Ph.D. Preliminary Examination

International Trade and Payments Theory

Spring 2011

Answer ALL three questions.

Please make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly. Questions have equal weight. You have 4 hours and 30 minutes to complete the exam.
Question 1. Risk sharing and international portfolios

Consider the following no-investment version of the BKK two-country, two goods international business cycle model. In each country there is a representative consumer with standard preferences

\[
E \left[ \sum_{t=0}^{\infty} \beta^t \log (c_t) - H(l_t) \right]
\]

\[
E \left[ \sum_{t=0}^{\infty} \beta^t \log (c^*_t) - H(l^*_t) \right]
\]

where \( c_t \) and \( c^*_t \) are consumption of a country specific final good and \( l_t, l^*_t \) are labor (from now on we use * denote foreign (or country 2) variables) and \( H \) is a convex, continuous and twice differentiable function. In each country there are two technologies: one for producing intermediate goods and one for producing final goods. Country 1 can produce intermediate good \( a \) using the following Cobb Douglas technology

\[ Y_t = z_t k^\alpha l_t^{1-\alpha} \]

where \( Y_t \) is world production of good \( a \), \( k \) is a fixed amount of country specific capital and \( z_t \) is a country specific, stochastic, exogenous productivity shock. Country 2 produces intermediate good \( b \) using a similar technology

\[ Y^*_t = z^*_t k^\alpha l_t^{1-\alpha} \]

where it is assumed that the fixed capital is the same across countries. Final goods in each country are produced using a country specific Cobb-Douglas technology

\[ X_t = a^\omega b_t^{1-\omega} \]

\[ X^*_t = a^{*\omega} b_t^{*\omega} \]

\[ 1 > \omega > 1/2 \]

The final goods in each country \( (X_t \text{ and } X^*_t) \) are used for private consumption and government consumption in that country. Government consumption follows an exogenous stochastic process, \( f_t, f^*_t \) in both countries.

i) Set-up an equal weight planning problem and write down the necessary first order conditions.

ii) Define the real exchange rate, the terms of trade and solve for them as a function of \( a, a^*, b, b^* \).

iii) Let \( e_t \) be the real exchange rate. Let \( \Delta c_t = c_t - e_t c^*_t \). Show that in the planning solution involves \( \Delta c_t = 0 \) for all dates and states. Give some intuition for this condition. Is it consistent with what is observed in the data? If not briefly discuss the reasons why.
Assume now that the intermediate technologies are operated by competitive firms which own the capital stock and hire workers and similarly the final goods technologies are operated by competitive firms which buy intermediate inputs. Assume that government spending is financed each period by lump sum taxation. Finally assume that households can trade two assets, i.e. claim to dividend streams of the intermediate producing firms in both countries.

iv) Write down the budget constraints of the households and define a competitive equilibrium for this set-up

vi) Let $q_a$ and $q_b^*$ be the prices of intermediate goods relative to the final goods in the two countries. Define $\Delta f_t \equiv f_t - e_t f_t^*$ and $\Delta Y_t \equiv q_a Y_t - e_t q_b^* Y_t^*$. Show that

$$\Delta Y_t = (2\omega - 1)(\Delta f_t + \Delta c_t)$$

holds in every date and every state.

vii) Using the budget constraints of the households and the condition you showed in point v) show that there is constant portfolio holdings that guarantees that $\Delta c_t = 0$ for all dates and states and solve for that portfolio.

viii) (Harder) Show that when $\omega = 1/2$ this constant portfolio does not exist. Give the economic intuition for why this is the case.
Question 2

Consider a model with two countries. Suppose labor is the only factor of production and assume that the number of workers $N$ is the same in each country. Each worker is endowed with one unit of time.

There are two sectors, manufacturing and services. In the service sector, one unit of labor produces one unit of services. (Note service productivity is the same in both countries).

The manufacturing sector follows Eaton and Kortum (2002). There are a continuum of differentiated manufacturing goods, $j \in [0, 1]$. Let $T_i$ be the productivity of country $i$. (For the first two parts of this question, we take this as exogenous, and make it endogenous in parts three and four.) This governs the distribution of productivity draws, so that the c.d.f. of productivity $z$ in country $i$ is

$$F_i(z) = e^{-T_i z^{-\theta}}.$$

Suppose there is a CES aggregator of the differentiated manufacturing goods $j$,

$$Q = \left[ \int_0^1 q(j) \frac{z^{1-\sigma}}{\sigma} dj \right]^{\frac{\sigma}{\sigma - 1}},$$

where $\sigma < 1 + \theta$, and $Q$ is a level of the manufacturing composite, while $q(j)$ is a quantity of differentiated good.

Finally utility of manufactured good composite and services is Cobb-Douglas,

$$U = Q^\mu S^{1-\mu},$$

where $S$ is the quantity of services.

Let $\tau \geq 1$ be the iceberg cost of shipping between the two locations.

Let $N_i^S$ and $N_i^M$ be the quantity of labor working in each sector at country $i$.

Let $w_i$ be the wage at location $i$, let $P_i^M$ be the price index for the manufactured good composite at location $i$. To simplify calculations, we review some of the results of EK that you can take as given. Define $\Phi_1$ and $\Phi_2$ by

$$\Phi_1 = T_1 w_1^{-\theta} + T_2 w_2^{-\theta} \tau^{-\theta},$$
$$\Phi_2 = T_1 w_1^{-\theta} \tau^{-\theta} + T_2 w_2^{-\theta}.$$

Then $P_i^M$ equals

$$P_i^M = \gamma \Phi_i^{-1/\theta},$$
for a constant $\gamma$. Also, let $n \neq i$. Then the probability that country $i$ is the lowest cost provider to country $n$ equals

$$\pi_{ni} = \frac{T_i w_i^{-\theta} \tau^{-\theta}}{T_n w_n^{-\theta} + T_i w_i^{-\theta} \tau^{-\theta}}$$

if $n \neq i$, and the probability that $i$ is the lowest cost provider to itself is

$$\pi_{ii} = \frac{T_i w_i^{-\theta}}{T_n w_n^{-\theta} \tau^{-\theta} + T_i w_i^{-\theta}}.$$

(1) Suppose $\tau = \infty$, so that each country is in autarky. Solve for the equilibrium, and determine the price level of composite manufactured goods. 

(2) Solve for the equilibrium levels of trade flows in the general case. Under what condition (conditions) is there no trade in the service good? Under what condition (conditions) is there no trade in manufactured goods? Why are these different questions?

(3) Let’s make things more interesting by adding dynamics and an external effect in productivity. Suppose that at time $t$, $T_{i,t} = g(N_{i,t-1}^M)$, where $g(\cdot)$ is a strictly increasing function and $g(0) > 0$. Define an equilibrium. Derive equations characterizing a symmetric, stationary equilibrium, i.e. a $N^M_*$ and $N^S_*$, where $N_{i,t}^M = N^M_*$, $N_{i,t}^S = N^S_*$, for all $i$ and $t$.

(4) Continuing the additional assumptions from part (3), suppose the government of each country can level a subsidy on manufactured goods and pay for this with a tax on services. Propose one way of formally modeling this competition between governments. Set up the government’s problem of policy choice in your formulation. Discuss the economics of why we may—or may not—observe subsidies imposed, in the equilibrium outcome of your model of competition between governments.
Question 3. Dynamic Heckscher-Ohlin Model

Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_1^t c_2^t).$$

The investment good is produced according to

$$k_{t+1} = dx_1^t x_2^t.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$c_1 + x_1 = \phi_1(k_1, \ell_1) = k_1,$$
$$c_2 + x_2 = \phi_2(k_2, \ell_2) = \ell_2,$$

where

$$k_1 + k_2 = k,$$
$$\ell_1 + \ell_2 = 1.$$ 

The initial value of $k_1$ is $k_0$. All of the variables specified above are in per capita terms. There is a measure $L$ of consumer/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner’s problem for this economy. Explain how solution to this social planner’s problem is related to that of the one-sector social planner’s problem

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

s.t. $c_t + k_{t+1} = dk_t^a$
$$c_t, k_t \geq 0$$
$$k_0 = \overline{k}_0.$$

[You can write done a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]

c) Solve the one-sector social planner’s problem in part b. [Recall that the policy function for investment has the form $k_{t+1}(k_t) = Adk_t^a$ where $A$ is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of $n$ different countries, all with the same technologies and preferences, but with different constant populations, $L_i$, and with different initial capital-labor ratios $\overline{k}_0$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.
e) State and prove versions of the factor price equalization theorem, the Stolper-
Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this
particular world economy.

f) Let $s_i = c_i / y_i$ where $y_i = p_i k_i + p_{2i} = d k_i^o$ is world GDP per capita. Transform the first-order conditions for the one-sector social planner’s problem in part b into two difference equations in $k_i$ and $s_i$. Use the first-order conditions for the consumer’s problem of the equilibrium in part d to show that

$$\frac{y_i' - y_i}{y_i} = \frac{s_i}{s_{i-1}} \left( \frac{y_{i-1}' - y_{i-1}}{y_{i-1}} \right) = \frac{s_i}{s_0} \left( \frac{y_0' - y_0}{y_0} \right).$$

g) Use the solution to the one-sector social planner’s problem in part c to solve for $s_i$. Discuss the economic significance of the result that you obtain.