Ph.D. Preliminary Examination

MACROECONOMIC THEORY

Spring 2013

Majors and Minors : Answer ALL FOUR parts.

Please read the instructions before each part and make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly.

You have 5 hours to complete the exam.
Part 1. Please answer the following question.

Overlapping Generations and Pareto Efficiency

Consider an overlapping generations economy in which the representative consumer born in period \( t, t = 1, 2, \ldots \), has the utility function over consumption of the single good in periods \( t \) and \( t + 1 \)

\[
u(c_t, c_{t+1}) = \log c_t + \log c_{t+1}
\]

and endowments \((w_t, w_{t+1}) = (w_1, w_2)\). Suppose that the representative consumer in the initial old generation has the utility function

\[
u^0(c_1^0) = \log c_1^0
\]

and endowment \( w_1^0 = w_2 \) of the good in period 1 and endowment \( m \) of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that \( m = 0 \). Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that \( w_2 < w_1 \). Is the equilibrium allocation in part (c) Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two goods in each period. The representative consumer born in period \( t, t = 1, 2, \ldots \), has the utility function over consumption of the two goods in periods \( t \) and \( t + 1 \)

\[
u(c_{1t}, c_{2t}, c_{1t+1}, c_{2t+1}) = \log c_{1t} + \log c_{2t} + \log c_{1t+1} + \log c_{2t+1}
\]

and endowments \((w_{1t}, w_{2t}, w_{1t+1}, w_{2t+1}) = (4,2,1,2)\). Suppose that the representative consumer in the initial old generation has the utility function

\[
u^0(c_{11}^0, c_{21}^0) = \log c_{11}^0 + \log c_{21}^0
\]

and endowment \((w_{11}^0, w_{21}^0) = (1,2)\) of the good in period 1 and endowment \( m \) of fiat money. Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

(f) Is the equilibrium allocation in part (e) autarky? Explain carefully why or why not.
Part 2. Please answer the following question.

This problem asks you to work with the version of the neoclassical growth model with human capital and exogenous labor supply.

Consider the following Planner’s Problem:

$$\max \quad \sum \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to:

(i) \( c_t + x_{kt} + x_{ht} \leq y_t = Ak_t^\alpha z_t^{1-\alpha} \),
(ii) \( k_{t+1} \leq (1-\delta_k)k_t + x_{kt} \),
(iii) \( h_{t+1} \leq (1-\delta_h)h_t + x_{ht} \),
(iv) \( z_t \leq n_th_t \)
(v) \( 0 \leq \ell_t + n_t \leq 1 \),
(vi) \( h_0, k_0 \) given.

(a) Show that the value function for this problem is homogeneous of degree \((1-\sigma)\) in the initial capital stocks, \((h_0, k_0)\).

(b) Characterize the homogeneity properties of the optimal decision rules in initial capital stocks, \((h_0, k_0)\). Specifically if the initial conditions on the capital stocks are \((\eta h_0, \eta k_0)\) instead of \((h_0, k_0)\) where \(\eta > 0\), how are the optimal time paths for consumption, labor supply, investment, etc., affected? Prove your claims.

Suppose now that the government has a fixed sequence of expenditures that it must finance, \(g_t\), and that it can use taxes on capital and labor income, \(\tau_{kt}\) and \(\tau_{zt}\).

(c) Define a competitive equilibrium for this environment.

(d) What is the Ramsey problem here for a benevolent government? In particular, carefully derive and explain the implementability constraint for this environment.

(e) Assume that \(\delta_k = \delta_h\). What is \(\frac{\tau_{kt}}{\tau_{zt}}\) in this case?

(f) Assume that \(\delta_k = \delta_h\). What can you say about \(\lim_t \tau_{kt}\)? About \(\lim_t \tau_{zt}\)?
Part 3. Please answer the following question.

A Cash-Credit Goods Model with Private Money

Consider a cash credit goods in which households have preferences of the form \( \sum_{t=0}^{\infty} \beta^t (\log c_{1t} + \phi \log c_{2t}) \), where \( c_{1t} \) and \( c_{2t} \) denote consumption of cash and credit goods respectively, \( \phi \) is a parameter, and \( 0 < \beta < 1 \) is the discount factor. Households supply labor inelastically. The resource constraint is given by

\[
c_{1t} + c_{2t} + k_{t+1} = k_t^\alpha + (1 - \delta)k_t
\]

where \( k_t \) denotes the capital stock, the capital share parameter \( \alpha \) and the depreciation rate \( \delta \) are both positive and less than 1.

It is convenient to let \( q_t \) denote the price of money (or the inverse of the price level). The representative household’s budget constraint is given by

\[
q_t M_{t+1} + c_{1t} + c_{2t} + k_{t+1} \leq q_t M_t + R_k k_t + w_t l_t + q_t T_t
\]

where \( M_t \) denotes cash balances, \( R_k k_t \) denotes the return to capital and \( T_t \) denotes lump sum transfers of cash by the government. The household can use fraction \( \theta \) of its capital stock to purchase cash goods. The cash in advance constraint is

\[
c_{1t} \leq q_t M_t + \theta k_t.
\]

(a) Define a competitive equilibrium.

(b) Assume that the aggregate stock of money grows at a constant rate, \( \pi \). Characterize the steady state.

(c) Show that if \( \pi \) is above a critical threshold, the economy has a nonmonetary steady state in which the price of money is zero.

(d) Show that below the critical threshold, comparing steady states, the capital output ratio rises as the growth rate of money rises.
Part 4. Stuff related to the Fourth Mini.

In the following there are 10 questions for 120 points. Answer questions for a total value of 100 points. Be as BRIEF as you can and good luck.

Growth Models and Vacations

Consider an economy with two types of agents indexed by $i \in \{A, B\}$ that differ in preferences and wealth and that have equal measure. There are two producible goods in the economy, a standard good $c$, and another good $h$ that we refer to as hotels. This other type of goods $h$ are peculiar in the sense that they need time to be enjoyed. Hotels only give utility if combined with time in equal amounts (the units are such that what matters is the minimum of the time used in enjoying $h$, and $h$ itself) and we denote the composite, $d_t$, vacation. Leisure, this, time that is used for nothing else also yields utility and we denote it $\ell_t$. Clearly leisure is different from vacation.

Type A agents are moody and have per period preferences given by $u^A(c_t, \theta_t d_t, \ell_t)$ where $\theta_t$ is a shock with transition $\Gamma$. Type B agents are not moody and do not care about vacations either.

Goods $h_t$ and $c_t$ are perfect substitutes in production, as is the investment good, and they are all produced with a CRS technology $F(K, N)$ where $K$ and $N$ are aggregate capital and labor respectively. Capital depreciates at rate $\delta$, has to be installed one period in advance, and it can be reallocated freely across sectors within a period.

1. (10 points) Define the set of feasible allocations and a social planner problem.

2. (20 points) Define Recursive Competitive Equilibrium with complete markets. Make sure that you not only define the required objects but also state the conditions that such objects must satisfy. Will there be state contingent trades in equilibrium?

3. (15 points) Define now a Recursive Competitive Equilibrium without state contingent markets. Does the price of good $h$ in terms of the numeraire $c$ change across realizations of the shock?

4. (10 points) Briefly discuss what are the likely implications of a high $\theta$ realization relative to a low one, all other things equal.

Monopolistic Competition

Imagine that preferences of a representative consumer in a static closed economy are given by

$$u\left(\{c(i)\}_{i \in [0, A]}, n\right) = \left(\int_0^A c(i)^{\gamma}di\right)^{\theta/\gamma} - \chi n^2$$
where 1 − n is leisure and n is time spent working. Output is produced with one unit of labor that is taken to be the numeraire.

5. (10 points) Give an expression for the price that each firm charges, as a function of the income of the consumer.

**Lucas Shopping Trees and Idiosyncratic Endowments**

Imagine an economy with a continuum of households that cannot borrow or issue state contingent assets but can own shares of a mutual funds that owns a measure 1 of Lucas trees each one of them with a unit of output each period. Households have a random endowment of a good that can only be consumed by the household itself, they cannot be traded or sold. Denote it b, it follows a Markov process with transtion matrix \( \Gamma \). The household has utility \( u(c, d) \) where \( d \) is search effort for and \( c \) is consumption.

There are two markets for consumption goods in this economy. One half of the trees are allocated to market 1 and are obliged to sell the goods at price \( p_1 \) determined by the government. The other half of the trees are allocated to market two and are free to charge \( p_2 \). Both prices are in terms of shares of the mutual fund. There is a matching function \( M(T_i, D_i) \) where \( T_i \) is the measure of trees in market \( i \) and \( D_i \) is the aggregate search effort in market \( i \).

6. (20 points) Write down the problem of the household including state variables.

7. (5 points) Will households search in one or in both markets?

8. (10 points) Write down a formula for the value of trees in each market.

9. (10 points) Define briefly a steady state with special attention to the stock market clearing condition.

10. (10 points) Imagine now that the trees are free to go to either market. What changes with respect to the previous question?