International Trade and Finance Prelim

Spring 2013

Instructions: Answer the question from Part I and two questions from Part II, for a total of three questions.

You will have five hours.
Part I
Answer the question in Part I.
Question I.1: Self-Fulfilling Debt Crises

Consider a small open economy whose government borrows from international lenders. In every period, the value of output is

\[ y(z) = Z^{1-z} \bar{y} \]

where \( 1 > Z > 0 \) is a constant and \( z = 0 \) if the government defaults in that period or has defaulted in the past and \( \bar{y} \) is a constant. The government’s tax revenue is \( \theta y(z) \) where the tax rate \( 1 > \theta > 0 \) is constant. The consumers in the economy consume \( c = (1 - \theta) y(z) \). The government is benevolent and makes choices to maximize the expected discounted value of

\[ u(c, g) = \log c + \gamma \log g \]

where \( \gamma > 0 \) and \( 1 > \beta > 0 \) is the discount factor. At the beginning of every period, the state of the economy is \( s = (B, z_{-1}, \zeta) \) where \( B \) is the level of government debt; \( z_{-1} = 0 \) if the government has defaulted in the past, and \( z_{-1} = 1 \) if not, and \( \zeta \sim U[0, 1] \) is the realization of a sunspot variable. The government first offers \( B' \) to international bankers. The international bankers have the same discount factor \( \beta \) as the government. They are also risk neutral and have deep pockets. These international bankers buy the bonds at a competitive auction that determines a price for \( B', q(B', s) \). The government finally chooses to default or not, which determines private consumption \( c \). Government spending \( g \) is determined by the government’s budget constraint

\[ g + zB = \theta y(z) + q(B', s)B'. \]

If the government defaults, setting \( z = 0 \), then assume that \( z_{-1} = 0 \) implies \( z = 0 \) thereafter; that is, the economy suffers from the default penalty \( 1 - Z \) forever. Furthermore, \( z_{-1} = 0 \) implies \( q(B', s) = 0 \); that is, the government is permanently excluded from credit markets.

**Question I.1 continues on the next page.**
Question I.1 continued:

(a) Define a recursive equilibrium.

(b) Assume that the bankers expect the government to default if $\zeta > 1 - \pi$ and if such an expectation would be self-fulfilling, where $1 \geq \pi \geq 0$ is an arbitrary constant. Find a level of debt $\bar{b}$ such that, if $B \leq \bar{b}$, no default occurs in equilibrium, but that, if $B > \bar{b}$, default occurs in equilibrium.

(c) Suppose that $B_0 > \bar{b}$, and the government chooses to run down its debt to $B_T \leq \bar{b}$ in $T$ periods. Prove that it cannot be optimal to set $B_T < \bar{b}$. Prove that it is optimal for the government to set $g_t$ constant as long as $B_t > \bar{b}$ and no crisis occurs. Find expressions for $g_t$ and $B_t$ that depend on $B_0$ and $T$. Find an expression for the expected discounted value of the utility of running down the debt that starts at $B_0$ to $\bar{b}$ in $T$ periods. Find the limit of these expressions when $T = \infty$.

(d) Using the answers to part (c), write down a formula that determines a value of debt $\bar{B}(\pi)$ such that the government would choose to default if $B > \bar{B}(\pi)$ even if international bankers do not expect a default.

(e) Using the answers to parts (a)–(d), construct a recursive equilibrium.

(f) Use this model to interpret events of the Mexican financial crisis of December 1994 through January 1995. Discuss the strengths and weaknesses of this model.
Part II
Answer two questions from Part II.
Question II.1: Default Risk and Income Fluctuations in Emerging Economies

Consider a small open economy in which households are identical, have strictly concave utility
\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)
\]
and a stochastic stream of endowments \(y\) which follow a Markov transition function \(\pi(y'|y)\) and \(y\) has a bounded support \(Y\). A government maximizes the utility of these households. It trade bonds with risk neutral competitive foreign lenders by selling one period bonds \(B'\) to the lenders at price \(q(B', y)\) that will be determined in equilibrium. It can also default. If the government decides to repay its debt at \(t\) then its budget constraint is
\[
c + q(B', y)B' = y + B
\]
while if it decides to default at \(t\) then its budget constraint is
\[
c = Y^{Aut}(y)
\]
where \(Y^{Aut}(y) < y\) is an increasing function of \(y\) and represents the net endowment in default. After defaulting the country regains access to the credit markets with probability \(\theta\). Let \(\delta(B', y)\) be the government’s decision function for default. Foreign creditors maximize expected profits and have cost of funds \(1 + r\) where \(\beta = 1/(1 + r)\).

The timing within a period is: At the beginning of the period a government that is currently in the state of default is allowed to reenter the credit market with probability \(\theta\). Any government that already has access to the credit market starts with incoming debt \(B\) (possibly zero for newly readmitted governments) then observes \(y\) and then decides whether to default. If the government decides to repay it chooses new debt \(B'\) facing the schedule \(q(B', y)\) where this schedule is consistent with foreign lenders earning zero expected profits. Consumption occurs.

Question II.1 continues on the next page.
Question II.1 continued:

(a) Set up and define a recursive competitive equilibrium

(b) Solve for the bond price schedule $q(B', y)$.

(c) What is the explanation of this model for the observed pattern of countercyclical interest rates and countercyclical capital flows experienced by emerging economies?

(d) Contrast that explanation for the pattern of capital outflows with that from standard two country real business cycle models.
Question II.2: Fiscal Devaluations

Consider a two country economy consisting of a home and foreign country. The representative home consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \sum_{s^t} \beta^t U(c_t(s^t), n_t(s^t), M_t(s^t) / P_t(s^t))$$

subject to the sequence of budget constraints

$$P_t(s^t)c_t(s^t) + M_t(s^t) + \sum_{s^t+1} Q_{Ht}(s^t+1)B_{Ht+1}(s^t+1)$$

$$+ e_t(s^t) \sum_{s^t+1} Q_{Ft}(s^t+1)B_{Ft+1}(s^t+1)$$

$$\leq B_t(s^t) + e_t(s^t)B_{Ft}(s^t) + M_{t-1}(s^t) + W_t(s^t)n_t(s^t) + \Pi_t(s^t) + T_t(s^t)$$

Let

$$U = \frac{c_t^{1-\sigma} - \frac{\kappa n_t^{1+\phi}}{1+\phi}}{1-\sigma} + \frac{b}{1-v} \left( \frac{M_t}{P_t} \right)^{1-v}$$

$$c_t = \left( \frac{1}{\gamma_h c_{Ht}} + \frac{1}{\gamma_f c_{Ft}} \right)^{\frac{1}{\theta_h}}$$

$$c_{kt} = \left( \int c_{kt}(i)^{\frac{\rho-1}{\rho}} di \right)^{\frac{\rho}{\rho-1}}$$

with the corresponding price indices. Let the foreign country problem be similar and denoted with asterisks (*). Consider monopolistically competitive firms that set prices in the currency of the market to which they are selling. Let the technology of each firm be given by

$$y_t(i) = a_t n_t(i)$$

where $a_t$ is a country specific technology shock for the home country. Firms set their prices in a flexible manner to maximize profits

$$\Pi_t(i) = P_{ht}(i)c_{ht}(i) + e_t P_{ft}(i)c_{ft}(i) - W_t n_t(i)$$

Question II.2 continues on the next page.
Question II.2 continued:

(a) Let the foreign consumer have access to complete state contingent nominal bonds in both the home and the foreign currency.

(i) Relate the nominal exchange rate to the contingent bond prices.

(ii) Relate the marginal rates of substitution of consumers in two two countries to each other and the nominal exchange rate.

Consider a deterministic version of the economy in which the equilibrium path of the exchange rate is \( e_t = e_0(1 + \delta_t) \) where \( \delta_t \) is a given sequence and let \( M_t \) follow a path consistent with this path for the exchange rate.

(b) Add a set of tax instruments for the government such that the government has enough tax instruments to replicate the real allocations associated with this path.

(c) Show how to set these tax instruments so as to replicate the given equilibrium path.

(d) Now assume that prices are set in a Calvo fashion, in that each producer can change its prices with a fixed probability. How does your answer to parts (b) and (c) change?
Question II.3: Explain One of the Outstanding Puzzles

Pick ONE outstanding puzzle in international macroeconomics, such as the Feldstein Horioka puzzle, the forward premium anomaly, the home bias in portfolios, the risk sharing puzzle, the Backus Smith puzzle relating the real exchange rate to marginal rates of substitution, the asset allocation puzzle, or another of your choosing.

(a) Carefully describe the set of facts that constitute this puzzle.

(b) Develop as carefully as possible a simple standard economy for which these set of facts are a puzzle.

(c) Describe as carefully as possible an economy which can produce the facts that constitute this puzzle.

(d) Describe precisely how you would evaluate the quantitative ability of your economy to generate these facts.

Include the equations of both the simple model that exhibits the puzzle and your proposed model that will generate these facts. It is important to develop the intuition of what goes wrong in the simple model and how your model will potentially account for these facts.