Ph.D. Preliminary Examination

MACROECONOMIC THEORY

Spring 2012

Majors and Minors: Answer ALL FOUR parts.

Please read the instructions before each part and make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly. You have 5 hours to complete the exam.
Part 1. Please answer the following question

**Infinitely lived consumers and dynamic programming**

Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer’s utility function is

\[
\sum_{t=0}^{\infty} \beta^t \log c_t
\]

Here \(0 < \beta < 1\). The consumer is endowed with 1 unit of labor in each period and with \(k_0\) units of capital in period 0. Feasible allocations satisfy

\[c_t + k_{t+1} \leq \theta k_t^{\alpha} \ell_t^{1-\alpha}\]

Here \(\theta > 0\) and \(0 < \alpha < 1\).

a) Formulate the problem of maximizing the representative consumer’s utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman’s equation.

b) Guess that the value function has the form \(a_0 + a_1 \log k\). Solve the dynamic programming problem.

c) Define an Arrow-Debreu equilibrium for this economy. Explain carefully how to use the solution to the dynamic programming problem in part b to calculate the Arrow-Debreu equilibrium.

d) Define a sequential markets equilibrium for this economy. Explain carefully how to use the solution to the dynamic programming problem in part b to calculate the sequential markets equilibrium.

e) Suppose now that there are equal amounts of two types of consumers in the economy. The two types of consumers have the same discount factor \(\beta\). They have different utility functions in each period, \(\log(c - c^1)\) and \(\log(c - c^2)\), \(c^1 \neq c^2, c^1 \geq 0, c^2 \geq 0\), different endowments of labor in each period, \(\ell^1\) and \(\ell^2\), and different initial endowments of capital, \(k^1_0\) and \(k^2_0\). Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

f) Does the equilibrium allocation for the economy in part e solve a dynamic programming problem like that in part a? Carefully explain why or why not. If it does solve such a problem, write down the appropriate Bellman’s equation.
Part 2. Please answer the following question

Consumption/Savings with Production Risk

Consider an economy with a continuum of consumers/entrepreneurs of measure 1, indexed by $i$.

Each consumer has access to a production technology that uses capital and labor to produce a homogeneous good. This production technology is specific to the entrepreneur who owns the technology. Letting $k_{it}$ denote the amount of capital entrepreneur $i$ has at date $t$, the production function is

$$ y_{it} = z_{it} f(k_{it}, n_{it}) $$

where $z_{it}$ is a random variable that is iid over time and across consumers, $f$ is a constant returns to scale function and $n_{it}$ denotes the amount of labor hired by entrepreneur $i$. The economy has no aggregate uncertainty. Capital fully depreciates after one period. Entrepreneurs can invest their capital only in their own production technology and cannot rent/sell it to other consumers. In addition to capital, entrepreneurs trade among themselves one-period risk-free bonds. Let $r_t$ denote the interest rate on bonds purchased at date $t$. The profits generated by entrepreneur $i$ are given by

$$ \pi_i(k_i, z_i; w) = \max_{n_i} z_i f(k_i, n_i) - w n_i $$

The consumer/entrepreneur’s problem is to maximize life-time utility

$$ \max_{c_{jt},k_{jt+1},b_{jt+1}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_{jt}) $$

subject to

$$ c_{jt} + k_{jt+1} + b_{jt+1} = \pi_i(k_{jt}, z_{jt}; w_t) + w_t + (1 + r_{t-1}) b_{jt} $$

where, note that the endowment of labor of each entrepreneur is normalized to be 1. Assume $u(c) = c^{1-\gamma} / (1-\gamma)$ with $\gamma > 1$. Let $\lambda^*$ denote the initial distribution of assets $(k_{i0}, b_{i0})$ across consumers. The market clearing condition for bonds is $\int b_{it} d\lambda = 0$ for all $t$. There is no constraint (other than the natural debt limit) on how much the consumer can borrow using the risk-free bond. We require however $k_{jt+1} \geq 0$.

1. Write the consumer’s problem recursively. Define a recursive competitive equilibrium.

2. Characterize the consumer’s decision rules in the stationary steady-state: derive expressions relating the consumption and portfolio decisions to primitives of the
model. Show that these consumption rules aggregate into an expression for aggregate consumption as a function of the aggregate capital stock. Use this aggregation result to characterize the steady state level of the capital stock. (Hint: You may find it useful to let human wealth be defined as the present discounted value of wages.)

3. Does $\beta (1 + r) = 1$ in this economy?
Part 3. Please answer the following question

Permanent Income Consumers in a Small Open Economy

Consider a small open economy which face a world interest rate $r$ and is inhabited by a continuum of measure 1 of infinitely lived consumers. In each period each consumer can have either high ($y_t = y_h$) or low income ($y_t = y_l$). Given the income in period $t$, the probability that income stays the same in period $t + 1$ is given by $p$. Consumers can only trade a non contingent bond which pays a fixed interest rate $r$, so that each consumer solves the following problem

$$\max \left\{ \{c_t, a_{t+1}\} \right\}_{t=0}^\infty E_0 \sum_{t=0}^\infty \beta^t u(c_t)$$

s.t.

$$c_t + a_{t+1} = (1 + r) a_t + y_t$$

$$a_0 = 0, (1 + r) \beta = 1, \text{ No Ponzi}$$

Suppose that in period 0 half of the consumers start with $y_0 = y_h$ and the remaining half have $y_0 = y_l$

1. Compute GDP in this economy and show that it is constant through time, 5p

2. Define permanent income of a consumer

$$P(y_t) = \frac{r}{1 + r} y_t + \frac{r}{1 + r} E_t \sum_{j=1}^\infty \left( \frac{1}{1 + r} \right)^j y_{t+j}$$

Guess that $P(y_h) = \alpha y_h + (1 - \alpha) y_l$, $P(y_l) = \alpha y_l + (1 - \alpha) y_h$ and solve for $\alpha$. (Hint: use the recursive representation of $P(y_h)$ and $P(y_l)$), 15p

3. Solve for the consumption function of consumers i.e. for consumption as a function of consumer’s wealth $a_t$ and current income $y_t$, 10p

4. Show that if a consumers gets the following sequence of income realization $(y_0 = y_l, y_1 = y_l, y_2 = y_l, ...)$, her consumption will fall through the first three period, despite her income being constant, 10p

5. Solve for the distribution of wealth changes in the economy, 10p

6. Show that net exports in this economy are constant through time, 10p

7. Solve for rate of change (over time) of the variance of the cross sectional distribution of income, wealth and consumption, 20p
8. Assume that in an arbitrary period $t$ the parameter $p$, unexpectedly, increases, i.e. the process for income becomes more persistent. Show the following two properties

1. Inequality in consumption (as captured by the variance of consumption) increases in the short run but it falls in the long run (relative to the case in which $p$ does not change). 10p

2. Inequality in wealth (as captured by the cross-sectional variance of wealth) falls (relative to the case in which $p$ does not change). 10p
Growth Models

Consider an economy with two types of people indexed by \( i \in \{a, b\} \) (arguers and bores). There is a continuum of each type of equal measure and they are both infinitely lived.

What makes a type is both preferences and technology. The types are not perfect substitutes in production. There is an aggregate production function \( f(z, K, H^a, H^b) \), where shocks follow a Markov chain with transition \( \Gamma_{z, z'} \), \( K \) is aggregate capital and \( H^a \) and \( H^b \) are total hours of each type worked. There is a maximum sustainable capital stock \( K \).

In addition, type \( a \) households like it more when other agents of their type are not working so that they can spend time together, and, you guessed it, argue. The utility of bores is \( u^b(c, h) \), increasing in the first and decreasing in the second argument, while the utility of arguers is \( u^a(c, H^b, h) \), increasing in the first and decreasing in the other two arguments (we are using the convention of big letters for aggregate variables and small letters for individual variables). Preferences are given by the expected discounted value (both types have the same discount factor, \( \beta \)).

1. (10 points) Set the value of the shock to its unconditional mean \( \bar{z} \), and then define what the steady state of this economy is. State the conditions that have to be satisfied in the steady state.

2. (5 points) Is the steady state unique?

3. (5 points) Is the Arrow Debreu equilibrium optimal? Make the case.

4. (15 points) Define Recursive Competitive Equilibrium. Make sure that you not only define the required objects but also state the conditions that such objects must satisfy.

5. (10 points) Will there be state contingent trades in equilibrium?

Industry Equilibria and Search

There is measure one of households that can become workers and own shares on a mutual fund. They have standard discounted expected utility over consumption
with a period utility function that is CRRA. There are also a measure one of firms owned by the workers that have idiosyncratic productivity $s \in S$ with transition $\Gamma$ and strictly concave production function $f$. Firms are matched to workers. Every period a fraction $\delta$ of the jobs is destroyed. Firms can post vacancies at a cost $\kappa$. There is a Cobb-Douglas matching function

$$M = A(V)^5 (1 - N)^5.$$  

where $V$ is the number of matches posted and $N$ is the measure of employed agents. There is no entry or exit of firms.

6. (10 points) Imagine that wages are exogenously determined at $\bar{w}$. Define the problem of a firm in a stationary equilibrium.

7. (10 points) Write the problem of the household (hint: it is like an Aiyagari household).

8. (10 points) Define stationary equilibrium.

9. (10 points) Imagine that firms bargained unilaterally with each worker their wage. Would it depend on the wealth of the worker? Write the problem that the bargaining process solves.

**Monopolistic Competition**

Imagine that preferences of a representative consumer in a static closed economy are given by

$$u\left(\{c(i)\}_{i \in [0, A]}, n\right) = \left(\int_{0}^{A} c(i)^{\gamma} \, di\right)^{\theta/\gamma} - \chi \frac{n^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}}$$

Where $1 - n$ is leisure and $n$ is time spent working. Output is produced with one unit of labor that is taken to be the numeraire.

10. (10 points) Give an expression for the price that each firm charges, as a function of the income of the consumer.