite integer) perfectly divisible commodities.

(a) For each model, define **competitive equilibrium**.

(b) For each economy, state conditions that guarantee existence of competitive equilibrium using the notation given above for each economy.

(c) Again using the notation given above for each economy, state the Second Fundamental Theorem of Welfare Economics.

(d) For each economy, what can be said about the set of competitive equilibrium prices? For this question, you should continue to make the assumptions you used in part (c) for each economy and you may make additional assumptions providing that you state them clearly, explain their economic significance, and explain why they help you to obtain sharper results here.

(e) Given the above, when should we use each model (if we should) in microeconomic theory? List and briefly explain considerations that would cause you to favor first a model with continuous preferences and second a model with twice continuously differentiable utility functions.
Part III

Answer one question from Part III
Question III.1

Define normal form and agent normal form of an extensive form game. Define a perfect equilibrium of an extensive form game (this is also known as Selten’s Trembling Hand equilibrium). Then:

(a) Prove that an equilibrium is a perfect equilibrium of the extensive form if and only if it is a perfect equilibrium of the agent normal form of the game.

(b) Show by an example that a perfect equilibrium of the extensive form does not induce a perfect equilibrium of the normal form.

(c) Show by an example that a perfect equilibrium of the normal form does not induce a perfect equilibrium of the extensive form.
Question III.2

Consider a finite normal form game. Define a correlated strategy and equilibrium.

(a) Define formally the **communication extension** of the normal form game, where an action profile is chosen according to a probability which is known to all players, and then a signal is announced to players with a probability over signals dependent on the action profile. Two extensions may be defined: (i) the same signal has to be announced to all players or (ii) the signal may be different.

(b) Prove that the equilibria of one of the two extended games described in option (i) and (ii) of point 1 above correspond to the set of correlated equilibria. State clearly which of the two games you are considering.

(c) What corresponds to the equilibria of the other game?
Part IV

Answer one question from Part IV
Question IV.1

State the four axioms of the Nash bargaining solution; and prove that the Nash bargaining solution is the only bargaining solution that satisfies these four axioms.
Question IV.2

Suppose there are two states of nature: \( \omega = 0 \) and \( \omega = 1 \). Let \( x \) and \( \hat{x} \) be two signals of \( \omega \). Signal \( x \) has \( I \) (\( I \) finite) possible realizations, and the probability of the \( i \)th realization conditional on state \( \omega \) is \( p^i_\omega \). Signal \( \hat{x} \) has \( J \) (\( J \) finite) possible realizations, and the probability of the \( j \)th realization conditional on state \( \omega \) is \( \hat{p}^j_\omega \). Let \( l^i := p^i_0 / p^i_1 \) and \( \hat{l}^j := \hat{p}^j_0 / \hat{p}^j_1 \). Let \( F(y) \) be a discrete distribution such that \( y = l^i \) with probability \( p^i_1 \), and \( \hat{F}(y) \) is a discrete distribution such that \( y = \hat{l}^j \) with probability \( \hat{p}^j_1 \).

Suppose \( x \) is more Blackwell-informative than \( \hat{x} \) as a signal of \( \omega \). Prove that \( F \) is a mean-preserving spread of \( \hat{F} \).

(P.T.O.)
Consider the following 2-period adverse-selection problem. The principal relies on an agent to produce multiple units of a good in each period. Her objective function is $V = S(q_1) - t_1 + \delta(S(q_2) - t_2)$, where $q_i$ (respectively $t_i$) is output (respectively transfer) in period $i$, $\delta \in (0, 1)$ is a common discount factor, and $S(q)$ is a gross profit function that satisfies $S' > 0$, $S'' < 0$, $S(0) = 0$, and $S'(0) = \infty$.

The agent’s total production cost is linear in output. Let $\theta_i$ be his marginal cost in period $i$. His objective function is $U = t_1 - \theta_1 q_1 + \delta(t_2 - \theta_2 q_2)$.

The marginal costs are unobservable to the principal. It is commonly known that, in period 1, $\theta_1 = \underline{\theta}$ (respectively $\theta_1 = \bar{\theta}$) with probability $\nu_1$ (respectively $1 - \nu_1$); and in period 2, conditional on $\theta_1$, $\theta_2 = \underline{\theta}$ (respectively $\theta_2 = \bar{\theta}$) with probability $\nu_2(\theta_1)$ (respectively $1 - \nu_2(\theta_1)$). We assume negative correlation: $0 < \nu_2(\underline{\theta}) < \nu_2(\bar{\theta}) < 1$.

The timing of contracting is as follows. First, $\theta_1$ is drawn and is observable only to the agent. Second, the principal offers a long-term contract to the agent for both periods. The agent then either accepts or rejects the contract. If he rejects, his reservation utility is 0. If he accepts, he cannot quit the relationship after the first period. Marginal cost $\theta_2$ is drawn at the beginning of the second period.

What is the principal’s optimal contract for different combinations of $\Delta \nu_2 \equiv \nu_2(\underline{\theta}) - \nu_2(\bar{\theta}) \in (-1, 0)$ and $\delta \in (0, 1)$?