Ph.D. Preliminary Examination

International Trade and Payments Theory

Spring 2012

Answer ALL four questions.

Please make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly. Questions have equal weight.

You have 5 hours to complete the exam.
Consider a two-sector growth model in which the representative consumer has the utility function
\[ \sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}). \]
The investment good is produced according to
\[ k_{t+1} = d x_{1t}^{a_1} x_{2t}^{a_2}. \]
Feasible consumption/investment plans satisfy the feasibility constraints
\[ c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}, \]
\[ c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}, \]
where
\[ k_{1t} + k_{2t} = k_t \]
\[ \ell_{1t} + \ell_{2t} = 1. \]
The initial value of \( k_t \) is \( \bar{k}_0 \). All of the variables specified above are in per capita terms. There is a measure \( L \) of consumers/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner’s problem for this economy. Explain how solution to this social planner’s problem is related to that of the one-sector social planner’s problem
\[ \sum_{t=0}^{\infty} \beta^t \log c_t \]
s.t. \( c_t + k_{t+1} = d k_t^{a_1} \)
\[ c_t, k_t \geq 0 \]
\[ k_o = \bar{k}_0. \]
[You can write down a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]
c) Solve the one-sector social planner’s problem in part (b). [Recall that the policy function for investment has the form \( k_{t+1}(k_t) = Adk^{a_1} \) where \( A \) is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of \( n \) different countries, all with the same technologies and preferences, but with different constant populations, \( L_i \), and with different initial capital-labor ratios \( \bar{k}_{i0} \). Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

e) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

f) Let \( s_t = c_t/y_t \) where \( y_t = p_{1t}k_t + p_{2t} = dk^{a_1}_t \) is world GDP per capita. Transform the first-order conditions for the one-sector social planner’s problem in part (b) into two difference equations in \( k_t \) and \( s_t \). Use the first-order conditions for the consumer’s problem of the equilibrium in part (d) to show that

\[
\frac{y_t^i - y_t}{y_t} = s_t \frac{y_t^i - y_{t-1}}{y_{t-1}} = s_t \left( \frac{y_0^i - y_0}{y_0} \right).
\]

g) Use the solution to the one-sector social planner’s problem in part (c) to solve for \( s_t \). Discuss the economic significance of the results that you obtain.
**Question 2:**

We start by reviewing a standard Eaton Kortum model, only with four sectors. We then tweak things to make a prelim question out of it.

*Four-Sector Eaton-Kortum*

Consider a model with two countries. Suppose labor is the only factor of production and assume that the number of workers \( N \) is the same in each country. Each worker is endowed with one unit of time.

There are four sectors, indexed by \( k \in \{1, 2, 3, 4\} \). Each sector follows Eaton and Kortum (2002). In sector sector \( T_k \) be the productivity parameter of country \( i \) in segment \( k \). This governs the distribution of productivity draws, so the c.d.f. of productivity \( z \) in country \( i \) and sector \( k \) is

\[
F_k^i(z) = e^{-T_k^i z^{1-\theta}}.
\]

Suppose composite sector \( k \) good is CES,

\[
Q_k = \left[ \int_0^1 q_k(j)^{\sigma-1} \frac{1}{\sigma} \, dj \right]^\frac{\sigma}{\sigma-1},
\]

where \( \sigma < 1 + \theta \), where \( q_k(j) \) is a quantity of differentiated good in sector \( k \).

Utility of function for the composite goods is Cobb-Douglas, with spending shares of \( \frac{1}{4} \) on each of the four sector goods.

Let \( \tau \geq 1 \) be the iceberg cost of shipping between the two locations.

Let \( w_i \) be the wage at location \( i \), and let \( P_k^i \) be the price index for sector \( k \) in country \( i \).

To simplify calculations, we review some of the results of EK that you can take as given. Define \( \Phi_1^k \) and \( \Phi_2^k \) by

\[
\Phi_1^k = T_1^i w_1^{-\theta} + T_2^i w_2^{-\theta} \tau^{-\theta},
\]

\[
\Phi_2^k = T_1^k w_1^{-\theta} \tau^{-\theta} + T_2^k w_2^{-\theta}.
\]

Then \( P_k^i \) equals

\[
P_k^i = \gamma \Phi_i^{-1/\theta},
\]

for a constant \( \gamma \). Also, let \( n \neq i \). Then the probability that country \( i \) is the lowest cost
provider to country $n$ in sector $k$ equals

$$
\pi_{ni}^k = \frac{T_i^k w_i^{-\theta} \tau^{-\theta}}{T_n^k w_n^{-\theta} + T_i^k w_i^{-\theta} \tau^{-\theta}},
$$

if $n \neq i$, and the probability that $i$ is the lowest cost provider to itself is

$$
\pi_{ii}^k = \frac{T_i^k w_i^{-\theta}}{T_n^k w_n^{-\theta} \tau^{-\theta} + T_i^k w_i^{-\theta}}.
$$

Finally, the overall price index in country $i$ is

$$
P_{overall}^i = \omega \prod_{k=1}^{4} \left( P_i^k \right)^{-\frac{1}{4}},
$$

for a constant $\omega$.

**Turning it into a Question**

First, let’s simplify by assuming the technology parameter is constant for each $k$ and $i$, i.e. $T_i^k = T$. But now let’s tweak things by adding a policy parameter that imposes a labor market inefficiency. Suppose that for sector 1 in country 1, there is a rule that for every one worker hired, another $\alpha$ workers must be employed to stand around and do nothing. That is, you need $(1 + \alpha)N_1$ workers to do what $N_1$ workers can do without the policy. Analogously, in country 2 and sector 2 there is the same rule with the same parameter $\alpha$.

There are no labor market efficiencies in sectors 3 or 4 in each country. In summary, country 1 has a labor market efficiency in sector 1, and country 2 has one in sector 2, and that’s it.

a) Solve for the equilibrium trade flows in each sector between country 1 and country 2. Explain how the within-sector and net across-sector trade flows depend upon $\alpha$, $\tau$ and $\theta$.

b) How does employment in sector 1 in country 1 vary with $\alpha$?

c) Suppose the iceberg trade cost is initially $\tau^0$, but is reduced to $\tau' < \tau^0$. Derive a formula for the welfare gain from the lower trade cost. Explain why the welfare gain depends on the policy parameter $\alpha$.

d) Suppose $\tau = 1$. Under what limiting case of the model is welfare independent of $\alpha$?
Question 3: Forward Premium Anomaly

In the data high interest rate currencies tend to appreciate so that

\[ \text{cov}(i_t - i^*_t, E_t \log e_{t+1} - e_t) \leq 0 \]

where \( i \) and \( i^* \) are home and foreign nominal interest rates and \( e \) is the nominal exchange rate.

a) Develop an economy which could generate this negative covariance.

b) Explain as carefully as you can what features are necessary to generate it. Can the money supply have a unit root? Can it be i.i.d? Can we get it with log-linear approximations to the first order conditions of some economy?

c) Suppose that consumption and inflation are normally distributed around some mean with constant conditional variances. Can this economy generate this negative covariance.

d) What would this covariance be in a simple monetary model with homogenous consumers and in which money is neutral (as it is in the original Lucas two currency paper)?
Consider a one good small open economy inhabited by an infinitely lived representative consumer with preferences given by the standard
\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{c^{1-\sigma}_t}{1-\sigma} \]
Let \( b_t \) be the quantity of one period bonds, maturing at \( t \), which the consumer trades internationally at a given world interest rate of \( r \), with \( \beta(1+r) = 1 \) and \( b_0 = 0 \). In each period consumer gets a deterministic endowment of the good, denoted by \( y_t \), given by
\[ y_0 = \bar{y}, y_1 = y_h, y_t = y_l \text{ for } t \geq 2 \]
with \( y_h > \bar{y} > y_l \)
a) Write down the consumers’ problem. 5p
b) Show that equilibrium consumption is constant and equal to
\[ c = \alpha_0 \bar{y} + \alpha_1 y_h + (1 - \alpha_0 - \alpha_1) y_l \]
and solve for \( \alpha_0 \) and \( \alpha_1 \) as a function of \( r \). 10p
c) Define, in terms of variables in the model, trade balance, current account and net foreign asset position and solve for equilibrium paths of these variables, as function of equilibrium consumption and endowments. 5p
d) Show that, for any \( r \), there exist values for \( y_0, y_1 \) and \( y_2 \) such that \( c_0 > \bar{y}, c_1 < y_h, c_t > y_l \) for \( t \geq 2 \), i.e. the consumer runs a trade deficit in period 0, trade surplus in period 1 and trade deficit from period 2 on. From now on assume that endowment values generate this pattern. 5p
e) Assume now that the representative consumer can, at the beginning of each period, default on foreign bonds. Assume that if default happens the consumer is excluded from future international borrowing but not from saving. Write the consumer’s problem in this case and solve for equilibrium consumption path in this case. 10p
f) Assume now that if the consumer defaults she is excluded from international borrowing and saving. Show that, in general, equilibrium path of consumption in this case is different from the one you solved for in point e. Explain why this is the case. 10p
g) Show that if \( \sigma \) is large enough exclusion from international borrowing and saving might be enough to sustain equilibrium constant consumption. Give some intuition for this result 5p