Monetary Economics

Spring 2014

Answer one question in Part I and one question in Part II, for a total of two questions.
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Part 1
Question 1: Optimal Maturity of Government Debt

Consider a one perishable good economy with production. The representative agent is endowed with one unit of labor in every period. Labor is the single factor of production, and the technology is given by

$$c_t + g_t \leq A_t(1 - x_t), \quad t = 0, 1, 2, \ldots, \text{ all } t^t$$

where $c_t$, $x_t$ and $g_t$ represent private consumption, leisure and government spending, respectively, and $A_t$ is a productivity shock. Government spending is exogenous and follows some stochastic process. $A_t$ and $g_t$ are the only source of uncertainty in this economy. At each $t$ there is a finite number $N$ of possible values for the pair $s_t = (A_t, g_t)$. Let $s^t = \{s_0, \ldots, s_t\}$ be the history of shocks up to time $t$. Preferences are represented by

$$E_0 \sum_{i=0}^{\infty} \beta^i U(c_t(s^t), x_t(s^t))$$

where $U$ is strictly increasing in both arguments and strictly concave. The only tax available to the government is a flat rate tax $\tau_t$ levied on labor income. Let $b_{-1}$ be the value of government liabilities in units of time zero consumption.

a. Assume that the government can issue one-period state contingent bonds. Set up the Ramsey problem. Show how the optimal one-period state contingent debt structure can be obtained from the solution to Ramsey problem.

b. Assume that the government can issue non-contingent bonds maturing at $J$ consecutive dates. State conditions under which the government can reproduce the optimal one-period state-contingent structure of debt.

Suppose now that there are two bonds: a one period uncontingent bond and a two period uncontingent bond. Let the initial amount of both bonds be zero. Productivity $A_t$ is constant at $\bar{A}$ and $g_t$ follows a Markov process with two values $g_H$ and $g_L$ with a Markov transition matrix $\Pi = (\pi_{ij})$. Let the initial state $g_0 = g_H$.

c. Does the solution to the Ramsey problem have a simple pattern? Can you describe the pattern of government surpluses $z(g_H)$ and $z(g_L)$.

d. Describe a formula that shows, given the consumption allocations from the Ramsey problem, what the optimal maturity of debt satisfies.

e. What condition do you need satisfied so that the model with two uncontingent bonds reproduces the equilibrium with state contingent bonds? What if it fails, is there any approximate implementation argument you can make?

f. Explain the intuition on how these bonds provide the government with the necessary “insurance” to implement the state contingent outcomes.
g. Suppose now that the government has only one nominal bond of one period maturity and we amend the economy so that money enters through a cash-in-advance constraint in which current money balances acquired in an asset market at the beginning of the period can be used to pay for current purchases. Suppose also there are many realizations of the shocks to productivity and to government spending. Can you implement the Ramsey equilibrium with the one period bonds? Describe verbally how money must enter the model in terms of distortions and so on for this to be possible.
Question 2: Sophisticated Monetary Policies

Consider a simplified version of the New Keynesian model. The representative household’s behavior is characterized by a linearized Euler equation:

\[ y_t = E_t[y_{t+1}] - \psi(i_t - E_t[\pi_{t+1}]) + \eta_t \]

and a linearized cash in advance constraint:

\[ \pi_t = \mu_t + (\gamma_t - \gamma_{t-1}) + \nu_t \]

Here \( \eta_t \) and \( \nu_t \) are exogenous shocks with mean zero and variance \( \text{var}(\eta) \) and \( \text{var}(\nu) \) respectively. There are two types of producers. A fraction \( \alpha \) have flexible prices, in the sense that they set their prices after the exogenous shocks are realized. A fraction \( 1 - \alpha \) are sticky price producers, and set their prices before the shocks are realized. A flexible producer \( j \) sets prices following the rule:

\[ p_{ft}(j) = p_t + \gamma y_t \]

A sticky price producer \( j \) sets prices according to:

\[ p_{st}(j) = E_{t-1}[p_t + \gamma y_t] \]

The aggregate price level is thus given by:

\[ p_t = \int_0^\alpha p_{ft}(j)dj + \int_\alpha^1 p_{st}(j)dj \]

The Central Bank can use either a money regime, setting the growth rate of money \( \mu_t \), or an interest rate rule, setting the nominal interest rate \( i_t \). The actions of the Central Bank are chosen after the sticky price producers have set their prices, but before the shocks are realized. The Central Bank has full commitment.

a. Explain the logic using the unsophisticated approach to monetary policy how a Taylor rule with a coefficient greater than one is supposed to implement a unique equilibrium.

b. Consider the King rule

\[ i_t(s^{t-1}) = i^*_t(s^t) + \varphi(x_t(s^{t-1}) - x^*_t(s^{t-1})) \]

where \( i^*_t(s^t) \) and \( x^*_t(s^{t-1}) \) are the interest rate and the sticky price producers’ choice of price relative to that of the aggregate price level associated with the equilibrium that the monetary authority wishes to implement. Can their be multiple equilibrium even if \( \varphi > 1 \)? Characterize the set of equilibria as best you can.

c. Find an expression for the inflation rate as a function of current output and the average price set by the sticky price producers.
d. Define a Competitive Equilibrium and a Continuation Competitive Equilibrium in this environment. (Start defining the relevant histories).

e. Define a Sophisticated Equilibrium in this environment. (Start defining the relevant histories).

f. Prove that any desired equilibrium outcome can be uniquely implemented with a sophisticated monetary policy involving one-period reversion to money after a deviation.

g. Explain the role of controllability of best responses and how this condition is used in the proof. What if this condition fails? Explain where the proof would break down.

h. Is it a necessary condition for unique implementation that the government reverts to money right after a deviation occurs?

i. (Important do not skip) Do regressions of interest rates on inflation recover parameters that are critical to ensuring unique implementation?

j. (Important do not skip) Is it possible to uniquely implement an equilibrium in which along the equilibrium path interest rates and sticky price producers choices satisfy

\[ i_t^{*}(s^{t-1}) = \bar{r} + \varphi (x_t^{*}(s^{t-1}) - \bar{x}) \]

with a passive Taylor rule, that is, with \( \varphi < 1 \)?
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Part 2
In this problem, you must solve a neoclassical growth model with a cash-in-advance constraint. There is no uncertainty. All households have common preferences, of the form

$$\sum_{t=0}^{\infty} \beta^t [U(c_t) - V(n_t)] , \quad \beta \in (0, 1)$$

where $c_t$ is consumption and $n_t$ is leisure. Assume this function to be strictly concave.

The technology is given by a constant returns to scale technology

$$y_t = f(k_t, n_t)$$

such that the marginal product of capital goes to zero as the capital labor ratio goes to infinity.

Firms rent capital and labor from households.

The law of motion for capital is given by

$$k_{t+1} = (1 - \delta)k_t + y_t - c_t.$$

Households can buy capital, government bonds or money. Government bonds pay a nominal return equal to $i_t$. The government issues bonds, money and can tax agents with lump sum taxes. There is no government expenditures.

Timing is as follows: At the beginning of the period, agents use their wealth to accumulate money, bonds or capital. A fraction $\gamma$ of current period consumption must be financed with the money holdings decided at the beginning of the period. The cash in advance constraint is then

$$M_t \geq \gamma P_t c_t$$

Current wages, bonds and the interest accrued plus un-depreciated capital and its rental rate become next period wealth.
1. Write the consumers problem and get the first order conditions.

2. Define a competitive equilibrium and find the equations that solve for an equilibrium.

3. Assume a constant rate of money growth and solve for a steady state.

4. Show that as the growth rate of money goes up, steady state output per capita goes down. How does the result change if \( V' = 0 \) (labor is inelastically supplied). Explain the result in economic terms.

5. Let \( \gamma \to 0 \). Show that there exists tax rates (there is more than one, right?) such that the economy is isomorphic to an economy with \( \gamma > 0 \), and zero tax rates.