Preliminary Examination

Growth and Development

Spring 2014

Answer two questions from Section I and two from Section II.
Section I: Question 1

Suppose there is a unit measure of individuals. Each has a unit of time. They use that to earn a wage (as an employee) or to start a business (and hire other individuals as employees).

All individuals are equally skilled as employees, but differ in their skill operating businesses. Let the business skill of an individual be indexed by $z \in [0, \bar{z}]$. Let $F(\cdot)$ be the c.d.f. over business skills.

There is a single consumption good. If a type-$z$ individual starts a business, then he produces $y$ units of the consumption good, where

$$y = Az^{1-\gamma}n^\gamma,$$

and where $n$ is the number of employees hired. Preferences are simply $u(c) = c$, where $c$ is the units of the consumption good consumed.

**a.** Let the single consumption good be the numeraire, and let $w$ be the wage paid to employees in units of that good. Define an equilibrium in this simple economy (it is best to think of an equilibrium as a pair $(z, w)$ satisfying certain conditions). Present two equations that any equilibrium must solve.

Now consider another economy, just like that above, except with the following restriction. If an entrepreneur hires $n$ units of labor, he again produces $y = Az^{1-\gamma}n^\gamma$ units of output. However, he is now required to hire other workers to “check” the output that “leaves” the plant. He must hire $m \geq \lambda n$ workers to check the output. In particular, the entrepreneur’s sales are $s = y = Az^{1-\gamma}n^\gamma$ if $m \geq \lambda n$ and $s = 0$ if $m < \lambda n$.

Restrictions like these are often referred to as featherbedding. The workers “checking” output are not adding to output. Such rules are usually thought to increase employment.

**b.** Define an equilibrium in this new economy (it is best to think of an equilibrium as a pair $(z, w)$ satisfying certain conditions). Present two equations that any equilibrium must solve.

**c.** How does the cutoff $z_*$ depend on the featherbedding parameter $\lambda$?

**d.** Give some intuition for your answer in **c.**
Section I: Question 2

Consider an industry that produces a good $y$ that sells for $(1+\tau)p_y$, where $p_y$ is the world price, and $\tau$ is the tariff rate faced by foreign firms selling in the domestic market. This good is produced with two inputs, an entrepreneur’s time and a machine. Entrepreneurs are indexed by $\gamma$. An entrepreneur of type $\gamma$ has access to the following production function

$$y = \gamma \min(n, I),$$

where $n$ is the entrepreneur’s time and $I$ is an indicator function, where $I = 1$ if the entrepreneur has the machine, $I = 0$ otherwise.

Entrepreneurs decide to produce this good or to work in their next best alternative occupation. In particular, the entrepreneurs have a unit time endowment which they devote to producing good $y$ or to the alternative occupation. Suppose all entrepreneurs have the same next best alternative, the value of which is $w$. Finally, the price of the machine is $q$.

a. Let the parameter $\gamma$ be distributed continuously with density $f(\cdot)$ on $[0, \overline{\gamma}]$. Under what conditions will entrepreneurs enter into the industry producing $y$?

b. Under these conditions, what happens to the set of entrepreneurs that produce $y$ as the tariff is increased?

c. What happens to the dispersion of labor productivity in the industry as the tariff is increased, in particular, what happens to the range of labor productivities?

d. What happens to the range of capital productivities as the tariff is increased?

e. We studied a number of papers in class where the authors proposed that one country, say country A, had more “distortions” on businesses than another country B if the distribution of productivities in country A was more “dispersed” than that in country B. If we interpret higher tariffs as meaning greater distortions, does this “idea” hold in the simple model above? Explain.
Section II: Question 1

Consider an economy with a representative consumer whose preferences over flows of agricultural products $A_t$, manufactured products $M_t$, and services $S_t$ are given by

$$
\int_0^\infty e^{-\rho t} \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} \right) dt,
$$

where $\rho$ and $\sigma$ are positive parameters and composite consumption $C_t$ is given by

$$
C_t = (A_t - A)^{\alpha A} M^{\beta M} (S_t + S)^{\beta S}.
$$

The parameters $A, S$ and $(\alpha A, \beta M, \beta S)$ are positive, and $\alpha A + \beta M + \beta S = 1$. Intermediate output $Y_t$ is produced using a constant returns to scale technology with capital and labor as inputs,

$$
Y_t \leq F(K_t, z_t N),
$$

where $N > 0$ and $z_t = ze^{\delta t} > 0$ grows at a positive rate. Capital can be accumulated according to

$$
DK_t \leq -\delta K_t + X_t,
$$

where $\delta$ is a positive. The consumption and investment goods can be produced from intermediate output, subject to the time-$t$ resource constraint

$$
A_t + M_t + S_t + X_t \leq E + Y_t,
$$

where $E$ is a flow of endowments of the intermediate good. The initial capital stock is positive.

a. Determine the cost-minimizing choices of $A_t, M_t$ and $S_t$ for a given level of composite consumption $C_t$. Verify that the cost function in units of intermediate good is $\frac{A - S}{P C_t}$, for some positive constant $P$.

b. Suppose it so happens that $E = A - S$. Give the equations that determine the balanced growth path, suitably defined.

c. What happens to consumer expenditure shares over time, assuming that $E = A - S$ is positive? If intermediate output is produced in the sector in which it is used, what can you say about sectoral labor shares when $E = A - S = 0$?
Consider an economy with a representative consumer endowed with one unit of labor whose preferences over consumption flows $C_t$ are determined by

$$\int_0^\infty e^{-\rho t} u(C_t)dt,$$

where $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$. There is a positive initial capital stock $K_0$. Capital can be used either to produce consumption goods or to produce more capital. If $X_t \in [0, K_t]$ units of capital are used together with one unit of labor to produce a flow $Y_t$ of consumption goods,

$$Y_t = F(X_t, 1),$$

then the capital stock grows according to

$$DK_t = A \cdot (K_t - X_t).$$

The production function $f(x) = F(x, 1)$ is given by

$$f(x) = \begin{cases} 
\alpha \xi^{1-\alpha} + (1 - \alpha)\xi^{-\alpha}x, & x \in [0, \xi] \\
\xi^{-\alpha}, & x \in [\xi, \infty) 
\end{cases}$$

All parameters are positive, $\alpha \in (0, 1)$, and $A > \rho > (1 - \alpha)(1 - \sigma)A$. Note that $f(0)$ is positive and that $f$ is continuously differentiable on $[0, \infty)$.

Let $q_t$ be the price of capital, and write $v_t$ for the rental price of capital, both in units of consumption. The interest rate is $r_t$, also in units of consumption.

**a.** A production function of the form $f$ is implied by cost minimization given (i) a Cobb-Douglas technology that uses capital and labor and (ii) a linear technology that uses only labor. Show this using a diagram that displays the isoquants for these two technologies. No need for algebra.

**b.** Explain why $v_t \geq Df(X_t)$ and $v_t \geq Aq_t$. When do these inequalities have to hold with equality? Explain why $r_tq_t = v_t + Dq_t$.  

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c. Conjecture that the equilibrium is described by a constant ratio $X_t/K_t \in (0, 1)$. Show that in such an equilibrium $DC_t/C_t = g$ and $r_t = r$, where

$$g = \frac{(1 - \alpha)(A - \rho)}{\alpha + (1 - \alpha)\sigma}, \quad r = \frac{\alpha\rho + (1 - \alpha)\sigma A}{\alpha + (1 - \alpha)\sigma}. \quad (1)$$

Verify that $X_t/K_t = (r - g)/A$ and note that $X_t \geq \xi$ corresponds to $K_t \geq K_* = A\xi/(r - g)$.

d. Describe the competitive equilibrium starting from an initial capital stock $K_0 \in (0, K_*)$. Provide details as time permits.
Section II: Question 3

Consider an economy with a representative consumer endowed with $H$ units of labor whose preferences over consumption flows $C_t$ are determined by

$$\int_0^\infty e^{-\rho t} \ln(C_t) dt,$$

where $\rho$ is positive and

$$\ln(C_t) = \int_0^1 \ln(C_{j,t}) dj.$$

Given the right blueprint, good $j$ can be produced using a linear labor-only technology with productivity $z_{j,t}$. The most productive blueprint is in the possession of the agent who created it, and becomes available to everyone when someone creates a more productive blueprint for good $j$. There are two technologies for creating a blueprint for good $j$ with productivity $\lambda z_{j,t}$, where $\lambda > 1$. On the one hand, anyone can hire $m_t \geq 0$ units of labor to create such an improvement at the Poisson rate $\gamma m_t$. On the other hand, the owner of the most advanced blueprint for any other good $j' \neq j$ can hire $x_t \geq 0$ units of labor to create such an improvement for good $j$ at the Poisson rate $f(x_t)$. The parameter $\gamma$ is positive, and the function $f$ is strictly increasing and strictly concave, and $f(0) = 0$.

Let $r_t$ be the interest rate and $w_t$ the wage. In the following, take for granted that the equilibrium will be symmetric and write $v_t$ for the profits from producing good $j$ and $q_t$ for the price of the most advanced blueprint for good $j$. The average rate at which the most productive blueprint for a particular good can be surpassed is denoted by $d_t$.

a. Explain why profits are given by $v_t = (1 - 1/\lambda)C_t$ and why $C_t = \lambda w_t L_t$ when $L_t$ is the aggregate amount of labor used to produce the goods $j \in [0, 1]$. At any point in time, what determines $w_t$?

b. What is the Bellman equation for $q_t$? Explain. Use the assumption that utility is logarithmic to derive a Bellman equation for $q_t/C_t$, and verify that it does not depend on the interest rate $r_t$.

c. State the first-order conditions for the amounts of labor that new entrants and the owners of frontier blueprints use to try to create better blueprints.
d. Conjecture that there a balanced growth path along which $M > 0$ units of labor are used by new entrants trying to create new blueprints, and $X > 0$ units of labor by the owners of frontier blueprints. Determine the equilibrium values for $M$, $X$, and $q_t/C_t$. What is the resulting growth rate of aggregate consumption?

e. Suppose blueprints created from existing blueprints (rather than from scratch by new entrants) stay within the same firm. Explain intuitively why this model cannot generate an empirically plausible distribution of employment across firms.