The exam consists of Questions I, II, and III

Question I: Diamond-Dybvig

Consider a standard Diamond-Dybvig economy with a continuum of agents in which agents have a probability of \( 0 < t < 1 \) of being impatient. There is the standard sequential service constraint, but all agents are required to show up at the bank and then individually decide whether or not to withdraw. Assume that banks are able to fully commit to the contracts they offer.

A. Suppose that when agents have to make their decision to withdraw or not they have no information other than their type. That is, they know do not know the number of agents who have showed up before them or how many have made withdrawals. Further, banks are not permitted to reveal this information to them. Is there a deposit contract that will prevent the bank run equilibrium from occurring? Explain why or why not and determine whether this contract is ex ante welfare maximizing.

B. Suppose that when agents have to make their decision to withdraw or not, they know the number of agents who have already showed up, but not how many have made withdrawals. That is, they know their place in line, but not the actions of other agents. Further, also was the case above, banks are not permitted to reveal this information to them. In this case, is there a deposit contract that will prevent the bank run equilibrium from occurring? Explain why or why not and determine whether this contract is ex ante welfare maximizing.

C. Now assume that banks cannot commit to the deposit contracts they offer. How would your answers to A and B change?
Question II: Optimal policy and the relationship between money and prices in a cash-in-advance model

Consider a representative consumer model with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c(1)_t + \ln c(2)_t \right]$$

Both consumption goods are perishable. The household is endowed with an exogenous income $y_t$ that can be transformed into consumption goods according to the technology

$$y_t = c(1)_t + c(2)_t$$

where

$$y_{t+1} = \rho y_t + (1 - \rho)\bar{y} + \varepsilon_{t+1}$$

and $\varepsilon \sim N(0, \sigma), \bar{y} > 0$. This is an economy with complete markets. However, cash is required to pay for consumption of good $c(1)_t$, so we impose the following restriction to the consumer’s problem

$$p_t c(1)_t = M_t$$

The government only issues money and nominal bonds that, if bought at time $t$, pay a gross return $R_{t+1}$ at period $t + 1$. Any profits (positive or negative) made by the government by issuing money are rebated to household in a lump-sum fashion.

1. Assume the rule for the money supply is exogenous. Define a competitive equilibrium for this economy. Characterize the equilibrium; that is, write all endogenous variables as a function of the exogenous variables.

2. Assume now that the government follows an inflation target, but that the quantity of money must be decided before observing the shock. So, let $\pi^*$ be the target. Thus, the quantity of money solves the problem

$$\min \frac{1}{2} E_t (\pi_{t+1} - \pi^*)^2$$

where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$. Note that this specification already makes explicit that monetary policy must be set before observing the shock. Define a competitive equilibrium for this economy. Characterize the equilibrium; that is, write all endogenous variables as a function of the exogenous variables.

3. Compare the relationship between money growth and inflation for the two equilibria described in questions 1 and 2.
Question III:

Consider the following money-in-the-utility function economy:

A continuum of infinitely lived households each has a constant endowment of \( y \) units of the single consumption good. Households can borrow or lend in terms of the consumption good. If a household lends one unit of the consumption good at date \( t \), it receives a payment of \( r_t \) units of the consumption good at date \( t + 1 \). (Likewise, if a household borrows a unit of the consumption good at date \( t \), it must pay \( r_t \) units of the consumption good at date \( t + 1 \).) Each household is also endowed at the beginning of time with \( M_{-1} \) units of money. (No money leaves or enters the economy after this.) Let \( p_t \) denote the price of money at date \( t \) in terms of the consumption good. There is no government (other than to enforce bond payments). Assume households face a borrowing constraint such that for all \( t \)

\[
M_{t-1} p_{t-1} + r_{t-1} b_{t-1} + \sum_{j=0}^{\infty} \frac{y}{\Pi_{s=0}^{t-1} r_{t+s}} \geq 0. 
\]  

(1)

A household’s allocation is a sequence \( \{c_t, M_t\}_{t=0}^{\infty} \) where \( c_t \) is the household’s consumption at date \( t \) and \( M_t \) is the household’s holdings of money at the end of date \( t \). A household with sequence \( \{c_t, M_t\}_{t=0}^{\infty} \) receives a payoff

\[
\sum_{t=0}^{\infty} \beta^t [\log(c_t) + \log(y + M_t p_t)].
\]  

(2)

1. Characterize all steady state equilibria of this economy. (those where \( p_t \) and \( r_t \) are constant.)

2. Characterize all other equilibria of this economy.