Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MINORS

Spring 2011

The time limit for this exam is 3 \( \frac{3}{4} \) hours. Notation:

\[ \mathbb{R} \] is the set of real numbers
\[ \mathbb{R}_+^n = \{ x \in \mathbb{R}^n : x_1 \geq 0 \& \ldots \& x_n \geq 0 \} \]
\[ \mathbb{R}_{++}^n = \{ x \in \mathbb{R}^n : x_1 > 0 \& \ldots \& x_n > 0 \} \].

For vectors \( x = (x_1, \ldots, x_n) \) and \( y = (y_1, \ldots, y_n) \) in \( \mathbb{R}^n \):

\[ x \succeq y \] means \( x_1 \geq y_1 \& \ldots \& x_n \geq y_n \).

Answer Question I.1 (required);

AND

Answer \textit{three} additional questions, at most one each from Parts II, III, and V. You can choose two questions from Part IV if you wish. The total number of questions to be answered for the exam is four.

You may use calculators to make calculations during the examination. However: in answering any question that requires you to justify your answer, if you do use a calculator, you must mention at what point in your answer you obtained results using a calculator, and what it was you were seeking to calculate.

Be sure you clearly define all \textbf{boldfaced/underlined} terms.

NOTE: This examination should have 16 pages including this one (Check to make sure!)
Part I

Answer Question I.1 from Part I.
Question I.1

Consider a two-consumer, two-good pure exchange economy. Consumer 1 has a rational, strongly monotone, strictly convex, and continuous preference relation that can be represented by the utility function:

\[ U^1(x_1, y_1) = x_1^\frac{1}{2} + y_1^\frac{1}{2} \]

for \( x_1 \geq 0 \) and \( y_1 \geq 0 \). Consumer 2 has a rational, strongly monotone, strictly convex, and continuous preference relation that can be represented by the utility function:

\[ U^2(x_2, y_2) = \left( x_2^\frac{1}{2} + y_2^\frac{1}{2} \right)^2 \]

for \( x_2 \geq 0 \) and \( y_2 \geq 0 \). Consumer 1’s initial endowments for each good are \( x_1^e = 10 \) and \( y_1^e = 10 \). Consumer 2’s initial endowments for each good are \( x_2^e = 15 \) and \( y_2^e = 6 \).

(a) Construct an Edgeworth box diagram (to scale, on the graph paper provided) showing (and labeling) the endowment allocation and typical indifference contours/sets including the directions of increasing preference for each consumer.

(b) Characterize the set of Pareto Efficient allocations and illustrate them on your diagram.

(c) Characterize the competitive (Walras) equilibrium allocation including the equilibrium prices and illustrate it on your diagram.
Part II

Answer at most one question from Part II.
Question II.1

Suppose a consumer’s rational and continuous preference relation \( \succeq \) on \( X = \mathbb{R}_+^2 \) can be represented by the utility function

\[
U(x_1, x_2) = \min\left\{ 2x_1^{\frac{1}{2}}, x_2^{\frac{1}{3}} \right\}.
\]

(a) Is the consumer’s preference relation monotone? Justify your answer while making sure to carefully define a **monotone preference relation**.

(b) Is the consumer’s preference relation convex? Justify your answer while making sure to carefully define a **convex preference relation**.

(c) Given the competitive prices \( p_1 > 0 \) and \( p_2 > 0 \) for \( x_1 \) and \( x_2 \) and income \( m \), derive the consumer’s Marshallian demands and indirect utility function.

(d) Use duality to derive the consumer’s expenditure function.

(e) Derive the consumer’s Hicksian demands from this expenditure function. How will an increase in \( p_1 \) affect the Hicksian demand for \( x_1 \)? Explain the intuition of this result given the preference relation represented by \( U(x_1, x_2) \).

Clearly define any **boldfaced/underlined** terms.
Question II.2

Consider a firm that produces a single output, \( q \geq 0 \), using two factors, \( z_1 \geq 0 \) and \( z_2 \geq 0 \) with an input requirement set that is closed, satisfies free disposal, and is strictly convex. Assume the firm operates with competitive output and factor markets. The profit function for the firm is

\[
\pi(p, r_1, r_2) = \frac{p^2}{4\alpha_1 r_1^\alpha_1 r_2^\alpha_2}
\]

where \( p > 0 \) is the price of output, \( r_1 > 0 \) and \( r_2 > 0 \) are factor prices, and \( \alpha_1 > 0 \) and \( \alpha_2 > 0 \) are constant parameters.

(a) What additional conditions, if any, on \( \alpha_1 \) and \( \alpha_2 \) are required for this profit function to satisfy the price homogeneity property for a valid profit function? Justify your answer.

(b) Derive the firm’s supply function and factor demands assuming this is a valid profit function.

(c) Use duality to derive the firm’s conditional factor demands. What are three properties that these conditional factor demands must satisfy? Note: You do not need to show that these properties are satisfied.

(d) Derive the firm’s cost function. What are four properties that this cost function must satisfy? Note: You do not need to show that these properties are satisfied.
Part III

Answer at most one question from Part III.
Question III.1

Consider an agent who consumes a single good at two dates, \( t = 0, 1 \), and has income \( m_0 > 0 \) at date 0, and \( m_1 > 0 \) at date 1. The agent’s preferences over consumption plans \((x_0, x_1)\), where \( x_0 \) is consumption at date 0 and \( x_1 \) is consumption at date 1, are represented by

\[
\ln(x_0) + \beta \ln(x_1),
\]

for \( x_0 > 0 \) and \( x_1 > 0 \). Discount factor \( \beta \) satisfies \( 0 < \beta < 1 \). The agent can save (but not borrow) using a risk-free asset with (gross) return \( \bar{r} \geq 1 \). Denote the saving by \( s \), where \( s \geq 0 \).

(a) Find the optimal consumption and saving choice \((x_0^*, x_1^*, s^*)\) as a function of \( \bar{r} \), \( \beta \), \( m_0 \) and \( m_1 \) using the Kuhn-Tucker approach. Clearly state conditions under which the optimal saving \( s^* \) is strictly positive. Justify that the solution you find is indeed optimal.

(b) Is the optimal consumption and saving choice \((x_0^*, x_1^*, s^*)\) homogeneous of degree 1 in income \((m_1, m_2)\)?
Question III.2

Consider an agent whose preferences under uncertainty have expected utility representation with von Neumann-Morgenstern utility function

\[ v(c) = \ln(c + \alpha) \]

for \( c \geq 0 \), where \( c \) denotes consumption and \( \alpha \) is a strictly positive parameter.

(a) Verify whether this agent’s **Arrow-Pratt measure of risk aversion** is increasing in the parameter \( \alpha \) or not. What does your answer imply about the risk compensation as a function of \( \alpha \)? If your answer relies on a general theorem about risk aversion, state that theorem clearly.

(b) Suppose that there are two assets: a risk-free asset with (gross) return \( \bar{r} = 1 \) and a risky asset with a return \( \tilde{r} \) that can take two values 2 and 0.5, each with probability 0.5. Find the optimal investment in the risky asset for the initial wealth \( w > 0 \) as a function of \( w \) and \( \alpha \). Justify that the solution you find is indeed optimal. Is the optimal investment an increasing function of wealth \( w \)? Is the optimal investment an increasing function of parameter \( \alpha \)?

Clearly define any **boldfaced/underlined** terms.
Part IV

Answer at most one question from Part IV.
Question IV.1

This question has two parts.

Part 1:

(a) Find all the sets that survive iterated elimination of weakly dominated strategies in the following game:

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<th>m</th>
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<th>e</th>
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<tbody>
<tr>
<td>T</td>
<td>0,0</td>
<td>1,1</td>
<td>0,0</td>
<td>-1,2</td>
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<td>M</td>
<td>3,1</td>
<td>2,4</td>
<td>-4,1</td>
<td>1,3</td>
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<tr>
<td>B</td>
<td>-1,-2</td>
<td>2,-3</td>
<td>-1,1</td>
<td>-1,4</td>
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Be sure that you consider all possible sequences of elimination.

(b) Find all the sets that survive iterated elimination of strictly dominated strategies in the same game.

Part 2: Find all the Nash equilibria of all the following games:

(1)

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<tr>
<td>T</td>
<td>-2,3</td>
<td>4,-5</td>
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<tr>
<td>B</td>
<td>-3,4</td>
<td>2,-3</td>
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(2)

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<tbody>
<tr>
<td>T</td>
<td>4,5</td>
<td>-1,4</td>
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<tr>
<td>B</td>
<td>3,5</td>
<td>3,7</td>
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(3)

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<tbody>
<tr>
<td>T</td>
<td>3,2</td>
<td>0,-1</td>
</tr>
<tr>
<td>B</td>
<td>0,0</td>
<td>5,2</td>
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</tbody>
</table>

(4)

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</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>2,5</td>
<td>1,4</td>
<td>2,0</td>
</tr>
<tr>
<td>B</td>
<td>9,8</td>
<td>3,2</td>
<td>0,-3</td>
</tr>
</tbody>
</table>
Question IV.2

This question has two parts.

Part 1:

(a) Write the extensive form for the repeated game which is repeated for two periods, with \( \delta = 1 \), and has stage game

\[
\begin{array}{c|cc}
& l & r \\
\hline
T & 3, 2 & -1, 6 \\
B & 5, 1 & 2, 3 \\
\end{array}
\]

(b) Find the subgame perfect equilibria of the repeated game. Prove your answer in detail.

Part 2: Below you will find several normal form games; these are exactly those for which you found the Nash equilibria in the past. For each of these games find:

(a) The set of Nash equilibria and Nash equilibrium payoffs for the stage game.

(b) The set \( \text{co}(F^0) \).

(c) The vector of minimax values \( (v^i)_{i \in I} \).

(d) The set of feasible and individually rational payoffs.

The games:

1. \[
\begin{array}{c|cc}
& l & r \\
\hline
T & 2, 2 & -1, -1 \\
B & 0, 4 & 0, 4 \\
\end{array}
\]

2. \[
\begin{array}{c|ccc}
& l & m & r \\
\hline
T & -1, 8 & 2, 5 & 4, 3 \\
B & 1, 2 & 2, 4 & 1, 9 \\
\end{array}
\]

3. \[
\begin{array}{c|ccc}
& l & m & r \\
\hline
T & 1, 1 & 0, 0 & 0, 0 \\
M & 0, 0 & 4, 4 & 8, 0 \\
B & 0, 10 & 9, 9 & 0, 0 \\
\end{array}
\]
Part V

Answer at most one question from Part V.
Question V.1

For a pure exchange economy with \( \ell \) goods and \( n \) consumers, indexed by \( i = 1, 2, \ldots, n \), each having consumption set \( \mathbb{R}_+^{\ell} \), initial endowment \( e_i \in \mathbb{R}_+^{\ell} \) and continuous utility function \( u_i : \mathbb{R}_+^{\ell} \to \mathbb{R} \) which is assumed to be strictly monotone and strictly concave, answer the following questions:

(a) Using the above notation, formally define the sets of weakly and strongly Pareto optimal allocations.

(b) Characterize the sets in (a) in terms of an expression involving a summation using utilities.

(c) State the first welfare theorem.

(d) Prove the first welfare theorem.

(e) Discuss the economic significance of the first and second welfare theorems.
Consider the following decision problem:

A referee (Rick) flips an unbiased coin and shows the result (heads or tails) to another person (Patty). Then you must pick heads or tails. If you pick the result of the coin flip, you win $100 and you get $0 otherwise.

(a) If you are risk neutral, what is the maximum amount of money that you would be willing to pay to participate?

(b) Assuming that you are risk neutral, how much would you be willing to pay for Patty’s (correct) information about the outcome of the coin flip (before you pick heads or tails)?

(c) How would your answers to (a) and (b) change qualitatively if you were strictly risk-averse? What if you were a risk lover? Explain briefly.

(d) Now suppose that Patty can choose to lie to you when she tells you her information. If Patty must pay herself for your winnings, what should she do? What should you be willing to pay for the information now (assuming that both you and Patty are risk neutral)?

(e) Again, suppose that Patty can lie to you but she receives exactly the same amount as your winnings. What should she do? What should you be willing to pay for the information (assuming both you and Patty are risk neutral)? What is the maximum amount you are willing to pay to participate? What is the maximum amount that Patty should be willing to pay to participate?
Question V.3

This question concerns noncooperative games in extensive form, also known as extensive form games, with finitely many players and finitely many pure strategies.

(a) Define an extensive form game without uncertainty and/or information.

(b) Define a subgame of an extensive form game.

(c) Define subgame perfect equilibrium in pure strategies.

(d) Discuss briefly the economic situations that should be analyzed using extensive form games and/or their subgame perfect equilibria.

(e) Now extend your definition in (a) to allow uncertainty and asymmetric information. What alterations and additions are needed in your definition?

(f) Define subgames of the games in (e).

(g) What assumptions are commonly made concerning information in extensive form games with uncertainty and asymmetric information? For each assumption you list, briefly define it at least informally and indicate its economic interpretation.