International Trade and Finance
Preliminary Exam

Spring 2014

Instructions: Answer the question from Section I, ONE of the questions from Section II, and the question from Section III, for a total of three questions.

You will have five hours to complete the exam.
SECTION I

Answer one question
Learning by doing in a dynamic Ricardian model

Consider an economy in which the consumption space is the set of functions \( c : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+ \). In \( c(x,t) \) the index \( x \) denotes the type of good and the index \( t \) denotes the date at which it is consumed. An individual consumer has preferences given by the functional

\[
    u(c) = \int_0^\infty e^{-\rho t} \left[ \int_0^c \log(c(x,t) + 1) dx \right] dt.
\]

Goods are produced using a single factor of production, labor:

\[
    y(x,t) = \ell(x,t) / a(x,t).
\]

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at \( \bar{\ell} \). The unit labor requirement \( a(x,t) \) is bounded from below, \( a(x,t) > \bar{a}(x) \), where

\[
    \bar{a}(x) = e^{-x}.
\]

At \( t = 0 \) there is a \( z(0) > 0 \) such that \( a(x,0) = e^{-x} \) for all \( x < z(0) \) and that \( a(x,0) = e^{x + z(0)} \) for all \( x \geq z(0) \). There is learning by doing of the form

\[
    \frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} 
    -\int_0^\infty b(v,t) \ell(v,t) dv & \text{if } a(x,t) > \bar{a}(x) \\
    0 & \text{if } a(x,t) = \bar{a}(x) 
    \end{cases}
\]

Here \( \dot{a}(x,t) \) denotes the partial derivative of \( a(x,t) \) with respect to \( t \) and \( b(v,t) = b > 0 \) if \( a(v,t) > \bar{a}(v) \) and \( b(v,t) = 0 \) if \( a(v,t) = \bar{a}(v) \). There is no storage.

(a) Provide a motivation for the both the utility function and the production technology described above.

(b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

(c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, \( z^1(0) > z^2(0) \). There is no borrowing or lending across countries. Define an equilibrium for this economy.

(d) Suppose that \( z^1(t) > z^2(t) \). Explain carefully and illustrate two of the five qualitatively different possible equilibrium configurations for production, consumption, and trade at time \( t \). (To make things easy, assume that \( z^1(t) \) and \( z^2(t) \) are sufficiently large so that good \( x = 0 \) is not produced in equilibrium.)
(e) Briefly describe the dynamics of this model, explaining the crucial role played by the sizes of the two countries, $\bar{\ell}^1$ and $\bar{\ell}^2$. In particular, how much larger than $\bar{\ell}^1$ does $\bar{\ell}^2$ have to be so that $z^2(t)$ passes $z^1(t)$. Discuss the economic significance of this result.
SECTION II
Answer one of two questions
Default Risk and Income Fluctuations in Emerging Economies

Consider a small open economy in which households are identical, risk averse and have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Households receive a stochastic stream of a tradable good $y$. The output shock is assumed to have a compact support and to be a Markov process with a transition function $f(y', y)$. The government is benevolent and its objective is to maximize the utility of households. The government has access to the international financial markets, where it can buy one period discount bonds $B'$ at price $q$, determined in equilibrium from risk neutral competitive foreign lenders. The government also decides whether to repay or default on its debt. If the government decides to repay its debt at $t$ then its budget constraint is

$$c_t = y_t + B_t - q_t(\cdot)B_{t+1}$$

where $q_t(\cdot)$ is a function that you will later describe. If it decides to default at $t$ then its budget constraint is

$$c_t = Y^d(y_t)$$

where $Y^d(y_t) < y_t$ and the function $Y^d(y_t)$ is an increasing function of $y_t$. After defaulting the country regains access to the credit markets with probability $\theta$. Let $\delta_t$ denote the government’s decision function for default. Foreign creditors maximize expected profits. The timing within a period is: At the beginning of the period a government that is currently in the state of default is allowed to reenter the credit market with probability $\theta$. Any government that already has access to the credit market starts with incoming debt $B_t$ then observes $y_t$ and then decides whether to default. If the government repays in period $t$ then it can choose new debt in that period. Lenders charge prices so that they earn zero expected profits. Consumption then occurs.

NOTE: Make sure you read carefully to see which questions assume commitment and which questions do not.
A. Suppose first that, contrary to the standard assumptions made in the literature, the government can commit to its actions.

i. Define a competitive equilibrium with commitment in a nonrecursive form. Start with the history of exogenous events. Be careful in explaining what you assume about the amount the government can borrow and the problem the risk neutral lenders solve. Do you allow for default in equilibrium? (Note there are multiple sets of assumptions that make sense here. Pick one set of assumptions and make sure you clearly state them clearly.)

ii. Under the assumptions you made, work out the bond price.

B. Now suppose that the government lacks commitment and that we model this lack of commitment as having the government solve a Markov problem. Define a recursive competitive equilibrium without commitment and solve for the bond price schedule \( q(B', y) \). Draw the 'Laffer Curve' for borrowing that the model generates when output shocks are assumed to be i.i.d. Specify bond prices, 'risky borrowing' region, and the range of \( B \) where the borrower optimally chooses a bond contract.

C. Assume now that the borrower can issue short- and long-term debt. Short-term debt is one period discount bond. The long-term debt is modelled as a perpetuity contract with coupon payments that decay geometrically. The borrowing country faces persistent income shocks and faces the cost of default in the form of lower income and temporary exclusion from international financial markets.

i. Assume the government can commit and that \( y_t \in \{y_L, y_H\} \) so that there are only two realizations of \( y_t \). Can the government achieve full risk sharing with these two bonds?

D. Describe the behavior of the term structure of interest rate spreads and maturity composition of government debt in emerging market economies. What properties should the probability of repayment have in order to be able to account for the observed dynamics of spreads?

E. Now suppose the government cannot commit and there are short and long bonds described above. The optimal maturity structure of debt in this model reflects a trade-off between the relative incentive benefit of short-term debt and the hedging benefit of long-term debt. What are the characteristics of the bond price functions that generate these benefits?
Sticky Prices, Volatility and Persistence of Real Exchange Rates

In the data, exchange rates are extremely volatile and persistent. Historically, most models have attempted to explain this using monetary shocks and sticky prices. However, they have difficulty matching the features in the data.

A. There are two ways of generating fluctuations in the real exchange rate: fluctuations in the relative prices of traded/nontraded goods or deviations from the law of one price for traded goods. Derive a relationship between the variance in the real exchange rate and these two possible sources. Which one does the data suggest we use?

B. Set up an economy which can generate volatile and persistent real exchange rates assuming there are no nontraded goods.

Derive a log-linear relationship between the real exchange rate in two countries and some variables the enter the utility function. Use this relationship to explain how the proposed economy does in generating the persistence and volatility of the real exchange rate. Explain the mechanisms.

C. Describe the main anomalies in the model. Pick what you consider to be the most important one. As carefully as you can describe an economy that attempts to overcome it. Does a quantitative version of this economy account for this anomaly. Discuss in detail.
SECTION III

Answer one question
Financial Development, Growth Rates, and Global Imbalances

Consider two countries with a continuum of agents that fact idiosyncratic risk and have preferences over consumption

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}) \]

where \( u'(c) > 0 \), \( u''(c) < 0 \), and \( u'''(c) > 0 \) and \( \beta < 1 \). Each agent receives an idiosyncratic stochastic endowment \( w_{it} \) which is equal to an \( \varepsilon_{it} \) share of the aggregate endowment \( w_t \) : \( w_{it} = w_t \varepsilon_{it} \). The idiosyncratic shock \( \varepsilon_{it} \) is i.i.d. and uniformly distributed between 0 and 1. The aggregate endowment \( w_t \) grows at a constant rate \( g \). The two countries differ in their financial development and the aggregate growth rate. In country 1, the developed country, agents have complete set of contingent claims and face a lower growth rate \( g^1 \). In country 2, the less developed country, agents only have access to un-contingent claims, and face a higher growth rate \( g^2 > g^1 \).

Financial Autarky

1. Set up the problem of agents in each country and derive their first order conditions.

2. Define the equilibrium of each economy and characterize it as much as possible.

3. Compare the resulting autarky interest rates in the two countries and provide intuition.

Financial Integration: Assume both countries financially integrate and that their financial frictions remain in place for the residents of the country.

4. Characterize the capital flows resulting from financial integration. How do the differential financial frictions and growth rates across countries shape the patterns of capital flows?

5. Is the model a good paradigm for the capital flows observed between the G-7 and the BRIC (Brazil, Russia, India, China) since 1985?