Labor Prelim Exam, May 2011

Answer only one of the three Blocks.

Alessandra Fogli (Block A)

If you choose this block you have to answer three questions out of the following five.

1. The evolution of female labor force participation over the last century in US.
   - Describe the evolution of aggregate female labor force participation (LFP) over the period.
   - Using cohort analysis, describe the evolution of female LFP by age, marital status and presence of children. What different information does cohort analysis convey?
   - How has LFP changed over the life cycle for married women of different cohorts? Has the evolution been geographically homogeneous?

2. Consider the following static problem of a married couple:

   \[
   \max \lambda_f \left[ \mu \log(c_f^1) + \nu \log(c_f^2) + (1 - \mu - \nu) \log(l_f) \right] + \lambda_m \left[ \mu \log(c_m^1) + \nu \log(c_m^2) + (1 - \mu - \nu) \log(l_m) \right]
   \]

   \[
   s.t. \quad c_f^1 + c_f^1 + qk \leq (1 - \tau_f)wl_f^1 + (1 - \tau_m)wl_m^1
   \]

   \[
   c_f^2 + c_m^2 = Ak^\theta(l_f^2)^{1-\theta}
   \]

   \[
   l_f + l_f^1 + l_f^2 = 1
   \]

   \[
   l_m + l_m^1 + l_m^2 = 1
   \]

   \[
   l_m^1 \geq 0, \quad l_m^2 \geq 0 \quad l_f^1 \geq 0 \quad l_f^2 \geq 0
   \]

   where \(c_f^1\) and \(c_m^1\) are the consumption of the market good by the women and the man, \(c_f^2\) and \(c_m^2\) are the consumption levels of home good, \(l_f^1\) and \(l_m^1\) are the hours they work in the market and \(l_f^2\) and \(l_m^2\) are the hours they work in the home.

   - Derive under which conditions on the parameters the solution implies \(l_m^2 = 0\). Given these conditions, show that hours worked by women in both market and home are independent of \(A\) (technological progress in the household).
   - How would you answer change if the utility functions were not logarithmic?
   - What assumptions in Greenwood, Seshadri and Yorukoglu guarantee that female work hours increase in response to an improvement in home technology?
3. Given a spatial dataset:

- Define spatial dependence and spatial heterogeneity.
- Given the general model:
  \[ y = \rho W_1 y + X \beta + u \]
  \[ u = \lambda W_2 u + \varepsilon \]
  \[ \varepsilon \sim N(0, \sigma^2 I_n) \]

- Define the matrix \( W_1 \) and discuss different ways to construct it.
- What special models are derived by imposing the following restrictions: (a) \( X = 0 \) and \( W_2 = 0 \), (b) \( W_2 = 0 \), (c) \( W_1 = 0 \)?
- Consider the case in which \( X = 0 \) and \( W_2 = 0 \). Show that the OLS estimator is biased and explain why.
- Write down the spatial model estimated in Fogli Veldkamp (2011) and interpret the coefficient on the spatial lag in light of the proposed theory.

4. What are the main theories about the evolution of female LFP that you know? Briefly describe their main ingredients and how their implications compare with the empirical evidence.

5. In the model of preference transmission of Bisin and Verdier (QJE), there are two possible types of cultural traits in the population \( \{a, b\} \) and the fraction of individuals with trait \( i \in \{a, b\} \) is denoted \( q^i \). All children are born without defined preferences or cultural traits, and are first exposed to their parent’s trait. Direct ”vertical” socialization to the parent’s trait, say \( i \), occurs with probability \( d^i(q^i) \). If a child from a family with trait \( i \) is not directly socialized, which occurs with probability \( 1 - d^i(q^i) \), he/she picks the trait of a role model chosen randomly in the population (i.e., he/she picks trait \( i \) with probability \( q^i \) and trait \( j \neq i \) with probability \( q^j = 1 - q^i \)).

- Write the transition probabilities \( P_{ii} \) and \( P_{ij} \) for all \( i, j \in \{a, b\} \) and the equation that describes the evolution of the fraction of the population with trait \( i \) in the continuous time limit.
- What are the conditions on the transmission mechanisms which induce heterogeneity in the long run stationary distribution of preferences in the population?
- For which cultural traits you think these conditions are more likely to be satisfied?
Fatih Guvenen

Labor Prelim Exam, BLOCK B

May 26 2011

There are 4 questions totaling 100 points in this block. You have 4 hours to complete the exam. Remember that if you choose a certain block you need to answer questions only in that block. Good luck!

1. [Education Market: 35 points] One view of the education market is that the final good (i.e., education or human capital) is not sold directly to customers (i.e., students) but is instead produced jointly with these customers. In this sense, customers are inputs into the production process and schools only sell an intermediate good, which is an education opportunity (e.g., a seat in a classroom). Consider a simple model of such a market with $T$ universities and $N$ student types. Let $Q_n$ denote the limited aggregate number of student type $n$ in the economy. We specify a Cobb-Douglas cost function (inverse technology):

$$Y_t = \left( s^1_t \right)^{\beta_1} \times ... \times \left( s^N_t \right)^{\beta_N} \times \left( H^1_t \right)^{\alpha_1} \times ... \times \left( H^N_t \right)^{\alpha_N}$$

where $Y_t$ is the total cost (of all operations) at university $t$; $s^t_n$ is the number of student type $n$ enrolled in $t$; $H^t_n$ is the human capital of type $n$ produced at university $t$ (measured in dollar units). We assume $\beta_n < 0$ and $\alpha_n > 0$, which ensure that $\partial Y^t / \partial H^t_n > 0$ (reasonable since human capital is an output); and $\partial Y^t / \partial s^t_n < 0$, which means that the total cost of producing the same amount of human capital decreases with the number of students of a given type (which seems reasonable since a higher $s^t_n$ implies lower per capita human capital that must be produced for a fixed $H^t_n$). Finally, students' preferences are such that they care only about the value of the human capital they acquire and the tuition they pay. [In what follows if you feel that something is not clear or you need to make additional assumptions, please state those assumptions clearly.]

(a) [10 points] Characterize the properties of efficient allocations in this model. In particular, how are students of different types allocated across schools and how is human capital production of different types allocated across schools?

(b) [15 points] Is there a price system that attains these efficient allocations? Decentralize these allocations assuming that universities are perfectly competitive and only charge tuition and do not sell human capital directly.

(c) [10 points] In this competitive equilibrium, can universities charge different tuition rates to different types of students? Can the same student type pay a different tuition rate at different universities?
2. [40 points] This question considers a model of occupational switching in the presence of human capital accumulation and learning about one’s own abilities. We approach this problem in several steps.

(a) First, consider the following human capital accumulation problem. Individuals start life with a human capital stock of \( h_0 \) and invest a fraction \( i \in [0,1] \) of their time endowment learning new skills. New human capital is created according to the production function: 
\[
h_{t+1} = A h_t^\gamma i_t^\delta,
\]
where \( A \) is the learning ability of the individual and \( \gamma \) and \( \delta \) are positive parameters. Wage income at age \( t \) is given by \( h_t \varepsilon_t (1 - i_t) \) where \( \varepsilon_t \) is an i.i.d shock to the price of human capital in that period.

i. [10 points] First assume that individuals know their own \( A \). State the dynamic program corresponding to this problem.

ii. [15 points] Now consider the case where \( A \) is initially unknown but is learned over time. Specifically, individuals begin with a prior belief about \( A \), which they then update as they observe their wage income in each period. State the dynamic programming problem paying special attention to equations that specify how beliefs evolve. [Hint: You may need to transform some variables judiciously in order to make standard Kalman filtering techniques applicable.]

(b) [Multi-dimensional skills and occupational switching] Now suppose that there are \( J \) types of human capital, each produced according to their individual production function: 
\[
h_{t+1}^j = A^j (h_t^j)^\gamma (i_t^j)^\delta.
\]
Notice that this specification allows the individual to have different learning abilities in different skills. There are \( K \) occupations, each of which differ in how they value different types of human capital. Specifically, let \( p_k^j \) be the price of skill \( j \) in occupation \( k \). The wage earnings of an individual in this occupation is given by 
\[
\left( \sum_j p_k^j h_t^j \varepsilon_t^j \right) (1 - \sum_j i_t^j),
\]
where \( \varepsilon_t^j \) is the i.i.d shock to skill \( j \)’s price. Now assume (as before) that an individual begins life without perfectly knowing his abilities \( A^1, A^2, ..., A^J \).

i. [15 points] Describe in words (but as precisely as you can) the decisions of individuals in this model. Specifically, What determines how much an individual invests in one skill versus another? What determines occupational switching? How do you expect the pace of switching to change over the life cycle?
3. [25 points] During the 1970s, the US economy has witnessed diverging trends in two measures of inequality. On the one hand, the college premium (i.e., between-group inequality) fell consistently during this decade. On the other hand, residual (or within-group) inequality rose during the same period. This question asks you to analyze if these joint trends are puzzling or not in the context of a “single skill index model.” Specifically, suppose that all market skills can be summarized by a one-dimensional skill index. Some people have more of this skill whereas others have less. To simplify further, suppose that this skill index can take on only two values: H (high) or L (low) and an individual can be one of these two types. The wage rate of a type H (respectively L) individual is $W_H$ (resp. $W_L$). Education itself has no intrinsic value in such a model, but suppose that the college graduates group have a larger fraction of high skill individuals (denote this fraction by $\phi_c$) compared to the non-college group ($\phi_n$).

(a) [10 points] In this setting, define the college premium and within-group inequality in terms of the parameters just introduced.

(b) [10 points] What does this framework imply about the connection between between- and within-group inequality? Can they go in opposite direction in response to a change in the skill premium ($W_H/W_L$). How is the skill premium and the average wage rate related?

(c) [5 points] How can you modify this framework to make it consistent with the divergent trends presented above?
Block C (Victor’s)

In the following there are 10 questions for 120 points. Answer all questions. Be as precise as you can and good luck.

Human Capital Accumulation

Imagine a household with finite life where health determines utility, labor income and survival probability. The evolution of health is affected by shocks and by investments in terms of effort (the investment is a disutility) and in terms of resources.

1. (25 points) Pose a household problem that implements this environment where the household has access only to savings at interest rate $r$. Write the problem recursively and state necessary conditions for optimality of the decision.

2. (15 points) Write the problem of this household if it also has access to annuities markets.

3. (10 points) Can you say something about how the solutions compare?

Firms and workers.

In a one period model with firms and workers posting vacancies costs $c_v$ per unit and workers suffer a search cost $c_s$. Describe the equilibrium allocations and how to find them with the following search protocols

4. (10 points) Nash Bargaining.

5. (10 points) Competitive Search.

6. (10 points) Wage posting.

Imagine a couple.

She has two units of endowment while he has one. He is in love with her and if together he gets 10 utils. She is indifferent to him. Both agents have log utility and the good is private (no public good).

7. (10 points) Solve a planner problem with equal (Pareto) weights to determine what to do.

8. (10 points) Solve a bargaining problem with with equal weights to determine what to do.
9. (10 points) If she were to give a take it or leave it offer, how much would she ask for?

Imagine now that there are economies of scale for living together like those described by the OECD. Imagine also that she does not like him.

10. (10 points) How much would she have to dislike being together (in utils) in order to prefer living alone?