Part 1

Answer both questions

**Question 1. International Relative Prices**

Empirical evidence finds that international relative prices (including the real exchange rate, terms of trade, and individual relative prices) are volatile and persistent. With respect to a standard two-country two-good model, their behavior has been termed a puzzle in Backus, Kehoe, Kydland (92).

Using equilibrium conditions where possible to clarify your answers:

a. Is it possible to generate volatile and persistent relative prices in this model from productivity shocks alone? If so, what does it imply about trade flows?

b. Explain how extending this basic model to allow for non-traded goods can increase the volatility of the real exchange rates. What types of productivity shocks can generate volatile real exchange rates in this model? What does this imply about the source of real exchange rate fluctuations? Are they consistent with the data?

c. A key feature of relative price dynamics is that the real exchange rate is more volatile than the terms of trade. Consider a two-country, two-good world with no home bias such that $P = P_D^{0.5} P_F^{0.5}$, $P^* = P_D^{0.5} P_F^{0.5}$.

Suppose prices are set as a destination specific markup over marginal cost (i.e. $P_D = \mu \omega$, $P_F = \mu \omega^*$, ...). Show how markup variation can account for the relative magnitude of real exchange rate fluctuations.

d. Explain different ways to get this "markup" to vary. In particular, consider sticky prices (Chari, Kehoe & McGrattan), bundled non-traded inputs (Corsetti-Dedola-Leduc 2007), and pricing-to-market (Alessandria 2009) or any others that you can think of. What do these models imply about the persistence of relative prices and volatility of trade flows?
Question 2. The forward premium anomaly

In the data high interest rate currencies tend to appreciate in the sense that
\[(1) \quad \text{cov}(i_t - i_t^*, E_t \log e_{t+1} - \log e_t) \leq 0 \]

where \(\exp(i_t)\) and \(\exp(i_t^*)\) are the nominal interest rate on dollar- and euro-denominated bonds, \(e_t\) is the dollar price of euros, and where \(E_t\) denotes conditional expectation.

(a) Define a notion of risk premium on euro-denominated bonds relative to dollar-denominated bonds. If (1) holds, how must fluctuations in risk premia relate to fluctuations in interest differentials? Prove your claim. If \(e_t\) is a random walk how must fluctuations in risk premia relate to fluctuations in interest differentials?

(b) Consider a domestic pricing kernel \(m_{t+1}\) for a dollar asset and a foreign pricing kernel \(m^*_{t+1}\) for a euro asset. Any asset purchased at \(t\) with a stochastic dollar return \(R_{t+1}\) between periods \(t\) and \(t+1\) must satisfy \(1 = E_t m_{t+1} R_{t+1}\) and any asset purchased at \(t\) with a stochastic euro return \(R^*_{t+1}\) must satisfy \(1 = E_t m^*_{t+1} R^*_{t+1}\). Derive a formula for \(i_t, i^*_t\), and \(\log e_{t+1} - \log e_t\) in terms of these pricing kernels.

(c) Assume the pricing kernel is conditionally log normal. Recall that this implies
\[
\log E_t m_{t+1} = E_t \log m_{t+1} + (1/2) \text{var}_t (\log m_{t+1}).
\]

What will the covariance in (1) be when the conditional covariance of the pricing kernel is constant? What will the fluctuations in the risk premium be when the conditional covariance of the pricing kernel is constant?

(d) Briefly sketch out what properties the pricing kernel must have for the kernel to generate (1). Do not sketch out a whole general equilibrium model, rather simply describe the mechanical properties the kernel must have to be consistent with (1). For such a kernel what are the properties of the associated risk premia on euro-denominated bonds?
Part 2
Answer question 1

Question 1. Debt and default in an AK model.

Consider a small open economy with an infinitely lived representative consumer with preferences

\[(1 - \beta) \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \log (c_t) \right],\]

The consumer has access to an ak technology

\[y = ak,\]

where a is an exogenous stochastic productivity parameter and k is capital invested at date \(t - 1\). Each period a is drawn from a continuous distribution with cdf \(F(a)\) and bounded support \([a, \overline{a}]\), with \(a > 0\).

The country issues one-period non-state-contingent bonds \(d'\) to a risk neutral world investor which discounts the future at rate \(\delta\). The price of these bonds is given by a function \(p(k', d')\). Each period the country first decides whether to repay or not outstanding liabilities \(d\).

If it repays, the country decides new bonds \(d'\) to issue, new capital \(k'\) and consumption \(c\), subject to budget and borrowing constraints

\[c + k' \leq ak - d + p(k', d')d'.\]

\[\frac{p(k', d')d'}{k'} \leq \eta < 1\]

If the country defaults it does not repay \(d\) and is in autarky forever.

Denote with \(V(k, a, d)\) the value function of the country with capital \(k\), debt \(d\) and shock \(a\) before taking the default decision, with \(V^A(k, a)\) the value of being in default and with \(V^R(k, a, d)\) the value of repaying.

1. Write \(V(k, a, d)\) as a function of \(V^A(k, a)\) and \(V^R(k, a, d)\)

2. Verify that

\[V^A(k, a) = v^A + \log (ak)\]

solve for \(v^A\) as a function of the structural parameters and solve for consumption and investment as functions \(k\) and \(a\) in autarky.
3. Guess that $V^R(k, a, d) = v^R + \log(ak - d)$ with $v^R > v^A$ (You do not need to verify it now). Show that a country with $k, a, d$ will default if and only if the productivity shock $a$ is below a threshold value given by $\theta \frac{d}{k}$ where $\theta$ is a function of $v^R$ and $v^A$. Solve for $\theta$.

4. Show that there exists a non zero pricing function for the bond $p(k', d')$ that a non-defaulting country with given $k, a, d$ faces and solve for it.

5. Write down the Belmann equation for $V^R(k, a, d)$ and outline how you would verify the guess for it given in point 3.
Part 3
Answer Question 1

Question 1. Monopolistic competition with heterogeneous firms and trade
Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max \ (1-\alpha) \log c_0 + \frac{\alpha}{\rho} \log \int_0^m c(z)^\rho \, dz$$

s.t. \( p_0 c_0 + \int_0^\mu p(z)c(z)dz = w\ell \)

$$c(z) \geq 0.$$  

Here \( 1 > \alpha > 0 \) and \( 1 > \rho > 0 \). Furthermore, \( m > 0 \) is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function \( y_0 = \ell_0 \).

a) Suppose that the producer of good \( z \) takes the prices \( p(z') \), for \( z' \neq z \), as given. Suppose too that this producer has the production function

$$y(z) = \max \left[ x(z) \left( \ell(z) - f \right), 0 \right].$$

where \( x(z) > 0 \) is the firm’s productivity level and \( f > 0 \). Solve the firm’s profit maximization problem to derive an optimal pricing rule.

b) Suppose that good 0 is produced with the constant-returns production function \( y_0 = \ell_0 \). Suppose that firm productivities are distributed on the interval \( x \geq 1 \) according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma},$$

where \( \gamma > 2 \) and \( \gamma > \rho/(1 - \rho) \). Also suppose that the measure of potential firms is fixed at \( \mu \). Define an equilibrium for this economy.

c) Find an expression for the productivity of the least productive firm that produces. That is, find a productivity \( \bar{x} > 1 \) such that no firm with \( x(z) < \bar{x} \) produces and all firms with \( x(z) \geq \bar{x} \) produce. Relate the measure of firms that produce \( m \) to the measure of potential firms \( \mu \) and the cutoff \( \bar{x} \).

d) Suppose now that there are two symmetric countries that engage in trade. Each country \( i \), \( i = 1,2 \), has a population of \( \ell_i \) and a measure of potential firms of \( \mu \). Firms’ productivities are again distributed according to the Pareto distribution, \( F(x) = 1 - x^{-\gamma} \). A firm in country \( i \) faces a fixed cost of exporting to country \( j \), \( j \neq i \), of \( f_e \) where \( f_e > f_d = f \) and an iceberg transportation cost of \( \tau - 1 \geq 0 \). Define an equilibrium for this world economy.
e) Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in part c. [You should explain carefully how to calculate any cutoff values, but you do not actually need to calculate it.] Compare this value, or these values, with that in part c. Draw a graph depicting what happens when a closed economy opens to trade.

f) Discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.