Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MAJORS

Spring 2012

The time limit for this exam is $3\frac{1}{4}$ hours.

Answer one question from each part, for a total of four questions.

You may use calculators to make calculations during the examination. However: in answering any question that requires you to justify your answer, if you do use a calculator, you must mention at what point in your answer you obtained results using a calculator, and what it was you were seeking to calculate.

Be sure you clearly define all **boldfaced/underlined** terms. Also, please be sure to define precisely any notation that you introduce.

Note: This examination should have 13 pages including this one (Check to make sure!)
Part I

Answer one question from Part I.
Question I.1

Let \( u: \mathbb{IR}^L_+ \rightarrow \mathbb{IR} \) be a continuous and strictly increasing utility function. Let \( u^*(p, M) \) be the indirect utility function derived from the maximization of utility \( u \) subject to the budget constraint at prices \( p \in \mathbb{IR}^L_+ \) and income \( M > 0 \).

(a) Show that if \( u \) is concave, then \( u^* \) is a concave function of income \( M \).

(b) Show that if \( u \) is strictly concave, then Walrasian demand \( x^*(p, M) \) is single-valued for every \( (p, M) \).

(c) Suppose that \( u \) is quasi-linear of the form

\[
u(x) = x_1 + v(x_2, \ldots, x_L),\]

for every \( x \in \mathbb{IR}^L_+ \). Function \( v: \mathbb{IR}^{L-1}_+ \rightarrow \mathbb{IR} \) is assumed continuous, strictly increasing, and strictly concave. Show that indirect utility \( u^* \) is linear in \( M \) on the domain of price-income pairs \( (p, M) \) where the Walrasian demand is interior.

If your proof relies on differentiability of the demand function and/or the indirect utility function, you should clearly state additional assumptions pertaining to differentiability of function \( v \) that are sufficient for these desired properties of demand and indirect utility, and justify sufficiency of your assumptions.
Question I.2

Consider an agent whose preferences over state-contingent consumption plans \( c \in IR^S_+ \) have state-separable representation of the form

\[
U(c) = \sum_{s=1}^{s} u_s(c_s),
\]

where \( S \) is a finite number of states of nature. Functions \( u_s : IR_+ \to IR \) are assumed strictly increasing and differentiable for every \( s \). Let \( \pi \) be a probability measure on states such that \( \pi_s > 0 \) for every \( s \).

(a) Show that if the agent is risk averse with respect to probability measure \( \pi \), then utility function \( U \) must have an expected utility representation under \( \pi \). That is, the preference relation induced by \( U \) on \( IR^S_+ \) must have an expected utility representation.

(b) Show that the von Neumann-Morgenstern utility function of the expected utility representation in part (a) must be concave. [You may use the Theorem of Pratt in your proof, but you need to state it clearly and explain how it implies the claim.]
Part II

Answer one question from Part II
Question II.1

It is frequently stated that convexity matters for one of the welfare theorems but not the other. This question asks you to explore this claim in more detail, in the context of a pure exchange economy with $\ell$ commodities and $n$ consumers (indexed by $i = 1, 2, \ldots, n$), each having preferences $\preceq_i$ defined on $\mathbb{R}^\ell_+$ which are assumed throughout to be continuous complete preorders, and initial endowment vectors $e_i \in \mathbb{R}^\ell_+$.

(a) For a complete preorder $\preceq_i$, define continuity.

(b) State the Second Welfare Theorem.

(c) State the First Welfare Theorem and give a proof.

Now suppose for the remainder of this question that consumers’ feasible consumption sets are $X_i \subseteq \mathbb{R}^\ell_+$.

(d) Does your statement of the Second Welfare Theorem require any additional assumptions? If so, state them and explain briefly why they are needed.

(e) Does your statement of the First Welfare Theorem require any additional assumptions? If so, state them, explain intuitively why they are needed, and indicate where the new assumptions would be used in your proof in part (c).

(f) Discuss. Is it true that one of the welfare theorems does not require convexity?
Question II.2

Local television news media have recently reported that Delta Air Lines has been offering tickets on the same flights and same days at different prices to different customers, in particular showing higher prices sometimes to people that Delta knows have frequent flyer accounts and travel often (because this happened to people who are logged into their accounts when purchasing tickets and fly at least 50,000 miles per year credited to Delta). This question asks you to think about how one might analyze this in a general equilibrium model.

(a) How would you specify any systematic differences (what might be such differences, if any?) between frequent flyers and other consumers in your model?

(b) How would you model the idea of different prices for different customers formally? Be precise about the economic environment and any relevant assumptions in a formal general equilibrium model. To simplify, you may decide to neglect production and use a pure exchange economy since in the short run, Delta has already decided what products (routes, seats, meals, etc.) to produce and how much to produce.

(c) Discuss the implications for the existence of competitive equilibrium and both welfare theorems. Identify precisely which, if any, of the standard assumptions would not be valid in this situation. Define equilibrium and Pareto optimal allocations.

(d) Now suppose that instead of systematic price differences based on frequent flyer status, Delta is simply experimenting with pure randomized prices. How would you model this?

(e) Again for the case of random prices not correlated with frequent flyer status, what would be the implications for the existence of equilibrium and the welfare theorems? Define equilibrium and Pareto optimality in your basic model.
Part III

Answer one question from Part III
Question III.1

(a) Define a sequential equilibrium of an extensive form game.

(b) Provide a proof of existence of sequential equilibria. A sketch will illustrate the main idea, and explain which the main technical difficulties would be and how you would deal with those.

(c) Show that for finite extensive form games, if a behavioral strategy $\tau$ is completely mixed, then

- Every information set is reached with positive probability;
- The assessment $(\mu, \tau)$ is consistent if and only if $\mu$ can be derived by Bayes’ rule.
- Is every sequential equilibrium trembling hand perfect? Prove or disprove.
Question III.2

(a) Define Iterated Elimination of Strictly Dominated Strategies (IESDS), and Iterated Elimination of Weakly Dominated Strategies (IEWDS);

(b) Prove or disprove (with an example): all IEWDS procedures yields a unique outcome;

(c) Prove or disprove (with an example): IESDS cannot eliminate a Nash Equilibrium;

(d) Prove or disprove (with an example): IEWDS cannot eliminate a Nash Equilibrium.
Part IV

Answer one question from Part IV
Question IV.1

An auctioneer is auctioning an indivisible object among two symmetric bidders. Both bidders have quasilinear preference, and each bidder’s valuation of the object, $t_i$, is drawn independently from a uniform distribution over $[0, 1]$, and is known only to bidder $i$ himself. The auctioneer considers using one of the following two auctions:

Auction 1: Bidders simultaneously submit nonnegative bids. The bidder who submits the highest bid wins the object. If both bidders submit the same bid, winner is determined by a fair coin. A bidder bidding $b \geq 0$ has to pay $b$ if he wins, and has to pay $b^2$ even if he loses.

Auction 2: Bidders simultaneously submit nonnegative bids. The bidder who submits the highest bid wins the object. If both bidders submit the same bid, winner is determined by a fair coin. A bidder bidding $b \geq 0$ has to pay $b$ if he wins, and will receive $b$ from the auctioneer if he loses.

For each of these auctions, determine whether there is an efficient Bayesian Nash equilibrium (BNE) (that is, a BNE where the winner of the object is always a bidder with the highest valuation). If your answer is yes, explicitly construct one (you have to show carefully that it indeed is an efficient BNE), and calculate the auctioneer’s expected revenue in that equilibrium. If your answer is no, explain why this is so.

If your answer is yes for both auctions, does the auctioneer collect the same expected revenue from the two auctions?
Question IV.2

Let $\Gamma$ be a finite set of alternatives, with $|\Gamma| \geq 3$. Let $E$ be a finite set of individuals, with $|E| = n$. Individual preferences are represented by a binary relation on $\Gamma$ which is complete, anti-symmetric, and transitive. Let $P_i$ denote such a binary relation for individual $i$, and $xP_iy$ means $i$ prefers $x$ to $y$. Let $\Sigma$ denote the set of all such binary relations on $\Gamma$. For a given $P_i$, let $xR_iy$ mean either $xP_iy$ or $x = y$. A profile of preferences is a vector $P = (P_1, \ldots, P_n) \in \Sigma^n$. A social choice function (SCF) is a mapping $f : \Sigma^n \rightarrow \Gamma$. An SCF is dictatorial if there exists $i \in E$ such that for any $x \in \Gamma$ and any $P \in \Sigma^n$, $f(P)R_i x$. A mechanism is a pair $M = (S, g)$ where $S = S_1 \times \cdots \times S_n$ and $g : S \rightarrow \Gamma$. For any mechanism $M$ and any preference profile $P \in \Sigma^n$, the pair $G = (M, P)$ constitutes a complete-information game. For a given $G = (M, P)$, a strong equilibrium is a profile of actions $s \in S$ such that for any coalition $C \subset E$ and any $\hat{s}_C = (\hat{s}_i)_{i \in C} \in \times_{i \in C} S_i$, there exists $i \in C$ such that $g(s)R_i g(\hat{s}_C, s_{-C})$, where $s_{-C} = (s_j)_{j \notin C}$. Let $SE(G)$ denote the set of all strong equilibria of $G$. An SCF $f$ is fully implementable in strong equilibrium if there exists $M = (S, g)$ such that for any $P \in \Sigma^n$, $g(SE(M, P)) = \{f(P)\}$.

Prove that, if an SCF $f$ is such that $f(\Sigma^n) = \Gamma$, then $f$ is fully implementable in strong equilibrium only if it is dictatorial.