Answer any three of the following four questions. The answers will be equally weighted in the examination grade.

Analytical solutions can be derived for some of the problems this examination contains. If the algebra involved proves too cumbersome, however, essentially full credit will be given for careful explanations of the steps that could be followed to derive analytical solutions. If you feel that additional assumptions are required before a unique solution to the problem can be found, specify what they are and why you need them.

Please indicate clearly which questions and which part of the question you are answering. Note also that irrelevant material, even if correct, will receive no credit.
Question 1

Consider a model of timing of exit in which at time \( t = 0 \) there are \( N_0 \) firms and there is no subsequent entry. Each period has two stages. In stage 1, each existing firm simultaneously decides whether or not to exit the industry or remain in the industry in the current period. The exit decision is permanent. In stage 2, there is a market game described further below.

Any existing firm that chooses to remain in the industry in stage 1 must pay a fixed cost \( f_t \geq 0 \) in the period that potentially varies with \( t \).

At the beginning of each period, the number of existing firms of each age is public information. Each existing firm draws a private shock about the it’s profitability if it chooses to remain in the industry in the period. Let \( \varepsilon_{i,t} \) be the private component of profitability for firm \( i \) at time \( t \). Suppose that \( \varepsilon_{i,t} \) is drawn i.i.d. across firms and across time from a continuous distribution with density \( g(\varepsilon_{i,t}) \) with full support, \( \varepsilon_{i,t} \in (-\infty, \infty) \). Firms make exit decisions for the period after observing \( \varepsilon_{i,t} \).

Suppose there are a finite number of periods \( T < \infty \).

We now turn to the market game. All firms have marginal cost equal to zero. The inverse demand curve in the industry is \( p(Q) = A - Q \). In stage 2, the existing firms who choose not to exit in stage 1, play a Cournot game.

If an existing firm chooses to exit, the firm receives a one-time exit payment of \( \psi \).

Let \( \beta \) be the discount factor.

(a) Define a Markov-perfect equilibrium in this model and characterize the solution.
(Restrict attention to symmetric equilibria where firms in the same situation behave the same way.)

(b) Suppose we change the model as follows. Rather than play a Cournot game, firms that are in the industry in stage 2 of period \( t \) receive a fixed payment \( \pi_t \) that is independent of the number of firms. Suppose \( \pi_t \) is constant over time and that \( f_t \) is
constant over time. Characterize the equilibrium. How does the probability of exit vary over time?

(e) Continue to assume the alternative model of part (b). Under what assumptions on how $\pi_t$ and $f_t$ vary over time can you obtain a monotonicity result for how the probability of exit varies over time?

(d) Now return to the original model. The state at the beginning of period $t$ is $(N_t, t)$, the count of firms still in the industry (before exit at time $t$) and the current period. Discuss how the ex ante probability of exit (before firms draw their private shocks) varies with the state.
Question 2

Consider the Logit Model of Product Differentiation. There are $n$ firms indexed by $j$ and an “outside good” labeled by 0. Each firm $j \geq 1$ has constant marginal cost equal to $c_j$. There is a unit measure consumers. Let $i$ index an individual consumer and suppose the utility of consumer $i$ from purchasing good $j$ and paying price $p_{i,j}$ has utility

$$U_{ij} = \xi_j - \alpha p_{i,j} + \varepsilon_{i,j} \text{ for } j = 1, 2, \ldots, n$$
$$= \varepsilon_{i,0} \text{ for good } 0.$$

A consumer is therefore summarized by his or her vector of draws $\varepsilon_i = (\varepsilon_{i,0}, \varepsilon_{i,1}, \ldots, \varepsilon_{i,n})$ since consumers are otherwise the same. Assume the $\varepsilon_{i,j}$ are drawn type 1 extreme value which delivers the logit choice probabilities.

We allow for perfect price discrimination. Each firm observes the entire vector of draws $\varepsilon_i$ for each consumer $i$ and can set prices contingent on $\varepsilon_i$ (i.e. can set a price $p_{i,j}$ specific to individual $i$.). The $n$ firms compete in a Bertrand fashion for each individual consumer.

(a) Take as given the $n$ firms in the industry and calculate the equilibrium of price competition when perfect price discrimination is feasible. Derive formulas for the market shares of each firm.

(b) Suppose the $n$ firms in the industry observe every component of $\varepsilon_i = (\varepsilon_{i,0}, \varepsilon_{i,1}, \ldots, \varepsilon_{i,n})$, EXCEPT $\varepsilon_{i,0}$, the draw for the outside good. Define equilibrium in this industry and discuss how it is characterized. Are market shares necessarily the same as in part (a)?
Problem 3

Consider a classic Hotelling line model where consumers have heterogeneous tastes distributed uniformly across [0,1]. This taste represents the location of that consumer’s favorite good. Let \( x \) denote their location (this taste) on the unit interval. Good (firm) 0 is located at the left endpoint and good (firm) 1 is at the right endpoint. \( t \) is the ”transportation” cost or the dislike associated with being away from ones optimal good. Assume a common demand intercept for both goods \( q_0 = q_1 = q^* \) and a common slope for price normalized to -1.

A. A consumer located at \( x \) values good 0 at
\[
q^* - p_0 - t * x
\]
and good 1 at
\[
q^* - p_1 - t * (1 - x).
\]
Solve for the indifferent consumer \( x^* \) as a function of \( p_0, p_1, q^* \).

B. Denote plant \( i \)'s technical efficiency \( c_i \), the unit cost of production. Let profits for plant 0 be given by \((p_0 - c_0) * x^* \) and plant 1 \((p_1 - c_1) * (1 - x^*) \). Assume the two firms compete Bertrand-Nash in prices. Solve for equilibrium prices and quantities.

C. Solve for consumer and producer surplus at equilibrium prices.

D. Let \( c_0 < c_1 \). Suppose the government decides it is going to tax the ”inefficient” firm’s revenue by \( \tau \) percent - so the inefficient firm gets \( 1 - \tau \) in revenue - and give it directly to consumers. Resolve for equilibrium prices, quantities and consumer and producer surplus. How do things change?
Problem 4

Consider the current television/high-speed-internet markets in the United States (ignore phone service throughout). Most consumers have several choices for both pay television and for high speed internet, some of which are offered by the same firm. For television in most markets consumers have a choice between no pay television, pay television through their local cable company (only one company per market), pay television through DirecTV (satellite dish), or pay television through DISH (another satellite dish company). In these same markets consumers have a choice of high speed internet through their cable company, DSL through their local telephone company (which is noticeably slower), or no high speed internet. The characteristic distinguishing TV is the number of channels and the characteristic distinguishing high-speed-internet is the speed of access.

1. Consumers choose one and only one of their tv choice and one and only one of their high-speed internet choices. These choices may be interrelated. Write down the set of product choices consumers have and use it to posit a discrete choice demand system for these markets. How many products are their in the market? What is the price for each product?

2. Suppose you observed market shares on each of the products. Explain how you proceed with estimation if you wanted to accommodate the possibility that unobserved quality (to the econometrician) is correlated with observed prices.

3. Suppose you had a long list of demographics that you thought might affect tastes for each of the products. Write down how you would extend the demand system to accommodate the fact that demographics may affect tastes. How would you estimate the model if product characteristics were also endogenous (as in 2).

4. In some markets in recent years ATT, the local telephone company, has
introduced high-speed fiber optic networks. In these markets it now offers both high speed internet and pay TV (UVerse), in addition to DSL. Rewrite the choice set to accommodate this new market structure.

5. ATT recently proposed that it wanted to buy DirecTV. Suppose you had the data to estimate the demand specification for question 4. Propose a method for solving for the marginal costs for each of the products. Describe how you would check to see how much prices may increase post-merger (assume no cost savings). What cross-price elasticity is likely to drive the magnitude of this price increase?