

# A Model of Aggregate Fluctuations

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## Abstract:

Following a recession, the aggregate labor market is slack—unemployment remains high and recruiting efforts of employers, as measured by vacancies, are low. Realistic models describing these aspects of aggregate fluctuations have eluded research in the modern general-equilibrium framework. Although the facts about persistent slack periods suggest some kind of decline in aggregate demand, it has proven difficult to translate the concept of aggregate demand into a coherent model based on fundamentals. A decline in productivity does create a slack labor market in general equilibrium, but the needed reduction in productivity is far greater than any actually found in the U.S. economy. The alternative pursued in this paper is a shift in preferences away from the output of one sector. Although a shift away from one product is necessarily a shift toward another, the power of the offsetting effect depends on the elasticity of product supply. The model developed here has sufficiently low elasticity—thanks to immobile labor—that the preference shift causes strong and realistic aggregate effects. The model is intended to describe the aggregate effects of the shift away from investment goods that occurs in a recession.

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## I. Introduction

Modern economies experience substantial fluctuations in aggregate output and employment. In recessions, employment falls and unemployment rises. In the years immediately after a recession, the labor market is slack—unemployment remains high and the vacancy rate and other measures of employer recruiting effort are abnormally low. This paper places some standard elements of modern thinking about the labor market in a novel general-equilibrium model of aggregate fluctuations.

Lilien [1982] proposed that reallocation of labor across sectors played a key role in persistent fluctuations in unemployment. He attributed episodes of high unemployment to shifts in the composition of demand. During the time when workers are moving from declining to advancing markets, unemployment is high. Abraham and Katz [1986] pointed out an apparent defect in Lilien's view—it could not explain the slackness of the labor market following bursts of reallocation. The expanding sectors should be recruiting the workers from the shrinking sectors, behavior contradicted by the decline in vacancies that actually occurs in the periods following recessions.

More recently, another defect in the reallocation view has become apparent. The rate at which unemployed workers find new jobs is almost two orders of magnitude faster than the decline of unemployment back to normal following a recession (Hall [1995] and Cole and Rogerson [1999]). It seems difficult to interpret the period following an adverse shock as the move back to the stochastic equilibrium of the employment-unemployment transition process. That move takes a few months, not the several years that an aggregate recovery typically requires. Unemployed workers find jobs at a rate of about 60 percent per month, while unemployment returns to normal at about one percent per month.

Abraham and Katz concluded that the likely driving force for fluctuations in unemployment was aggregate demand, not reallocation. They viewed the slow movements

of unemployment after a recession as reflecting the gradual recovery of aggregate demand. But modern macroeconomics—devoted to the clarity that formal general-equilibrium modeling brings to the subject—has not found a satisfactory way to characterize aggregate demand in a GE model. Instead, a standard driving force in modern formal models is productivity. Because productivity shifts the labor demand curve—a coherent object in GE models—early researchers in formal GE macro thought that productivity could stand in for the elusive concept of aggregate demand. But the productivity driving force now seems an unlikely candidate. First, as Summers [1986] pointed out early in the development of modern GE macro modeling, technical regress is an unlikely candidate for the force causing a recession—we normally think of technical progress as monotonic. Rather, a more reasonable hypothesis is that measured productivity declines in recessions are induced by other adverse forces and are mainly measurement errors. Cochrane [1994] and Hall [1997] showed that productivity shocks could account for only the tiniest fraction of total movements of the economy, especially employment. Experience in the U.S. economy since the onset of the current recession in early 2001 confirms that productivity declines cannot be the primary driving force of recessions, because productivity rose as employment fell. Finally—and closest to this paper—Shimer [2002] shows that the productivity decline needed to explain the rise in unemployment during a recession is implausibly large, in a modern model of unemployment.

In my 1997 paper, I observed that the driving force for aggregate fluctuations looked more like a preference shift than any other shift that one could insert into a GE model. That paper, like most others in the GE tradition, did not treat unemployment as a distinct use of time. It interprets a rise in unemployment as a shift of work effort out of the labor market. I did not advocate this interpretation as a reasonable stopping point in the quest for an understanding of aggregate fluctuations, but rather as a demonstration of the need to treat unemployment as a productive activity, rather than as a form of leisure.

A line of research starting with Diamond [1982], Mortensen [1982], and Pissarides [1985]—nicely summarized in Pissarides's [2000] book and in Shimer [2002]—provides a

full account of unemployment as a productive use of time. I adopt their model—the DMP model—essentially intact in this paper.

The driving force I consider here is a preference shift among produced products. I take labor supply, in the sense of the time that workers divide between unemployment and employment, as completely inelastic, so there is no direct effect of the preference shift on unemployment. One might expect that the resulting model would resemble Lilien’s and suffer from the same inability to explain low vacancy rates. But I treat workers as having permanent affiliations with productive sectors, so there is actually no reallocation in the model. Instead, when preferences shift against one type of product, the labor market associated with that product becomes slack in a way described by the DMP model. A shift in the marginal rate of substitution schedule between two products might be thought to stimulate the demand for another product as much as it reduces the demand for one product. I show that this insight does not mean that a multi-sector GE model must have a boom in another sector whenever it has a recession in one sector. The strength of the spillover depends on the elasticity of product supply. I give what I believe is a plausible example where the spillover is tiny. An aggregate recession follows from a decline in the intensity of preferences for one product. The labor market for that product becomes slack without fully offsetting tightening of the labor markets associated with other products.

I consider the demand shift in a setup without physical capital, in order to keep the model transparent. I view the resulting model as a reasonable analog to a more realistic but much less transparent model with physical capital, where a decline in investment is the main change in the composition of output. I believe that the same analysis would apply to that type of change. In other words, I believe this approach can help understand recessions—such as the one that started in the U.S. in 2001—where the leading causal event seems to be a collapse of investment.

I start the paper with a simple accounting of labor-market dynamics, to clarify the distinction between the fairly rapid movement toward the stochastic equilibrium of the market and the slow movement of that equilibrium in response to changing economic

determinants. I then turn to the DMP model, which I develop in some detail even though I do not introduce any new elements. None of the published accounts of the model treats the case of arbitrary dynamic driving forces, which I need to embed the model in general equilibrium. I calibrate the model to data from the U.S. labor market and show how it incorporates speedy movement to stochastic equilibrium and slow movements of the equilibrium itself. Then I embed the DMP model in a GE model with two sectors. Preferences include a shift factor that controls the intensity of preferences for the first product. I show that a decline in this factor that is reversed over the three years following the decline induces persistent unemployment in the market for that sector. Little happens in the other sector, so the slackness of half the labor market translates into half that about of slack in the aggregate data. I display the resulting aggregate data in terms of the unemployment-vacancy rate phase diagram. Most of the movements in that phase diagram are along the economy's Beveridge curve—vacancies are low during the time when unemployment is high.

Here is the story of a recession and recovery within the model: A shift of preferences hits one major sector. Jobs are destroyed initially, but the main effect is that the labor market in that sector slackens. Workers spend more time looking for work and less time working. Employers find it easy to locate new workers and spend less effort recruiting, so vacancies fall. These slack conditions gradually wear off as the preference shift wears off. Other sectors enjoy only a small favorable spillover of demand from the preference shift. Their labor markets tighten, but not by nearly enough to prevent a substantial increase in aggregate unemployment and decline in vacancies.

## II. Dynamics of Fluctuations

### A. Basic Dynamics

The following is a simple model of labor-market dynamics that will help explain some of the issues in this paper. Let  $u_t$  be the unemployment rate,  $\phi_t$  be the job-finding rate (the fraction of unemployed workers in month  $t$  who are employed in month  $t+1$ ), and  $s_t$  the separation rate (the fraction of employed workers in month  $t$  who are unemployed in month  $t+1$ ). With the labor force normalized at 1, the law of motion of unemployment is

$$u_t = (1 - \phi_{t-1})u_{t-1} + s_{t-1}(1 - u_{t-1}). \quad (2.1)$$

I linearize around stationary values  $u$ ,  $\phi$ , and  $s$ :

$$u_t - u = (1 - \phi - s)(u_{t-1} - u) + (1 - u)s_{t-1} - u\phi_{t-1}. \quad (2.2)$$

The weighted average of separations and new hires,  $f_t = (1 - u)s_{t-1} - u\phi_{t-1}$ , is the driving force for unemployment. Higher separations raise unemployment and an increase in new hires lowers unemployment. I will use the following vocabulary to describe movements of unemployment in this framework: *job destruction* occurs when  $s$  shifts upward, *job preservation* occurs when  $s$  shifts downward, *market slackening* occurs when  $\phi$  shifts downward, and *market tightening* when  $\phi$  shifts upward. Notice that the measure of job destruction introduced by Davis and Haltiwanger [1990] does not isolate job destruction in the sense that I (and many others) use the term—they measure declines in employment at the plant level, which could occur either because of job destruction in my sense (workers losing jobs) or because of market slackness (fewer workers hired, so normal attrition results in lower employment).

The matching frictions captured by  $\phi$  and  $s$  make unemployment a convolution of the driving process. Consider the hypothesis that separations and new hires are driven by a common shock  $\varepsilon_t$ :

$$f_t = A(L)\varepsilon_t. \quad (2.3)$$

Then the time-series process for unemployment is

$$u_t = \frac{1}{1 - (\phi + s)L} A(L)\varepsilon_t. \quad (2.4)$$

If the observed time-series process for unemployment is

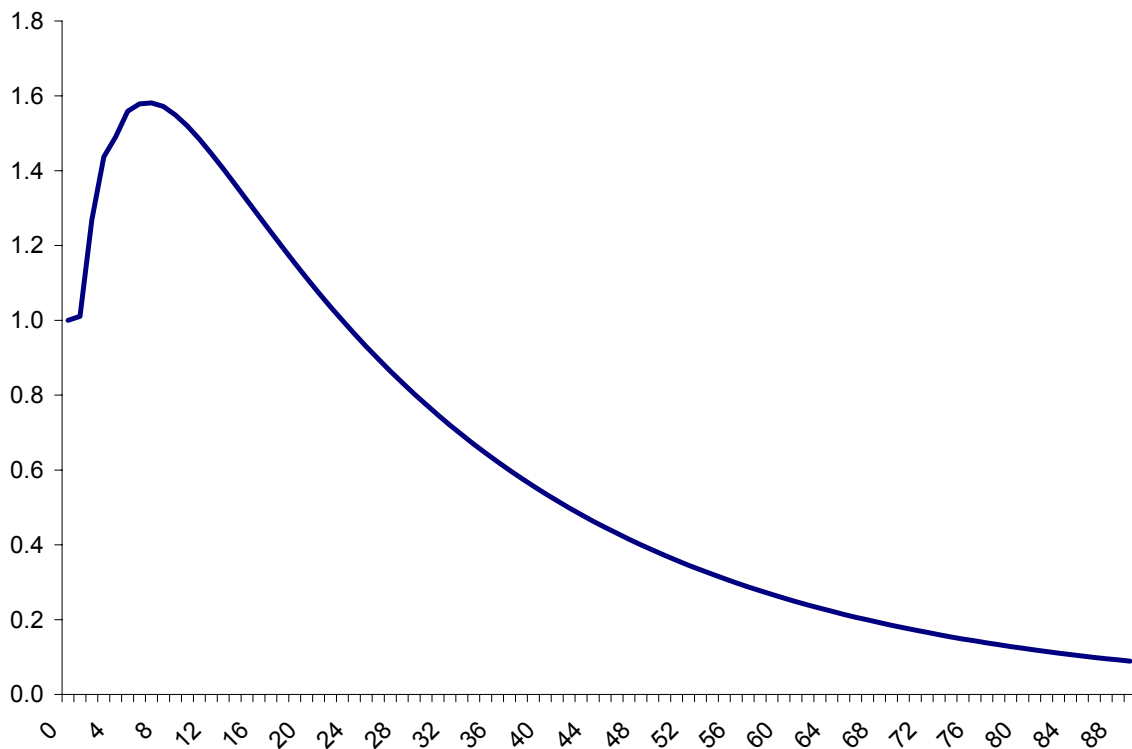
$$u_t = B(L)\varepsilon_t, \quad (2.5)$$

the coefficients of the underlying forcing process can be recovered from

$$A(L) = [1 - (\phi + s)L]B(L). \quad (2.6)$$

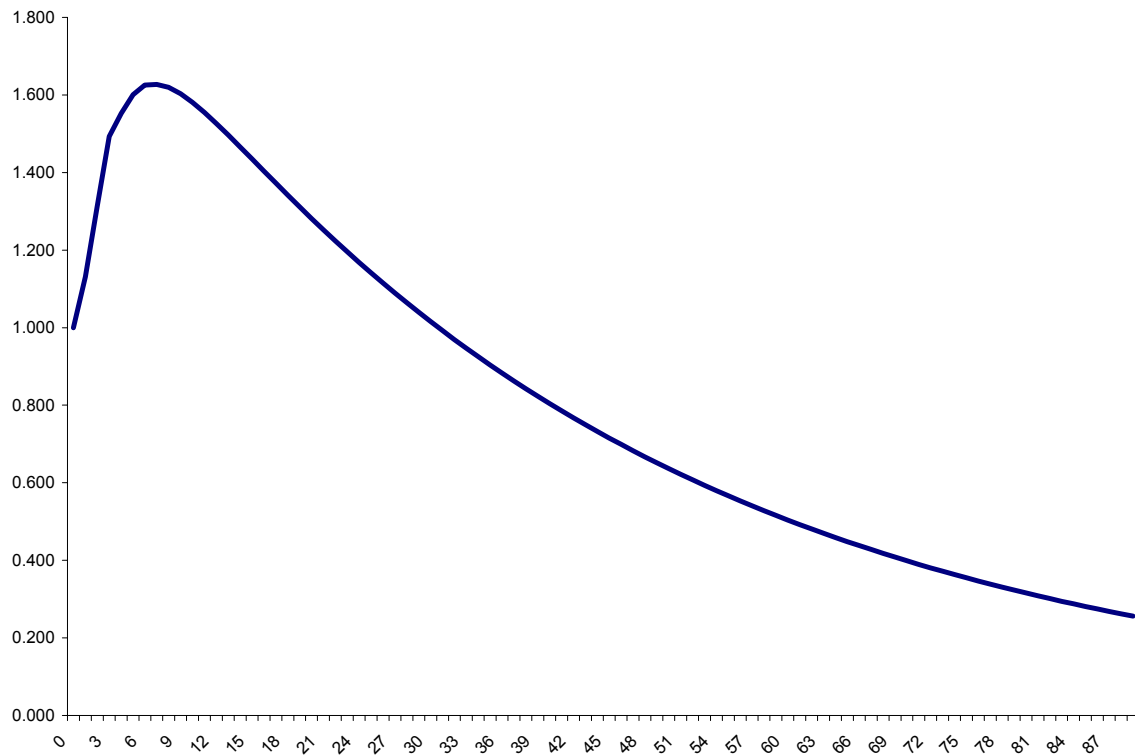
## B. Relative Importance of Persistent Driving Forces and Persistence Induced by Friction

Figure 1 shows the estimated moving-average coefficients for the time-series representation of monthly unemployment rate in the U.S. since the household survey began at the beginning of 1948. I derived these coefficients from a 4th-order autoregression, which seemed the most satisfactory of a variety of alternatives, all of which had similar MA representations. The effect of an innovation in unemployment rises to a peak 7 months after the innovation occurs and then subsides quite slowly. Unemployment is highly persistent—its monthly autocorrelation is about 0.99. It takes 42 months—three and a half years—for the effect of an innovation to decline to half its initial value. After 67 months, the effect is still 20 percent of the initial effect.



**Figure 1. MA Representation of the Process for National Unemployment**

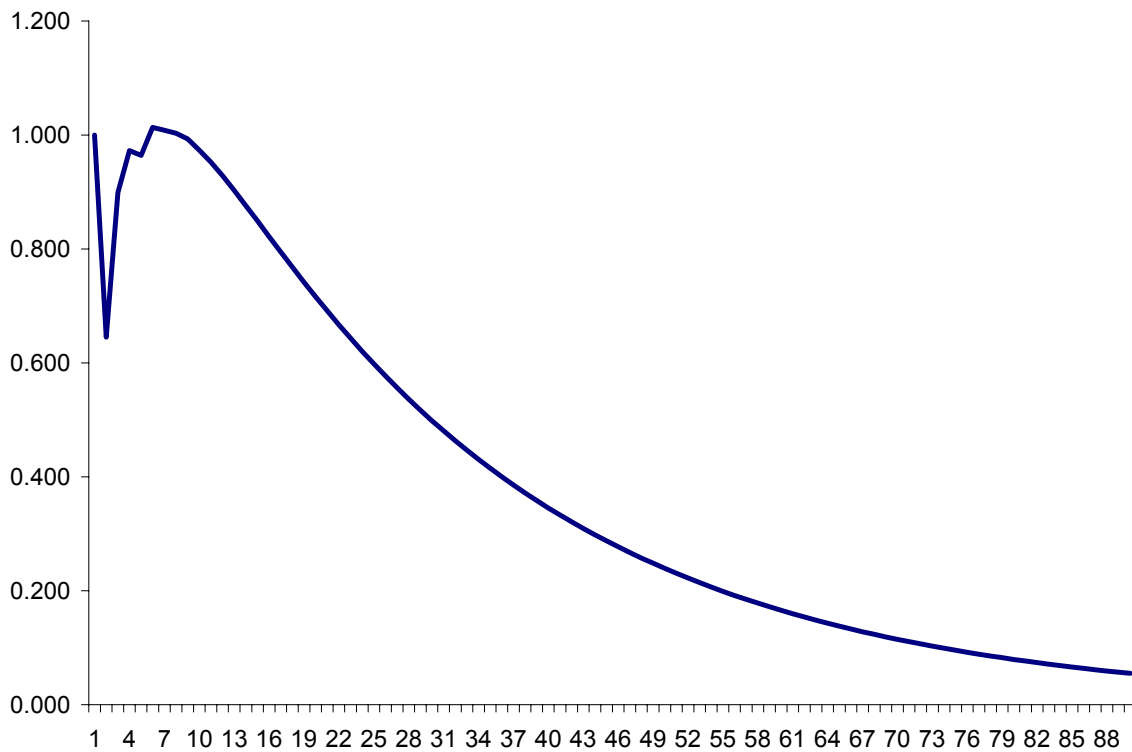
Figure 2 shows the MA representation of the process for unemployment measured at the state level. These figures are derived from a fourth-order autoregressive process with state-specific constants in a panel of monthly data starting in 1978. Unemployment at the state level has much the same persistence as national unemployment.



**Figure 2. MA Representation of the Process for State Unemployment**

To measure the separation and job-finding rates, I draw on a new body of data—the Bureau of Labor Statistics’ Job Openings and Labor Turnover survey, which started in December 2000. The survey reports that the average separation rate from December 2000 through November 2001 was 3.4 percent per month. It also reports the number of new hires over the period, which I divided by the corresponding seasonally unadjusted figures for total unemployment to calculate the average job-finding rate, 62 percent per month.

Figure 3 shows the moving-average representation for the driving variable, derived according to equation (1.6) using these separation and job-finding rates.



**Figure 3. Moving-Average Coefficients for the Process for the Forcing Variable**

An innovation in the forcing variable results in an immediate movement of the variable, normalized at one. The rest of the response is hump-shaped, rising from about 60 percent of the initial effect in the following month, back up to a response about as large as the initial response, and followed by a long, slow decline. According to these calculations, almost all of the persistence of unemployment arises from persistence in the driving force rather than persistence induced by matching friction. This feature of realistic dynamic labor-market models has been discussed earlier by Hall [1995] and Cole and Rogerson [1999].

A one-month burst of job destruction could raise unemployment substantially, but the unemployment would melt away at a monthly rate of  $\phi + s = 0.634$  and would disappear in just a few months. Prolonged periods of high unemployment are the result of forces that

raise job destruction for extended periods or forces that create slacker markets. Models of labor markets generally relate job destruction to changes in the variables that determine labor-market equilibrium, such as product and input prices. To generate continuing job destruction, product prices would need to fall continuously relative to input prices, for example. The models relate market slackness to the levels of the same variables—the market becomes slack when the product price remains at the same low level relative to input prices. In these models, there is a presumption that an innovation triggers a burst of job destruction followed by a long period of slack. I will develop a model with these implications later in the paper. Figure 3 invites the interpretation that the immediate spike corresponds to a transitory increase in separations while the hump shape of the rest of the MA coefficients corresponds to the persistent slackening of the labor market. The model supports this interpretation.

### **III. Model of the Labor Market**

#### **A. Theory**

The following is derived fairly directly from Pissarides [2001] and Shimer [2002], but extended to consider the full dynamic equilibrium with variations over time in driving forces. Pissarides considers only dynamics with unchanging driving forces and Shimer considers only the dynamics of stochastic productivity. I also use discrete time to facilitate computations, and I state prices in Arrow-Debreu time-0 form rather than using an interest rate. Finally, I posit that workers receive a single lump-sum payment,  $W$ , at the time they begin work, instead of receiving a wage payment each period. As Shimer [2002] points out, the way that compensation is spread over a job is irrelevant to the allocations generated by the model. The implicit employment contract has the worker and employer agree that employment will continue, without further compensation, until continuation would be inefficient. Pissarides and Shimer show how to extract the implicit wage each

period under the view that the Nash bargain is struck continuously. For the present purposes, nothing is gained by adding these calculations.

Let  $x_t$  be the ratio of vacancies to unemployment in the market and let  $\phi(x_t)$  be the per-period probability that a searching worker will find a job (the underlying matching technology has constant returns). Let  $\rho(x) = \frac{\phi(x)}{x}$  be the per-period probability that an employer will fill a vacancy.  $\phi$  is an increasing function and  $\rho$  is a decreasing function. I let  $\lambda_t$  be the value a worker enjoys when searching (leisure value or unemployment compensation). The price of output is  $p_t$ . Other inputs needed to produce the unit of output cost  $c_t$ . And it costs  $k_t$  in recruiting costs to hold a vacancy open for one period.

I distinguish  $N$  employment states, each with productivity  $z_i$ , with  $z_1 = 1$ . I assume that workers either remain in their previous state or advance to the next state each period—the probability of advance is  $s_i$ . All workers begin jobs in state 1.

The model is conveniently specified in terms of Bellman value-transition equations. Let  $U_t$  be the value a worker associates with being unemployed and searching for a new job and let  $E_{i,t}$  be the value of being employed in state  $i$ .  $E_{1,t}$  is the value the worker associates with being in a new job, *after* receiving the lump-sum wage payment. Let  $J_{i,t}$  be the value the employer associates with a filled job in state  $i$ .  $J_{1,t}$  is the value associated with a filled initial job, after making the wage payment. I assume, as is standard in this literature, that there is free entry to the creation of vacancies, so the value associated with an unfilled vacancy is zero.

I assume a random firm-specific component of output,  $\varepsilon$ , distributed independently over time and across firms, with mean zero and cumulative distribution function  $F$ . For convenience and without losing anything important, I assume that there is no random component for state 1 jobs. Some states may not generate positive joint value—that is, the breakup value  $U_t + \lambda_t$  may exceed the joint value of the continued job,

$E_{i,t} + J_{i,t} + z_i p_t - c_t + \varepsilon$ . In that case, the job should end. I restrict the driving forces so that this never occurs in state 1. I let  $q_{i,t}$  be the fraction of jobs that continue:

$$q_{i,t} = 1 - F(U_t + \lambda_t - E_{i,t} - J_{i,t} + z_i p_t - c_t) \quad (3.1)$$

Decreases in the  $q$ s generate job destruction and increases in the  $q$ s generate job preservation. In a stationary state, at least one of the productivity states will have a surplus close to zero. Its  $q$  will be around 0.5. If an adverse shock such as a price decline occurs, the surplus will cross into negative territory and  $q$  will fall. A fraction  $-\Delta q$  of the workers in that category will lose their jobs. If a favorable shock occurs, job preservation will be the result—a fraction  $\Delta q$  of workers who would have lost their jobs will be retained instead. For small shocks, job destruction and preservation are symmetric. For large shocks, job destruction can be more rapid than job preservation, because a number of states can become uneconomic at once and all the jobholders will lose their jobs, whereas if a number of previously uneconomic states suddenly have positive surpluses, they will only fill over time. This asymmetry does not appear in the model, nor does it appear to be important for fluctuations in the aggregate U.S. economy.

The value transition equations are:

$$U_t = \phi(x_t)(W_{t+1} + E_{1,t+1}) + (1 - \phi(x_t))(U_{t+1} + \lambda_{t+1}) \quad (3.2)$$

$$E_{i,t} = (1 - s_i) \left[ q_{i,t+1} E_{i,t+1} + (1 - q_{i,t+1})(U_{t+1} + \lambda_{t+1}) \right] \\ + s_i \left[ q_{i+1,t+1} E_{i+1,t+1} + (1 - q_{i+1,t+1})(U_{t+1} + \lambda_{t+1}) \right] \quad (3.3)$$

$$J_{i,t} = (1 - s_i) q_{i,t+1} (J_{i,t+1} + z_i p_{t+1} - c_{t+1}) + s_i q_{i+1,t+1} (J_{i+1,t+1} + z_{i+1} p_{t+1} - c_{t+1}) \quad (3.4)$$

$$0 = \rho(x_t)(J_{1,t+1} + z_t p_t - c_t - W_{t+1}) - (1 - \rho(x_t))k_{t+1} \quad (3.5)$$

The lump-sum wage payment is the result of a Nash bargain between worker and firm. If a potentially matched worker and firm do not form a match, the worker achieves a value of  $U_t + \lambda_t$  and the firm a value of zero. They achieve a joint value of  $J_{1,t} + p_t - c_t + E_{1,t}$  if they do form a match, so the surplus from the match is  $J_{1,t} + p_t - c_t + E_{1,t} - U_t - \lambda_t$ . The worker receives a value  $W_t + E_t$  from the match. The condition for a symmetric Nash bargain is that the worker receive the non-match value,  $U_t + \lambda_t$ , plus half the surplus:

$$W_t + E_{1,t} = U_t + \lambda_t + \frac{1}{2}(E_{1,t} + J_{1,t} + p_t - c_t - U_t - \lambda_t) \quad (3.6)$$

This model can be solved in reverse time. Given values of  $U_{t+1}$ ,  $E_{i,t+1}$ ,  $J_{i,t+1}$ ,  $W_{t+1}$ , and  $x_t$ , the first three equations give  $U_t$ ,  $E_{i,t}$ , and  $J_{i,t}$ . The Nash bargain gives the lump-sum wage,  $W_t$ . Then the zero-profit equation for time  $t$  can be solved for the new value of the vacancy/unemployment ratio,  $x_{t-1}$ . The reverse solution essentially forms present discounted values by adding up values stated in time-0 prices. Thus one can find an accurate dynamic path by starting at a fairly distant horizon with low terminal values of the variables and iterating back to the present. I take stationary-state values multiplied by  $\beta^T$  as my terminal values, where  $\beta$  is the ratio of successive values of the driving prices in the long run (the discount factor).

The result of this calculation is the equilibrium path for  $x_t$ . To find the resulting paths of unemployment, employment, and vacancies, I iterate forward from the given initial unemployment rate. Suppose that the labor force is normalized at one. Then the laws of motion of employment are:

$$n_{1,t} = \phi(x_{t-1})u_{t-1} + (1-s_1)n_{1,t-1} \quad (3.7)$$

$$n_{i,t} = q_{i,t} \left[ (1-s_i)n_{i,t-1} + s_{i-1}n_{i-1,t-1} \right], \quad i > 1 \quad (3.8)$$

and unemployment is:

$$u_t = 1 - \sum n_{i,t} . \quad (3.9)$$

The vacancy rate is

$$v_t = x_t u_t . \quad (3.10)$$

## B. Functional Forms and Parameter Values

Job destruction occurs when the surplus in a productivity state crosses from positive to negative and job preservation when the surplus transits zero from below. For small price movements, job destruction and preservation occurs around the state that has stationary surplus closest to zero. Hence the model with only two productivity states conveys most of the behavior of a more elaborate model. I calibrate the model so that state 2 has an average surplus of zero in the stationary state, so that  $q_2 = 0.5$ . A price decrease drives the average surplus in this state below zero and destroys some of its jobs and a price increase raises it above zero and preserves more state-2 jobs.

I take the job-finding probability to be

$$\phi(x) = \omega x^{1/2} \quad (3.11)$$

and the idiosyncratic cdf to be

$$F(\epsilon) = \frac{1}{1 + e^{-\chi\epsilon}} . \quad (3.12)$$

The model operates at a weekly frequency, to avoid the danger that either the job-finding rate or the job-filling rate might exceed one. I calibrate to the following data on the U.S. labor market:

<i>Symbol</i>	<i>Concept</i>	<i>Value</i>	<i>Source</i>
$\phi$	Job-finding rate	0.62 per month	JOLTS and Household Survey
$v$	Vacancy rate	0.028	JOLTS
$u$	Unemployment rate	0.056	Household Survey historical average
$n_2$	Stationary fraction of workers in state 2	0.007	Roughly calibrated to initial spike in Figure 3

Notice that the value of the vacancy/unemployment ratio,  $x$ , is 0.5. I calibrate or estimate the following parameters:

<i>Parameter</i>	<i>Interpretation</i>	<i>Value</i>	<i>Source</i>
$\omega$	Intensity of matching	0.212	Calibration
$s_1$	Weekly transition rate from state 1 to state 2	0.00815	Calibration
$s_2$	Weekly transition rate from state 2 to unemployment	0.25	Calibration
$\lambda$	Flow value while searching (leisure or unemployment compensation)	0.4	Approximate replacement rate for unemployment compensation
$c$	Flow cost of other inputs	0.5	Approximate labor share in revenue in typical industry
$k$	Flow cost of a vacancy	0.255	Calibration
$\beta$	Discount factor	0.999014	Corresponds to 5 percent annual rate
$\chi$	Tightness of distribution of idiosyncratic shock	400	Rough match of job destruction/preservation effect in Figure 3

I normalize the stationary level of the price,  $p$ , to one. The calibration solves the model comprising 12 equations: (3.1), (3.2), two each of (3.3) and (3.4), (3.5) through (3.8), (3.11), and the following equation that makes state 2 sensitive to a shock by setting its surplus to zero:

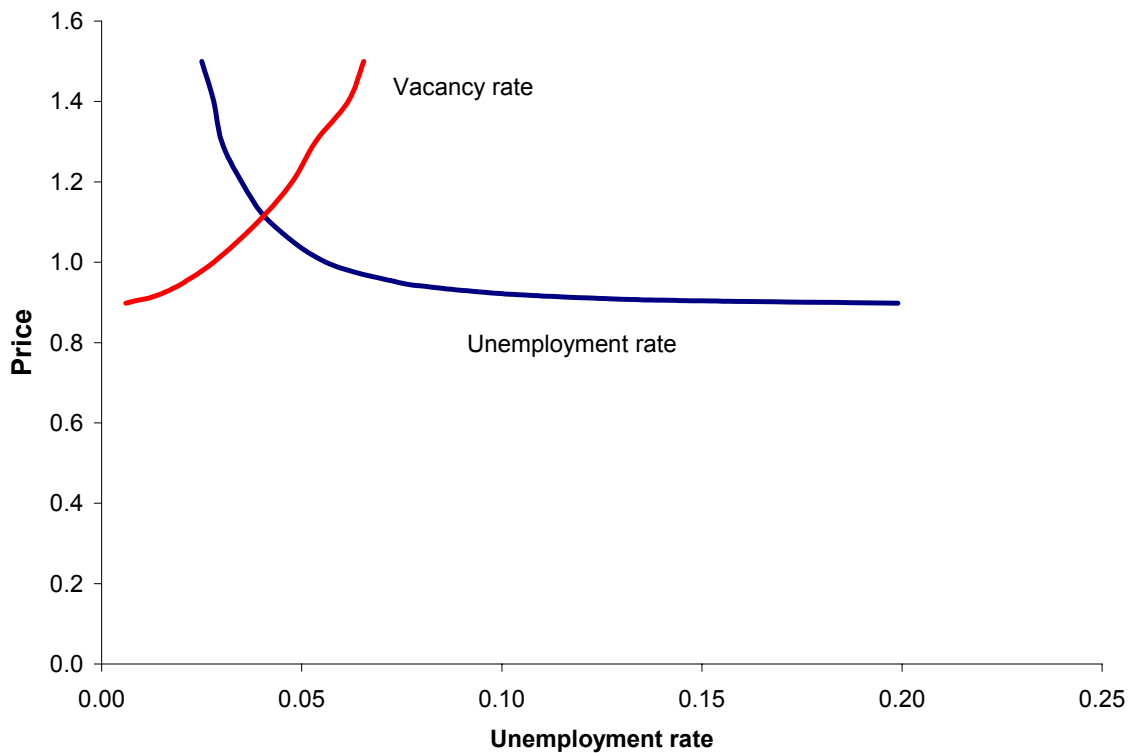
$$J_2 - z_2 p - c + E_2 - U - \lambda = 0 \quad (3.13)$$

I define the stationary state by setting the time  $t+1$  values equal to  $\beta$  times the time  $t$  values. The solution gives the stationary values of 7 endogenous variables:  $U$ ,  $E_1$ ,  $E_2$ ,  $J_1$ ,  $J_2$ ,  $W$ , and  $q_2$ , and 5 calibrated parameters:  $\omega$ ,  $k$ ,  $s_1$ ,  $s_2$ , and  $z_2$ . The values of the variables are:

<i>Variable</i>	<i>Interpretation</i>	<i>Value</i>
$U$	Value while searching	495
$E_1$	Value of future work while working in a job in state 1	444
$E_2$	Value of future work while working in a job in state 2	495
$J_1$	Value of worker to the firm in state 1	52.0
$J_2$	Value of worker to the firm in state 2	0.1
$W$	Lump-sum wage	51.9

Figure 4 shows the relation between the product price,  $p$ , and unemployment in the stationary state. The relation abstracts from transitory matching frictions and from effects from changing driving forces. In the stationary state, all prices and costs decline in proportion  $\beta$  each period. The relation is distinctly nonlinear. Unemployment becomes perfectly elastic with respect to the price at a critical price that is just below  $c + \lambda$ , the sum

of the non-labor input cost and the outside opportunity cost of labor (just below because there is a tiny amount of option value associated with the idiosyncratic disturbance  $\varepsilon$ ). As the product price becomes higher, the unemployment rate falls, because the product price determines the inside opportunity cost of labor. With more expensive labor, the market uses less unemployment and more vacancies to accomplish matching.



**Figure 4. Stationary-State Relations among Price, Unemployment, and Vacancies**

The curves in Figure 4 display properties that are central to the view of the labor market developed in this paper. Although the full model takes account of the aspects of the labor market not embodied in the figure—matching dynamics triggered by job destruction and job preservation and the effects of expected future changes in driving forces—the curves tell the main story of the model. In labor markets where product demand is weak, unemployment can rise to high levels. In markets where product demand is strong,

unemployment can only fall a relatively small amount below its normal level. Product supply is relatively inelastic. A decline in price raises unemployment and lowers vacancies—it moves the economy along its Beveridge curve.

### C. Dynamic Responses to Price Shocks

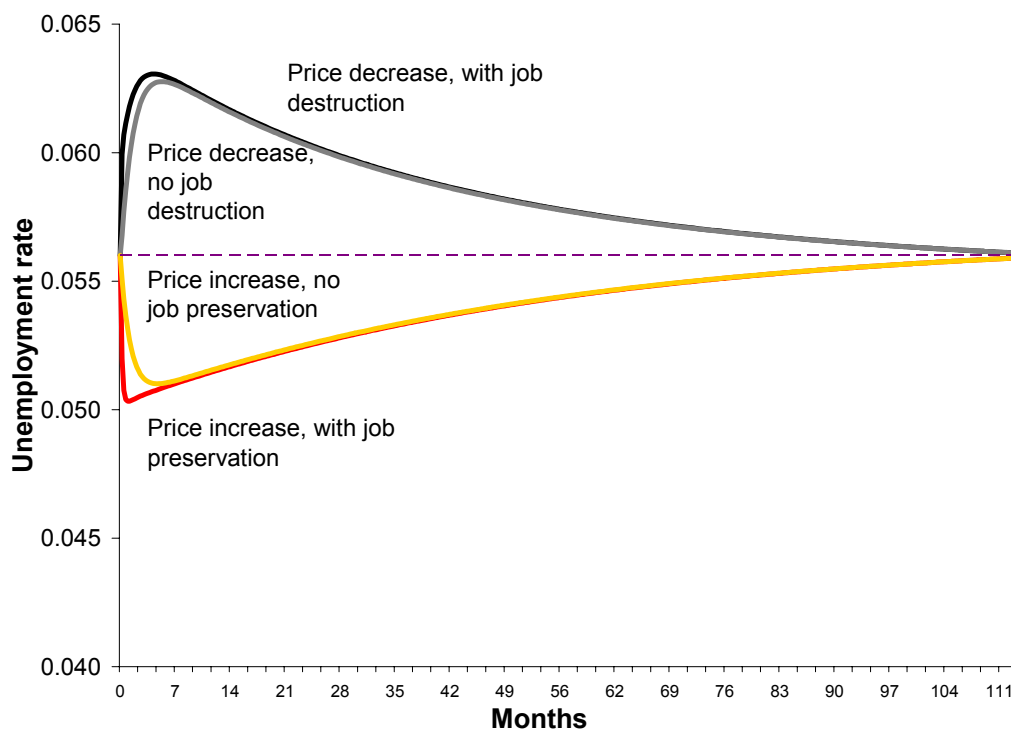
I calculate four impulse response functions for unemployment, when the product price receives a persistent shock of the form:

$$p_t = \left( 1 + \delta \frac{\theta^t - \theta^T}{1 - \theta^T} \right) \beta^t \quad (3.14)$$

The price jumps by the amount  $\delta$  in the first period and converges to the path  $\beta^t$  as time passes, at a rate controlled by the parameter  $\theta$ . The perturbation goes to zero at the horizon,  $T$ , so I can set the terminal values to the stationary values. I start the model from its stationary distribution of the labor force among the two job states and unemployment (93.7 percent, 0.7 percent, and 5.6 percent). Two of the response functions are for the model described above, with positive and negative price shocks, with  $\delta = -.05$  and  $.05$ . The other two response functions are for the same price shocks, but in models where job destruction and preservation is blocked ( $q_2 = 0.5$  is imposed, independent of the actual surplus for state-2 jobs).

Figure 6 shows the four response functions. At the top are the two for price decreases. The darker curve includes job destruction and the lighter one does not. Job destruction makes a substantial contribution to unemployment in the first few months. The retention rate,  $q_2$ , falls from its stationary level of 50 percent to 21 percent, which immediately releases 0.29 percent of the labor force into unemployment. But job destruction ends quickly and the extra unemployment it causes is quickly absorbed by new hires. The response function with job destruction coincides with the one without

destruction after about 7 months. The longer-run dynamics are controlled entirely by market slackening.

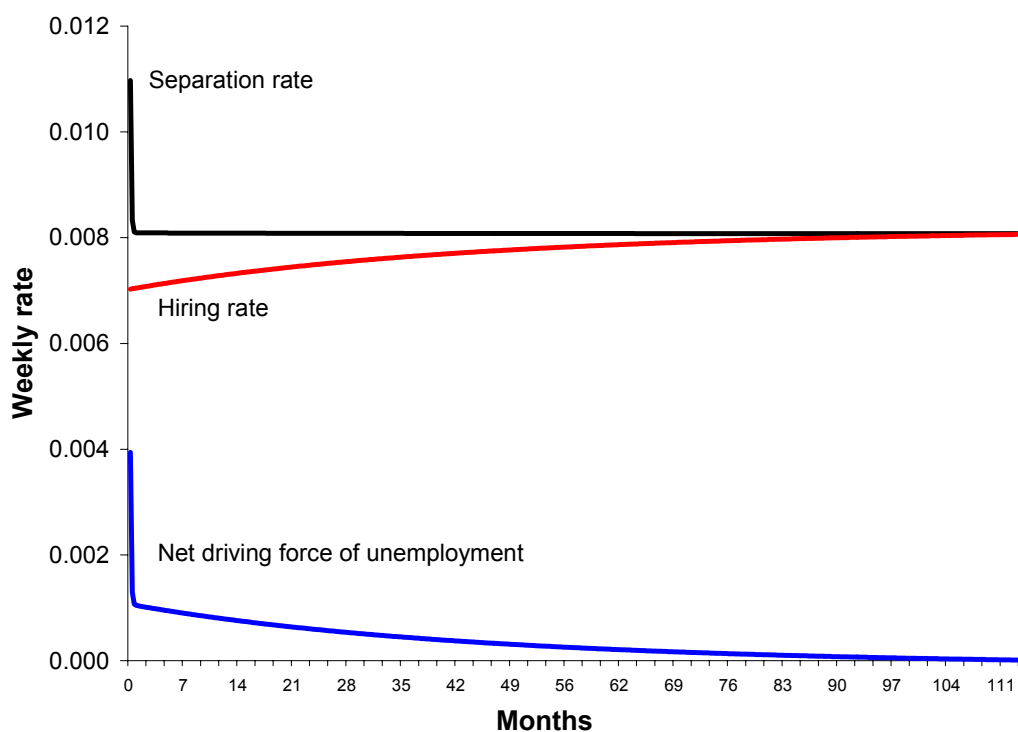


**Figure 5. Responses to Price Increases and Decreases, with and without Job Destruction and Job Preservation**

The lower part of Figure 5 shows responses to the corresponding price increases, with and without job preservation. Again, the two lie virtually atop one another after the first few months. The asymmetry of the job preservation and job destruction responses is a result of the coarseness of the model—lowering the retention rate in state 2,  $q_2$ , from 0.5 to a low value (the job-destruction effect) reduces state-2 employment by less than an increase to a value close to one (the job-preservation effect) increases state-2 employment. Although, in the stochastic equilibrium, state-2 employment has little effect on

unemployment—it is absorbed by state-1 employment—there is a strong transitory effect when state 2 either empties (job destruction) or fills up (job preservation).

Figure 6 shows the underlying driving forces in the model corresponding to Figure 3—the separation and hiring rates that enter the matching dynamics. The figure considers a price decline of the type described by equation (3.14). Unemployment is a geometric moving average of the net driving force, the separation rate less the hiring rate. As suggested by Figure 3, the net driving force consists of a spike from the initial job destruction caused by the price shock, followed by a long period of lower hiring rate that returns to normal at the same rate that the price returns to normal. While Figure 3 suggests that the price dynamics are hump shaped, Figure 6 is based on a first-order autoregression, with geometric decline.



**Figure 6. Separation Rate, Hiring Rate and Net Driving Force of Unemployment**

## IV. General Equilibrium

### A. Demand

To create a general-equilibrium environment that is not needlessly complex and opaque, I proceed in the following way: The economy has three goods. One, numbered zero, is a flow endowment to households. The other two are produced in industries with labor markets described above. The economy has no physical capital. Households order consumption streams according to the utility function,

$$U = \sum_{t=1}^{\infty} \beta^t u(y_{0,t}, y_{1,t}, y_{2,t}, \gamma_t). \quad (4.1)$$

$\gamma_t$  is a preference shift. I will consider paths of the form

$$\gamma_t = 1 + \delta \frac{\theta^t - \theta^T}{1 - \theta^T}. \quad (4.2)$$

Output satisfies

$$y_{i,t} = n_{1,i,t} + z_2 n_{2,i,t}. \quad (4.3)$$

Each industry uses  $y_0$  as a direct input and also to pay unemployment compensation. Thus

$$y_{0,t} = Y_0 - c(n_{1,1,t} + n_{1,2,t} + n_{2,1,t} + n_{2,2,t}) - \lambda(u_{1,t} + u_{2,t}). \quad (4.4)$$

I deal with allocations and do not exhibit income flows directly—unemployment compensation is financed by a non-distortionary tax system.

The preferences associate demand prices (marginal rates of substitution) with each allocation:

$$p_{i,t} = \frac{\frac{\partial U}{\partial y_{i,t}}}{\frac{\partial U}{\partial y_{0,0}}}. \quad (4.5)$$

The labor-market model of the previous section associates a labor allocation with a triplet of price vectors. General equilibrium is

*Definition:* An equilibrium comprises three price vectors,  $p_{i,t}$ , four employment vectors,  $n_{i,j,t}$ , three product vectors,  $y_{i,t}$ , two vacancy/unemployment ratios,  $x_{i,t}$ , ten Bellman value vectors,  $U_{i,t}, E_{1,i,t}, E_{2,i,t}, J_{1,i,t}$ , and  $J_{2,i,t}$ , and two lump-sum wage vector,  $W_{i,t}$ , satisfying the marginal rate of substitution condition, equation (4.5), the production function, equation (4.3), the material-balance condition for the endowment, equation (4.4), the labor-market value transition equations, (3.2) through (3.5), the Nash bargain condition, equation (3.6), and the laws of motion of employment, equations (3.7) and (3.8).

From the equilibrium, I calculate the unemployment rate from equation (3.9) and the vacancy rate from equation (3.10). I find the equilibrium by (i) setting the output vectors to their stationary-state values, (ii) finding the corresponding prices from the marginal rate of substitution condition, equation (4.5), (iii) finding the levels of employment and output from the labor-market model corresponding to those prices, (iv) updating the output vectors by a fraction of the difference between the new and old vectors, (v) returning to step (ii) if convergence has not been achieved.

I specify the utility kernel as:

$$\log y_{0,t} + \gamma_t \log y_{1,t} + \log y_{2,t} \quad (4.6)$$

I will work out the simplest possible case to bring out the basic issue in the effect of preference shifts on general-equilibrium allocations. Suppose that it takes one unit of  $y_0$  to

produce one unit of either  $y_1$  or  $y_2$ . Then equation (4.4) becomes  $y_0 = Y - y_1 - y_2$ . From the constant expenditure shares property of log preferences,

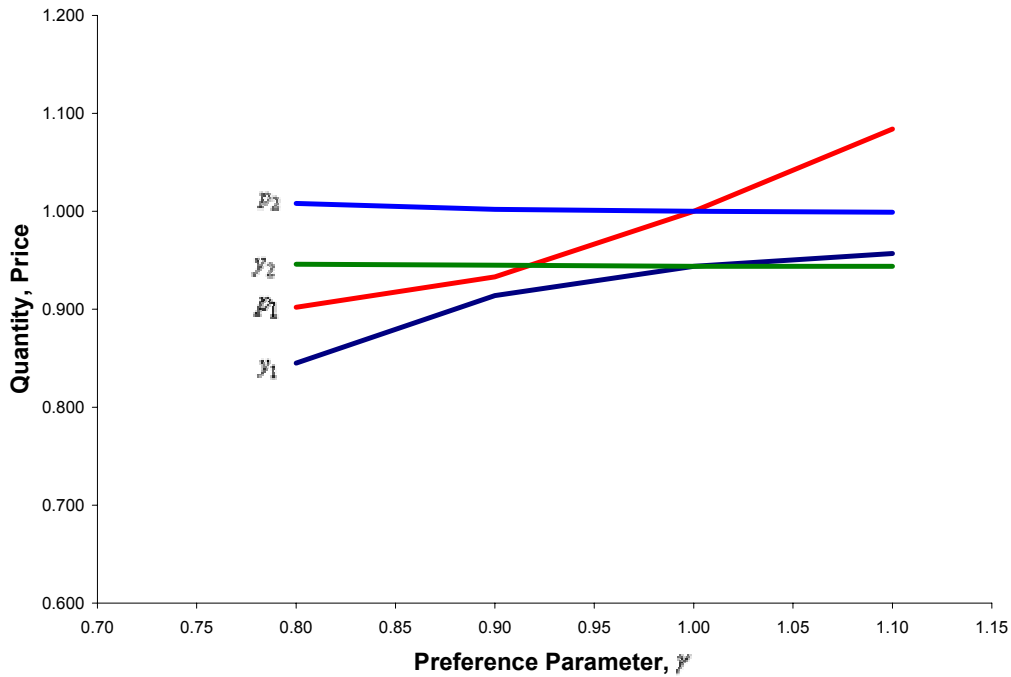
$$\frac{p_2 y_2}{Y - y_1 - y_2} = 1. \quad (4.7)$$

Thus, at the point  $y_2 = y_2 = p_2 = \gamma = 1$ ,

$$\frac{dy_2}{d\gamma} = -\frac{1}{2 + \frac{1}{\varepsilon_2}} \frac{dy_1}{d\gamma}. \quad (4.8)$$

Here  $\varepsilon_2$  is the price elasticity of supply for good 2. If the supply is infinitely elastic, good 2 changes by half the amount of good 1 when the preference parameter shifts (the endowment good absorbs the other half). If supply is less than infinitely elastic, the response is smaller. As I noted earlier, the elasticity of supply is fairly low in the model of this paper, because labor does not move from one market to another and unemployment can only be driven to low levels by quite a high product price.

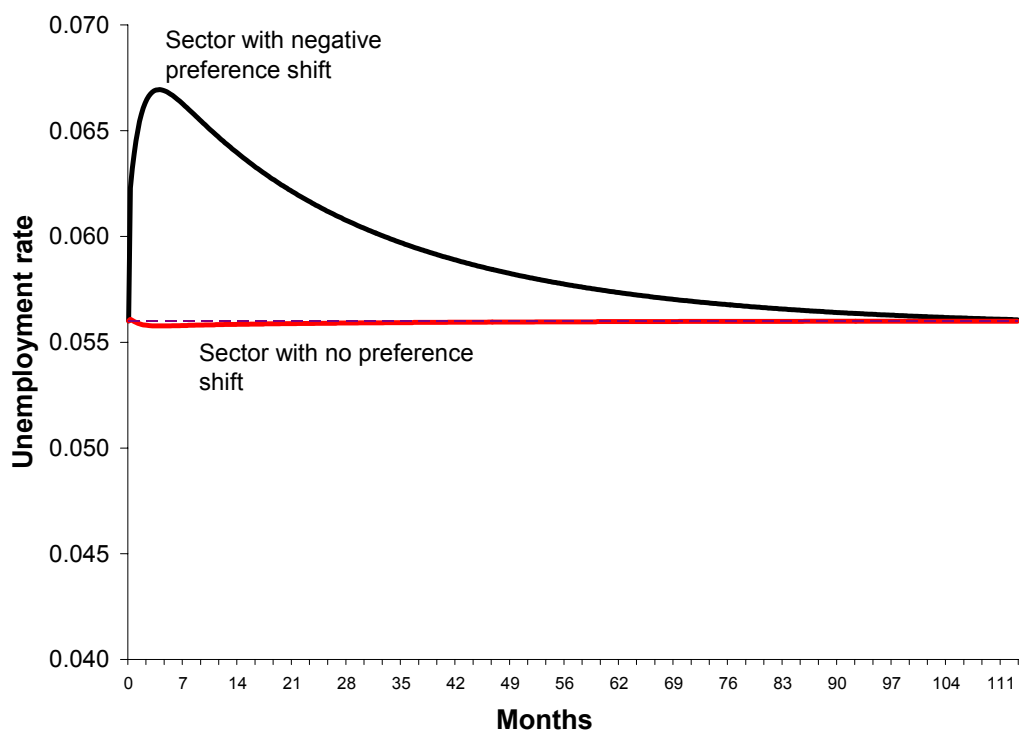
To bring in the basic supply elasticity from the labor-market model developed earlier, I examine the stationary values of the variables as functions of the parameter shift. The horizontal axis of Figure 8 is the value of the parameter  $\gamma$ , the intensity of preference for the first produced good. As expected, a higher preference intensity for the good raises both its price,  $p_1$ , and quantity,  $y_1$ . The curve for the price is convex and that for output is concave, reflecting the same curvature as in Figure 4. The supply function for good 1 is more elastic at lower levels of output and less elastic for higher levels, when unemployment in that sector is pressing against its lower limit of zero.



**Figure 8. Stationary Output and Price in the Two Sectors as Functions of the Preference Parameter**

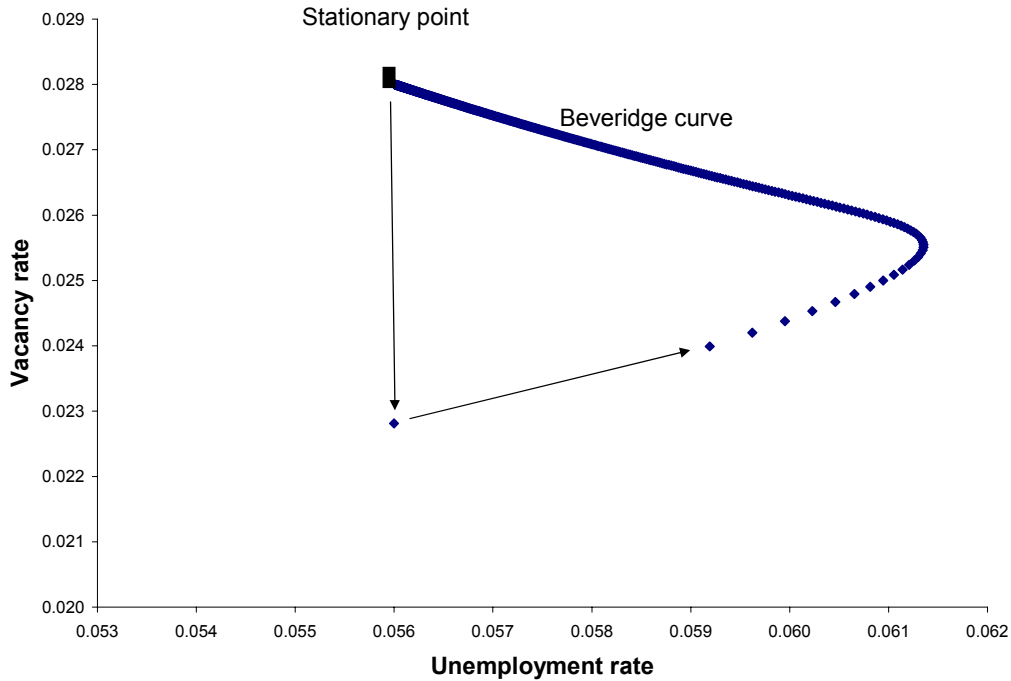
The important message of Figure 8 is that a decrease in the intensity of preference for the first good does not stimulate output in the other sector nearly as much as it decreases output in the first sector. As equation (4.8) illustrates, the cross response is limited by the elasticity of supply in the other industry.

Figure 8 shows the dynamic responses of unemployment in the two sectors to the persistent shift in the intensity parameter for good 1, perturbed according to equation (4.2). The preference parameter drops initially by 5 percent, then recovers at a weekly rate of 0.7 percent. Sector 1, under the influence of the negative preference shift, shows a large increase in unemployment. Sector 2 shows a small decrease. The net effect of the shift is to raise total unemployment.



**Figure 8. Dynamic Responses of Unemployment to Persistent Preference Shift**

Figure 9 shows the dynamics of the economy, aggregated over the two sectors, in the form of an unemployment-vacancy rate phase diagram. The challenge to general-equilibrium models of aggregate fluctuations has been to find a driving force that moved the economy along its Beveridge curve. In a recession, the labor market softens, unemployment rises, and the vacancy rate falls. Figure 9 shows that the model captures this property, once the brief period of initial dynamics is completed. The vacancy rate—a jump variable in the model—moves immediately to its lower level, while the unemployment rate—a state variable in the model—moves quickly to its stochastic equilibrium. From that time onward, the aggregate labor market moves along its Beveridge curve, with gradually falling unemployment and rising vacancies, as the preference shock gradually subsides.



**Figure 9. Unemployment-Vacancy Rate Phase Diagram for the Persistent Preference Shock**

Figure 9 shows that the model replicates the key feature of aggregate movements in the labor market that has eluded other models that do not rely on counterfactual large movements in productivity. A negative shock raises unemployment and lowers vacancies on average across the economy.

## V. Concluding Remarks

The key ingredients in the view of aggregate fluctuations developed in this paper are, first, the received analysis of unemployment as a productive use of workers' time, governed by substitution toward unemployment when the opportunity cost declines; second, the immobility of labor among sectors over the relevant period of time (a few

years); and, third, the specification of a preference shift among the two types of output as a driving force. Plainly immobility is not tenable over the longer run. But the importance of industry-specific skills, especially among experienced workers, and the high costs of geographic mobility introduce enough friction to make the assumption a useful starting point.

As I have stressed, I view the preference shift as a stand-in for shifts in the composition of demand that come from other sources, such as fluctuations in investment. A more complex model would consider investment explicitly, with, I believe, similar conclusions to those reached here. A deeper question is the source of the fluctuations in investment in that model. The forces that led to the collapse of investment—especially investment in computers and communications networks—are not well understood. I believe that this model helps understand the consequences of the collapse if not its cause.

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