Public Announcements, Adjustment Delays and the Business Cycle*

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Abstract

I study the effects of a lack of common knowledge on nominal adjustment in a dynamic price-setting game with incomplete information. In particular, I show how the speed of price adjustments following a nominal or real shock depends on the information structure among price-setters. The provision of public information leads to a reduction of higher-order uncertainty, and hence to more rapid price adjustments, but it potentially comes at the cost of an increased exposure to informational noise. I extend my analysis to allow for other disturbances, showing that higher-order uncertainty may account for the persistence of any kind of shock. Finally, I reconsider the role of monetary policy and discuss how the central bank's policy actions may act as a focal point for market beliefs and hence affect nominal and real adjustment through its "coordination effect".

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1 Introduction

What is the relation between the supply of money, prices and real output in the short run? In particular, why do prices not adjust immediately after a money supply shock? The theoretical literature around these two questions has emphasized two potential causess of incomplete nominal adjustment, each of which has led to important subsequent insights on the dynamics of price adjustments and expectations about the conduct of monetary policy: lack of information or "misperceptions" originally developed by Phelps (1970) and Lucas (1972), and adjustment costs or real rigidities that prevent an immediate adjustment of pricing decisions.

Despite its theoretical success, the incomplete information model runs up against a powerful criticism, when used as a descriptive model of business cycles: The theoretical model predicts that prices should fully adjust once the information about aggregate shocks becomes available; however most macro-economic data is available after only short delays, and incomplete information can therefore not account on its own for the observed delays of price adjustment.

A recent article by Woodford (2001a) introduces strategic pricing into an incomplete information model similar to Lucas. Woodford also alters the information structure, assuming that information comes in the form of private signals to the price-setting decision makers, but the true state never becomes common knowledge. His analysis develops a simple intuition why monetary shocks have persistent real effects, even when they are accurately observed by price-setters: although firms may have precise information about the policy shock, they lack information about each other's beliefs; in fact, they have no information at all about what their beliefs are relative to the population average. In an environment of strategic complementarity, however, precisely such higher-order beliefs are necessary to forecast the behavior of other agents, and Woodford shows that the existence of higher-order uncertainty can lead to substantial nominal adjustment delays.

The purpose of this paper is to expand and develop Woodford's idea, and more generally, to study the effects of the market's information structure on the inflation/output dynamics. My first objective is to explore the effects of higher-order uncertainty in as simple and accessible a model as possible, that is also sufficiently flexible to extend or adapt to other contexts. The second objective is to study the role of the information structure in detail: Since the composition of the information structure determines the degree of higher-order uncertainty, as quantified by the departure from common knowledge, Woodford's results suggest that it should also have an influence on nominal adjustment. Using some recent insights from the theory of global games, which emphasizes the coordinating effect of public information (cf. Morris and Shin, 2000; Hellwig 2002), this paper then explores this link between the parameters of the information structure, in particular the precision of public and private information, and the process of nominal adjustment. For these purposes, the paper develops a version of the Lucas-Woodford model that is sufficiently simple and flexible to study the dynamic implications of a whole range of information structures.

My analysis of higher-order uncertainty in price adjustment leads to a reconsideration of the role and the conduct of monetary policy. The central bank is an important source of information for market participants, either through disclosures, or through policy actions. A key aspect of this information is that it is mostly public. Consequently, the conduct of monetary policy (i.e. the choice of targets and instruments, as well as the policy rules and information disclosures) affects the market information structure, and hence the macroeconomic adjustment process. Whereas most of the literature on the conduct of monetary policy emphasizes the desirability of "monetary transparency" (interpreted here to mean the provision of public information) for monitoring purposes in a principal-agent setting, taking the macro-economic dynamics as given, this paper argues that the central bank's information policy influences price adjustments through the market information structure.

Before highlighting the paper's main results, it will be useful to motivate my approach towards modelling the information structure, in particular the separation of information into public and private signals. Woodford studies an environment, in which individuals have access only to private information. He bases his information structure on the famous ”island” paradigm, which is meant to represent the informational differences between agents. Moreover, he appeals to limits in individual information processing capacities. As argued by Sims (2001), this can account for a ”private signal” information structure like Woodford's, even when the relevant economic data is publicly observed. While it is important to emphasize the role of differential information, the island paradigm has the drawback that it allows for no informational interaction among decisionmakers; in other words, in Woodford's economy a price-setter has no clue about how his information compares to the population average. More realistically, information processing within a market environment relies to a large extent on interaction and communication, and in the process, decisionmakers do learn about each other. In this respect, public disclosures and the processing of information by the media play an important role, and Morris and Shin (2001) emphasize the importance of such publicly available information as focal points for beliefs. As we shall see here, publicly observed policy actions have a similar role. The hypothesis that a decisionmaker has access to idiosyncratic and public signals can therefore by literally interpreted as capturing the informational differences across agents, whether they are the result of decisionmakers using different sources of information, or the result of limited information processing a la Sims, at the same time as taking into account the fact that various channels of communication serve to coordinate expectations. The degree to which the population is capable of processing information is captured by the parameters of the information structure, in particular the relative importance of public information, and the overall degree of noise. Alternatively, the separation of signals into public and private information may be motivated on theoretical grounds: as discussed in Hellwig (2002), these informational parameters are related to the degree of common p-belief, which quantifies the departure from common knowledge, and hence the degree to which individual decisionmakers are capable of efficiently coordinating their decisions.

The main results of this paper then discuss, how the inflation-output trade-off depends on the importance of higher-order uncertainty, measured as a function of (i) the degree to which pricing decisions are strategic complements, and (ii) the parameters of the information structure. In particular, the provision of public information reduces higher-order uncertainty and therefore leads to a faster adjustment of prices and smaller, less persistent effects of monetary shocks on output; on the other hand, a higher precision of public information may increase the macro-economic exposure to informational noise. This second effect is important in particular when public information is relatively noisy. The informational noise effect is at the heart of the static model by Morris and Shin (2001); indeed the formal analysis in this paper extends some of their results into a context that is of interest to dynamic macroeconomic theory.

In the second half of the paper, I then explore some of the welfare implications of changes in the market's information structure. The key insight of this section is, as stated above, that the monetary policy framework and the central bank's information disclosures influence the adjustment process of output and prices by altering the market's information structure. Augmenting the model by an objective function for the central bank along the lines of Kydland and Prescott (1977) and Barro and Gordon (1983), I show that the provision of precise public information may serve as an implicit commitment device against inflationary biases: By committing to disclose public information, the central bank reduces the effects of monetary shocks on output, thereby reducing the temptation to use monetary policy to stimulate output. However, the provision of public information may come at the cost of increasing informational noise, if there is a lower bound on the precision of public information.

The model provides closed-form solutions for prices and output in response to the underlying aggregate demand and supply disturbances, as well as informational noise; the underlying informational parameters can potentially be inferred from the data, and the model itself leads to some interesting testable implications. Moreover, the modelling approach appears to be sufficiently flexible to be applied in other macroeconomic contexts, in which strategic complementarities play a role, for instance investment or demand spill-overs. The paper thus makes the additional methodological contribution of proposing a solution technique for embedding higher-order uncertainty into dynamic macroeconomic models, and the arguments proposed here suggest that higher-order uncertainty coupled with strategic complementarities may be the cause of persistent effects not only of monetary shocks, but of other aggregate disturbances as well.

The remainder of the paper is organized as follows: Section 2 introduces the main model, the informational assumptions, and discusses the main analytical building blocks. Section 3 presents the paper's main theoretical results regarding the link between the information structure about monetary shocks and the inflation-output trade-off. Section 4 extends the analysis to allow for higher-order uncertainty regarding other disturbances; in particular, it is argued that the Woodford's insight regarding the persistence of monetary shocks applies also to supply shocks. Section 5 augments the initial model to discuss the welfare implications of information provision by the central bank, and informally discusses the role that the monetary policy regime, and in particular explicit monetary targets, have in reducing higher-order uncertainty. Section 6 concludes by discussing the paper's main implications and potential other applications.

2 The Model

2.1 Set-up

There is a large number of price-setters in monopolistic competition a la Dixit-Stiglitz.[1] When solving the price-setter's optimization problem, the first-order condition implies that each price-setter sets the log of his own price p_t^i according to

$$p_t^i = E_t^i (p_t) + (1 - r) E_t^i (y_t) \tag{1}$$

y_t denotes the log of real output relative to its steady-state value (which here is normalized to 0), p_t denotes the population average of log-price, and $E_t^i (\cdot) = E \left(\cdot \mid \Im_t^i\right)$ denotes the expectations operator conditional on i's information set as of date t, $\Im_t^i$, and $r \in (0, 1)$. The monetary authority targets the log of nominal output, denoted θ_t, which is assumed to be generated as an (exogenous) linear process from a sequence of monetary policy shocks $\{\varepsilon_t\}_{t=-\infty}^{\infty}$. Allowing for some finite degree k of integration,

$$\Delta^k \theta_t = \sigma \left[\varepsilon_t + \sum_{s=1}^{\infty} b_s \varepsilon_{t-s}\right], \tag{2}$$

where $\varepsilon_t \sim N(0, 1)$, and ε_t is iid over time. Substituting $\theta_t = y_t + p_t$ into (1) yields

$$p_t^i = r E_t^i (p_t) + (1 - r) E_t^i (\theta_t). \tag{3}$$

r thus measures the degree to which individual pricing decisions are strategic complements. As a consequence of the Dixit-Stiglitz model, r is increasing in the elasticity of substitution between different goods (i.e. in the degree of competition), and decreasing in the degree of convexity of the cost function. As the economy becomes perfectly competitive (or as the cost function becomes linear), r converges to 1.

In period t, a price-setter has access to noisy information about θ_t, to be precise, one process of private information $\left\{x_{t-s}^i\right\}_{s=0}^{\infty}$ and a process of public information $\{z_{t-s}\}_{s=0}^{\infty}$:

$$x_t^i = \theta_t + \sigma_u u_t^i;\ u_t^i \sim N(0, 1)$$

[1] To emphasize the paper's main contributions, I abstract from the micro-foundations of market interaction and the information structure in a rational expectations equilibrium, and simply take the price-setters' first-order condition as given. See Woodford (2001a) for a detailed discussion motivating the underlying informational assumptions, and Woodford (2001b) for an analysis of the micro-foundations of household and firm behavior, on which this model is based.

and

$$z_t = \theta_t + \sigma_v v_t;\ v_t \sim N(0,1),$$

where $\{u^i_{t-s}\}_{s=0}^{\infty}$ and $\{v_{t-s}\}_{s=0}^{\infty}$ are iid processes, independent of each other, as well as of $\{\varepsilon_{t-s}\}_{s=0}^{\infty}$. Finally, θ_t becomes commonly observable with a delay of T periods, that is, at time t, $\{\varepsilon_{t-T-s}\}_{s=0}^{\infty}$ (or equivalently, $\{\theta_{t-T-s}\}_{s=0}^{\infty}$) is common knowledge among all price-setters. I make this last assumption for pure convenience, and for computational reasons. Nothing prevents T from being very large, in which case we approach an environment, in which θ never becomes fully observable.

It will be convenient to introduce a vector notation for both the fundamental process and the processes of public and private information: define Θ_t as the column vectorof realisations of θ from period $t-T+1$ up to period t; let $m_{t-\tau}$ denote the expectation of $\theta_{t-\tau}$, based on the common knowledge of $\{\varepsilon_{t-T-s}\}_{s=0}^{\infty}$, and let M_t denote the vector of $m_{t-\tau}$, for $\tau = 0, ..., T-1$. Finally, let E_t be the vector of monetary policy shocks from period $t-T+1$ up to period t. Then,

$$\Theta_t = \begin{pmatrix} \theta_t \\ \theta_{t-1} \\ . \\ \theta_{t-T+1} \end{pmatrix},\ M_t = \begin{pmatrix} m_t \\ m_{t-1} \\ . \\ m_{t-T+1} \end{pmatrix} \text{ and } E_t = \begin{pmatrix} \varepsilon_t \\ \varepsilon_{t-1} \\ . \\ \varepsilon_{t-T+1} \end{pmatrix};$$

and the monetary policy process can be expressed as

$$\Theta_t = M_t + \sigma B E_t,$$

where B is some $T \times T$ upper-triangular matrix whose entries are derived from (2), with $b_{ii} = 1$ for $i = 1, ..., T$. Similarly, it will be convenient to express the signal process in vector form. Let

$$X^i_t = \begin{pmatrix} x^i_t \\ x^i_{t-1} \\ . \\ x^i_{t-T+1} \end{pmatrix},\ Z_t = \begin{pmatrix} z_t \\ z_{t-1} \\ . \\ z_{t-T+1} \end{pmatrix} \text{ and } V_t = \begin{pmatrix} v_t \\ v_{t-1} \\ . \\ v_{t-T+1} \end{pmatrix},$$

i.e. X^i_t and Z_t denote the vectors of public and private signals available to price-setter i. The vector of public signals is then written as

$$Z_t = \Theta_t + \sigma_v V_t.$$

At time t, the state of the economy is summarized by $(\{\theta_{t-T-s}\}_{s=0}^{\infty}, \Theta_t, Z_t)$, while i's information set $\Im_t^i = \left(\{\theta_{t-T-s}\}_{s=0}^{\infty}, X_t^i, Z_t\right)$.

I now return to the pricing equation. Averaging (3) over i, and substituting forward yields

$$p_t^i = (1-r)\sum_{s=0}^{\infty} r^s E_t^i \left[\overline{E_t}^{(s)}(\theta_t)\right], \tag{4}$$

where $\overline{E_t}^{(s)}(\theta_t)$ denotes the s-th order *average expectation*, i.e. $\overline{E_t}^{(0)}(\theta_t) = \theta_t$, $\overline{E_t}^{(1)}(\theta_t) = \overline{E_t}(\theta_t)$ denotes the population average expectation over θ_t, and $\overline{E_t}^{(s+1)}(\theta_t) = \overline{E_t}\left[\overline{E_t}^{(s)}(\theta_t)\right]$. In words, $\overline{E_t}^{(s)}(\theta_t)$ is the population average expectation of the population average expectation of the ... (repeat s times) ... of the population average expectation of θ_t. Note that the average expectations operator in general does not satisfy the law of iterated expectations; in fact it satisfies it if and only if all available information is public. The average price is given by

$$p_t = (1-r)\sum_{s=0}^{\infty} r^s \overline{E_t}^{(s+1)}(\theta_t) \tag{5}$$

and the log of real output y_t is given by

$$y_t = (1-r)\sum_{s=0}^{\infty} r^s \left[\theta_t - \overline{E_t}^{(s+1)}(\theta_t)\right]. \tag{6}$$

The deviation of real GDP from its trend level is therefore a weighted average of the deviation of all average higher-order expectations. In order to fully derive the dynamics of price and output adjustments following a monetary policy shock, we need to work out the dynamics of higher-order average expectations. This is done in two steps: I first derive a linear filtering equation for $E_t^i(\Theta_t)$ as a function of the signal processes X_t^i and Z_t. By averaging over the filtering equation, I then find a linear relation between Θ_t and $\overline{E_t}(\theta_t)$, which is iterated to solve for (5) and (6). These steps are carried out in the two subsequent lemmas.

2.2 Optimal Filtering

We begin the analysis by deriving a linear filtering equation for $E_t^i(\Theta_t)$. Standard results imply that the signal process $\left\{X_t^i, Z_t\right\}$ can be aggregated into

$$\Xi_t^i = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2} X_t^i + \frac{\sigma_u^2}{\sigma_v^2 + \sigma_u^2} Z_t$$

For further reference, it is useful to define $\Sigma \equiv \frac{\sigma_v^2 \sigma_u^2}{\sigma_v^2+\sigma_u^2}$ as the ex post noise in the signal process Ξ_t^i, and $\alpha \equiv \frac{\sigma_v^2}{\sigma_v^2+\sigma_u^2}$ as the relative importance of private information.

A simple way to express $E_t^i(\Theta_t)$ in the required matrix form is to proceed by maximum likelihood estimation. The log likelihood function $L\left(\Theta_t;\Xi_t^i.M_t\right)$ for the inference problem is given by

$$\begin{aligned} L\left(\Theta_t;\Xi_t^i.M_t\right) &= -\frac{1}{2\sigma^2}\left(\Theta_t - M_t\right)'\left[B^{-1}\right]' B^{-1}\left(\Theta_t - M_t\right) \\ & -\frac{1}{2\Sigma}\left(\Theta_t - \Xi_t^i\right)'\left(\Theta_t - \Xi_t^i\right). \end{aligned} \tag{7}$$

Maximizing L with respect to Θ_t to solve for $E_t^i(\Theta_t)$ yields as a first-order condition

$$\left(BB'\right)^{-1}\left(E_t^i(\Theta_t) - M_t\right) + \frac{\sigma^2}{\Sigma}\left(E_t^i(\Theta_t) - \Xi_t^i\right) = 0$$

which has as a solution

$$E_t^i(\Theta_t) - M_t = \left[I_T + \frac{\Sigma}{\sigma^2}\left(BB'\right)^{-1}\right]^{-1}\left(\Xi_t^i - M_t\right)$$

We have shown

Lemma 1 *For an information structure satisfying the assumptions of the previous section, the posterior expectation of individual i about Θ_t satisfies*

$$E_t^i(\Theta_t) - M_t = \alpha\Delta\left(X_t^i - M_t\right) + (1-\alpha)\Delta\left(Z_t - M_t\right) \tag{8}$$

where

$$\Delta = \left[I_T + \frac{\Sigma}{\sigma^2}\left(BB'\right)^{-1}\right]^{-1}. \tag{9}$$

It should be noted that maximum likelihood methods can be used to obtain a similar linear filtering equation far more generally, for instance to account for "learning", i.e. a gradual increase of the public and private signal precisions over time. Also, note that, if v is an eigenvector of BB', with corresponding eigenvalue λ, then v is also an eigenvector of Δ, corresponding to an eigenvalue of $\widehat{\lambda} = \lambda\left(\lambda + \frac{\Sigma}{\sigma^2}\right)^{-1}$. Since BB' is positive definite, it follows that all eigenvalues of Δ are positive and strictly between 0 and 1. Again, this property can be shown to hold generally.

The matrix Δ determines the weights that a Bayesian estimate of Θ_t attributes to past observations. The coefficients in Δ only depend on the ratio between Σ and σ^2, i.e. the importance of signal noise *relative to fundamental noise*. We can thus separate the effects resulting from the *composition of the information structure* (parametrized by α) from the effects coming from signal noise in the inference problem, parametrized by σ^2/Σ, and the effects of fundamental shocks, i.e. σ^2.

2.3 Higher-order expectations

In the next lemma, I use the linear filtering equation (8) to recursively derive an expression for $(1-r)\sum_{s=0}^{\infty} r^s E_t^i\left[\overline{E_t}^{(s)}(\Theta_t)\right]$:

Lemma 2 *Suppose that i's Bayesian posterior of Θ_t satisfies*

$$E_t^i(\Theta_t) - M_t = N_1\left(X_t^i - M_t\right) + N_2(Z_t - M_t)$$

where N_1 is positive definite and all its eigenvalues are strictly smaller than 1. *Then,*

$$(1-r)\sum_{s=0}^{\infty} r^s\left[E_t^i\left[\overline{E_t}^{(s)}(\Theta_t)\right] - M_t\right] = [I_T - rN_1]^{-1}\left[(1-r)N_1\left(X_t^i - M_t\right) + N_2(Z_t - M_t)\right]. \tag{10}$$

Proof. From (8), we determine the average expectation:

$$\overline{E_t}(\Theta_t) - M_t = N_1(\Theta_t - M_t) + N_2(Z_t - M_t) \tag{11}$$

and i's expectation of the average expectation equals

$$\begin{aligned} E_t^i\left[\overline{E_t}(\Theta_t)\right] - M_t &= N_1\left[N_1\left(X_t^i - M_t\right) + N_2(Z_t - M_t)\right] + N_2(Z_t - M_t) \\ &= N_1^2\left(X_t^i - M_t\right) + [I_T + N_1]N_2(Z_t - M_t). \end{aligned}$$

Iterating the procedure provides expressions for all higher-order expectations,[2]

$$E_t^i\left[\overline{E_t}^{(s)}(\Theta_t)\right] - M_t = N_1^{s+1}\left(X_t^i - M_t\right) + [I_T - N_1]^{-1}\left[I_T - N_1^{s+1}\right]N_2(Z_t - M_t)$$

[2] Since $I_T - \lambda N_1$ is invertible, for $|\lambda| \leq 1$, all the matrix operations below are well-defined.

and

$$\overline{E_t}^{(s+1)}(\Theta_t) - M_t = N_1^{s+1}(\Theta_t - M_t) + [I_T - N_1]^{-1}\left[I_T - N_1^{s+1}\right] N_2 (Z_t - M_t). \tag{12}$$

Substituting into (4) and (5) yields

$$\begin{aligned}
& (1-r)\sum_{s=0}^{\infty} r^s \left[E_t^i\left[\overline{E_t}^{(s)}(\Theta_t)\right] - M_t\right] \\
= \; & (1-r)\sum_{s=0}^{\infty} r^s N_1^{s+1}\left(X_t^i - M_t\right) \\
& + (1-r)\sum_{s=0}^{\infty} r^s [I_T - N_1]^{-1}\left[I_T - N_1^{s+1}\right] N_2 (Z_t - M_t) \\
= \; & (1-r)[I_T - rN_1]^{-1} N_1 \left(X_t^i - M_t\right) \\
& + [I_T - N_1]^{-1} N_2 (Z_t - M_t) \\
& - (1-r)[I_T - N_1]^{-1} N_1 [I_T - rN_1]^{-1} N_2 (Z_t - M_t) \\
= \; & (1-r)[I_T - rN_1]^{-1} N_1 \left(X_t^i - M_t\right) \\
& + [I_T - rN_1]^{-1} N_2 (Z_t - M_t) \\
= \; & [I_T - rN_1]^{-1}\left[(1-r) N_1 \left(X_t^i - M_t\right) + N_2 (Z_t - M_t)\right].
\end{aligned}$$

■

p_t^i is then given by the first entry of (10). As a first result, one observes that this dynamic model delivers (qualitatively and formally) the same implication as the static model of Morris and Shin (2001): *when taking his pricing decision, the price-setter discounts his private information by a factor* $(1-r)$*, and he thus over-reacts to public information* (i.e. reacts more than if the coordination motive were absent).

2.4 Impulse Responses

Applying the previous lemma to the present information structure and averaging over i, we then find an expression for p_t as a function of the processes of the fundamental and the public information. The dynamics of price and output adjustment then depend on the dynamic processes of Θ_t, Z_t, and X_t^i, which can now be substituted to compute the impulse response functions of y_t, p_t^i, and p_t with respect to the shocks ε_t, u_t^i, and v_t. Using the above results, the average price is given by the first entry of

$$(1-r)\sum_{s=0}^{\infty} r^s \left[\overline{E}_t^{(s+1)}(\Theta_t) - M_t\right] = [I_T - r\alpha\Delta]^{-1}\Delta\left[(1-r)\alpha(\Theta_t - M_t) + (1-\alpha)(Z_t - M_t)\right]$$
$$= \sigma(1-r\alpha)[I_T - r\alpha\Delta]^{-1}\Delta B E_t + (1-\alpha)\sigma_v[I_T - r\alpha\Delta]^{-1}\Delta V_t \quad (13)$$

and output is the first entry of

$$(1-r)\sum_{s=0}^{\infty} r^s \left[\Theta_t - \overline{E}_t^{(s+1)}(\Theta_t)\right] = \sigma\left[I_T - (1-r\alpha)[I_T - r\alpha\Delta]^{-1}\Delta\right]BE_t - (1-\alpha)\sigma_v[I_T - r\alpha\Delta]^{-1}\Delta V_t \quad (14)$$

The first row of the matrix $(1-r\alpha)[I_T - r\alpha\Delta]^{-1}\Delta B$ therefore measures the response of prices to a current or past monetary shock. The impulse response of output and prices to informational shocks is given by the first row of $(1-\alpha)\sigma_v[I_T - r\alpha\Delta]^{-1}\Delta$. Using (9) to solve for $(1-r\alpha)[I_T - r\alpha\Delta]^{-1}\Delta$ gives

$$(1-r\alpha)[I_T - r\alpha\Delta]^{-1}\Delta = \left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1}, \quad (15)$$

where $\gamma \equiv \frac{\Sigma}{\sigma^2}\frac{1}{1-r\alpha}$. Thus, up to a scaling effect of σ, the impulse response of prices and output to monetary shocks only depends on γ, which I interpret as an index measuring the importance of higher-order uncertainty: If $\alpha r = 0$, then higher-order uncertainty is either irrelevant ($r = 0$ implies that there is no coordination motive) or inexistent ($\alpha = 0$ implies that all information is common, hence there is no higher-order uncertainty). In that case, $\gamma = \frac{\Sigma}{\sigma^2}$, and the impulse responses correspond to the ones obtained if price-setters set their prices equal to their bayesian estimate of θ_t; formally, impulse responses are given by the filtering matrix Δ. Alternatively, if $\alpha = 1$, we find ourselves in the environment of maximal higher-order uncertainty, studied by Woodford (2001), in which $\gamma \equiv \frac{\Sigma}{\sigma^2}\frac{1}{1-r}$. γ is (*i*) increasing in $\frac{\Sigma}{\sigma^2}$, i.e. the relative importance of signal noise, (*ii*) increasing in α, i.e. the relative importance of private information, and (*iii*) increasing in r, the importance of strategic complementarities. In the extreme case, where r is close to 1 and $\alpha = 1$, i.e. in a highly competitive market with no public information, γ can become arbitrarily large, even for low values of Σ. Since r is a function of the degree of competition, we thus conclude that more competition lead to more higher-order uncertainty. Furthermore, γ is increasing in σ_v.(since an increase in σ_v raises both α and Σ), and in σ_u: Taking the derivative of γ with respect to σ_u, while holding σ_v and r fixed, we find

$$\begin{aligned}\frac{\partial\gamma}{\partial\sigma_u} &= \frac{\partial}{\partial\sigma_u}\left[\frac{\sigma_v^2}{\sigma^2}\frac{1-\alpha}{1-\alpha r}\right] = \frac{\sigma_v^2}{\sigma^2}\frac{\partial}{\partial\alpha}\left[\frac{1-\alpha}{1-\alpha r}\right]\cdot\frac{\partial\alpha}{\partial\sigma_u} \\ &= -\frac{\sigma_v^2}{\sigma^2}\frac{1-r}{(1-\alpha r)^2}\cdot\frac{\partial\alpha}{\partial\sigma_u} > 0, \text{ since } \frac{\partial\alpha}{\partial\sigma_u} < 0.\end{aligned}$$

When changing σ_u, the effect of improving information always dominates the compositional effect, due to an increase in the private information component. A reduction of σ_u therefore leads to an overall reduction in higher-order uncertainty, and thus reduces adjustment delays and the exposure to informational noise.

The impulse responses of output to informational shocks, on the other hand, depend on the above matrix, as well as a scaling factor $(1-\alpha)\,\sigma_v$. Solving as a function of γ and σ_v, the impulse response to informational shocks is given by the first row of

$$(1-\alpha)\,\sigma_v\,[I_T - r\alpha\Delta]^{-1}\,\Delta = \frac{\sigma^2}{\sigma_v}\gamma\left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1}. \tag{16}$$

Apart from a scaling factor σ, the impulse response function thus depends on the one hand on higher-order uncertainty through γ, on the other hand directly on $\frac{\sigma}{\sigma_v}$, i.e. the informativeness of public information relative to the fundamental process. Changing the composition of the information structure thus has both a direct effect and an indirect effect on the impulse response of prices and output to informational shocks.

3 Main Results

General solutions are now easily computable for any given matrix B. Using the previous computations, the processes for prices and output are written as

$$\begin{aligned}(1-r)\sum_{s=0}^{\infty} r^s\left[\overline{E_t}^{(s+1)}(\Theta_t) - M_t\right] &= \sigma\left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1} BE_t \\ &\quad + \frac{\sigma^2}{\sigma_v}\gamma\left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1} V_t\end{aligned} \tag{17}$$

$$(1-r)\sum_{s=0}^{\infty} r^s \left[\Theta_t - \overline{E}_t^{(s+1)}(\Theta_t)\right] = \sigma\gamma\left(BB'\right)^{-1}\left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1} BE_t - \frac{\sigma^2}{\sigma_v}\gamma\left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1} V_t \quad (18)$$

(17) and (18) illustrate the effect of higher-order uncertainty on the delays in price adjustment: If $\gamma = 0$, i.e. if information is disseminated infinitely quickly, and the fundamental immediately becomes common knowledge among the price-setters, then prices adjust to the full information level without delay, and monetary shocks have no effect on output. If $\gamma > 0$, then there is uncertainty and a lack of common knowledge of fundamentals. Only then output is affected by monetary shocks, and higher values of γ lead to longer adjustment delays and more important output effects. How important output effects are depends on the ex post noise in the information structure and on the importance of private information: The more important private information is, the longer the adjustment delays are, due to higher-order uncertainty. In this respect, the benchmark case where all information is common and prices are set equal to the Bayesian posterior in each period, provides an upper bound for the speed of adjustment. *Since $\gamma \geq \frac{\Sigma}{\sigma^2}$, prices adjust less than they would, if there was no higher-order uncertainty. Higher-order uncertainty thus amplifies price-stickyness.*

In addition to the monetary shocks, informational shocks affect prices and output. These shocks have effects similar to "supply shocks", insofar as any increase of prices also leads to a corresponding decrease in output. The impact of informational shocks depends positively on γ: The more important higher-order uncertainty is, the more important is the influence of noise in public signals. Moreover, holding γ fixed, a decrease in σ_v also leads to an increase of exposure to informational noise. As emphasized by Morris and Shin (2001), changes in σ_v therefore have an ambiguous effect, for fixed values of r and σ_u: On the one hand, a decrease in σ_v decreases higher-order uncertainty, and therefore reduces delays of price adjustment and informational noise indirectly; on the other hand, improved public information may lead to an over-exposure to informational noise, since price-setters over-react to public information. Note that the overall exposure to informational noise is non-monotonic with respect to σ_v: If σ_v is very large, price-setters pay little attention to public signals, and hence, informational noise has little effect on output. When σ_v is very small, there is little first and higher-order uncertainty about Θ_t, which means that price-setters are able to react almost immediately to monetary shocks, and coordinate their price adjustments. [3]

To complete the discussion, I briefly comment on the effects of r close to 1, i.e. a high degree of strategic complementarities, or market competition. We have already observed that this leads to large values of γ, and hence to more delays in price adjustment, however note that in the special case where $\alpha = 1$, i.e. in a highly competitive market with no common information, prices can take arbitrarily long to adjust, even when the private information is very precise.

In the remainder of this section, I illustrate these points explicitly, by setting $T = 1$ and $T = 2$, and solving in closed form. I then show numerical solutions for impulse responses, when T is large.

3.1 $T = 1$

The case $T = 1$ provides a simple extension of the static model of Morris and Shin (2001) into a dynamic contexts. In this case, $B = [1]$, and it can easily be checked that $\left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1} = (1+\gamma)^{-1}$. Writing the current average price and output as functions of the aggregate shocks, we find

$$p_t - m_t = \frac{1}{1+\gamma}\sigma\varepsilon_t + \frac{\sigma^2}{\sigma_v}\frac{\gamma}{1+\gamma}v_t \tag{19}$$

$$y_t = \frac{\gamma}{1+\gamma}\sigma\varepsilon_t - \frac{\sigma^2}{\sigma_v}\frac{\gamma}{1+\gamma}v_t \tag{20}$$

Substituting (19) into (20), one obtains

$$y_t = \gamma\left(p_t - m_t\right) - \frac{\sigma^2}{\sigma_v}\gamma v_t \tag{21}$$

Since $p_t - m_t$ is unexpected inflation, (21) is an expectations-augmented Phillips curve.[4] The slope of the Phillips curve depends on higher-order uncertainty. Thus, as higher-order uncertainty increases (γ increases), output becomes more sensitive to unexpected inflation.

[3] The same discussion applies, if we consider a reduction of σ_u and σ_v in equal proportions, i.e. reduce Σ while holding α fixed, or if we hold Σ fixed, while reducing α, i.e. increase the public information component of the information structure.

[4] Similar expressions can be derived in the general case. In that case, m_t no longer corresponds to the common expected price level, since some public information was revealed, without the fundamental becoming common knowledge. Lagged price and output levels therefore also enter into the equation to account for the unanticipated current effects of past monetary shocks.

The short-run volatilities of unexpected inflation and output are

$$\begin{aligned} E\left(p_t - m_t\right)^2 &= \frac{\sigma^2}{\left(1+\gamma\right)^2}\left[1+\gamma^2\left(\frac{\sigma}{\sigma_v}\right)^2\right] \\ E y_t^2 &= \frac{\gamma^2\sigma^2}{\left(1+\gamma\right)^2}\left[1+\left(\frac{\sigma}{\sigma_v}\right)^2\right] \end{aligned}$$

whereas the correlation between output and unexpected inflation is

$$E\left[y_t\left(p_t - m_t\right)\right] = \frac{\gamma\sigma^2}{\left(1+\gamma\right)^2}\left[1-\gamma\left(\frac{\sigma}{\sigma_v}\right)^2\right] \geq 0.$$

The variance of output is increasing in γ. The effect of γ on the variance of inflation is ambiguous: If γ is large, inflation volatility is mostly due to informational shocks, in which case, reducing γ reduces the volatility of inflation. If γ is small, inflation volatility is mostly due to noise in the monetary policy process, and reducing γ increases the short-run response of prices to monetary shocks. Finally, we observe that the model generally predicts a positive correlation between output and unexpected inflation, unless $\alpha = 0$, i.e. when there is no element of private information.

Full information revelation after one period precludes the discussion of any meaningful dynamics and persistence of shocks, since prices fully adjust after one period. To understand the effects of the informational parameters on adjustment dynamics, I therefore turn to the case, where $T = 2$.

3.2 $T = 2$

Let b denote the effect of a past monetary shock on the current value of θ, or

$$\Theta_t = M_t + \sigma \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} E_t$$

In this case,

$$\left(BB'\right)^{-1} = \begin{bmatrix} 1 & -b \\ -b & 1+b^2 \end{bmatrix}$$

and

$$\left[I_T + \gamma\left(BB'\right)^{-1}\right]^{-1} = \frac{1}{\left(1+\gamma\right)^2 + b^2\gamma}\begin{bmatrix} 1+\gamma+\gamma b^2 & \gamma b \\ \gamma b & 1+\gamma \end{bmatrix}$$

Substituting into (17) and (18), we find

$$\begin{aligned} p_t - m_t &= \sigma \left\{ \frac{1+\gamma+\gamma b^2}{(1+\gamma)^2 + b^2\gamma}\varepsilon_t + \left[1 - \frac{\gamma^2}{(1+\gamma)^2+b^2\gamma}\right] b\varepsilon_{t-1} \right\} \\ &\quad + \frac{\sigma^2}{\sigma_v}\gamma \left\{ \frac{1+\gamma+\gamma b^2}{(1+\gamma)^2+b^2\gamma} v_t + \frac{\gamma}{(1+\gamma)^2+b^2\gamma} b v_{t-1} \right\} \qquad (22) \\ y_t &= \sigma \left\{ \frac{(1+\gamma)\gamma}{(1+\gamma)^2+b^2\gamma}\varepsilon_t + \frac{\gamma^2}{(1+\gamma)^2+b^2\gamma} b\varepsilon_{t-1} \right\} \\ &\quad - \frac{\sigma^2}{\sigma_v}\gamma \left\{ \frac{1+\gamma+\gamma b^2}{(1+\gamma)^2+b^2\gamma} v_t + \frac{\gamma}{(1+\gamma)^2+b^2\gamma} b v_{t-1} \right\} \qquad (23) \end{aligned}$$

It is straight-forward to show that

$$\begin{aligned} \frac{d}{d\gamma}\left(\frac{(1+\gamma)\gamma}{(1+\gamma)^2+b^2\gamma}\right) &= \frac{(1+\gamma)^2+\gamma^2 b^2}{\left[(1+\gamma)^2+b^2\gamma\right]^2} > 0 \\ \frac{d}{d\gamma}\left(\frac{\gamma^2}{(1+\gamma)^2+b^2\gamma}\right) &= \frac{2(1+\gamma)+\gamma b^2}{\left[(1+\gamma)^2+b^2\gamma\right]^2} > 0 \end{aligned}$$

A decrease in γ increases the response of prices to both current and past monetary shocks. On the other hand, the coefficients on v_t and v_{t-1} are increasing in γ so that reducing higher-order uncertainty also leads to a reduction of the effects of current adn past informational shocks on output. However, as we have observed in the case where $T = 1$, a reduction in σ_v may actually lead to an increase in the effect of informational noise on output, if public information is very diffuse. We conclude that the results that were highlighted before apply to both current and past monetary and informational shocks.

3.3 Large T

In this section, I numerically solve for the impulse response function in the case of an example where $T = 30$ is set sufficiently large, so that by the time a monetary shock becomes common knowledge, it has almost entirely been factored into pricing decisions. I follow Woodford in the specification of the monetary policy process, assuming that

$$\Delta\theta_t = \rho\Delta\theta_{t-1} + \sigma\varepsilon_t \qquad (24)$$

i.e. the first difference of nominal GDP follows an AR(1)-process. To illustrate the effects, I fix $r = 0.85$ (the value used by Woodford) and $\rho = 0.9$. I then vary the informational parameters to

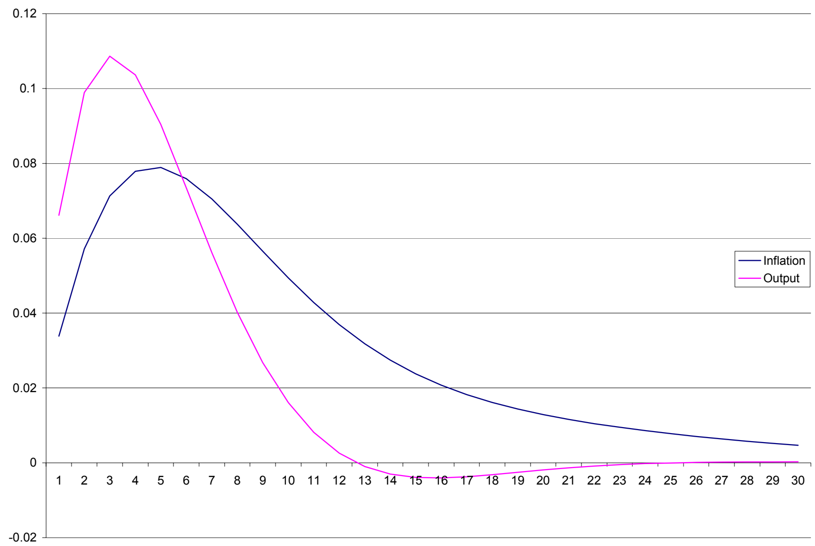

Figure 1: Monetary Shocks, $(5, 10)$

illustrate the comparative statics effects that were identified above. The figures below plot impulse responses to monetary and informational shocks for values of (σ_u, σ_v) of $(5, 10)$, $(3.33, 20)$, $(5, 1)$ and $(5, 100)$. The first two pairs of parameters yield a value of $\gamma = 62.5$, and hence illustrate the effects of compositional changes that leave the degree of higher-order uncertainty unchanged, the last two pairs illustrate the effect of changes in σ_v, when compared with the first. We observe that the reduction in higher-order uncertainty associated with improved public information leads to faster price adjustment, we also observe that the exposure to informational noise is non-monotonic, and tends to be largest for intermediate values of σ_v.

As was already observed by Woodford, it comes as a property of any incomplete information model of price adjustment that output peaks before inflation does, since it takes time for a monetary policy shock not only to become knowledge, but common knowledge among price-setters. The above graphs illustrate that this feature, which is in line with empirical VAR estimations, for example by Christiano, Eichenbaum and Evans [6], is robust to changes in the information structure. As a quantitatively testable description of nominal adjustment following a monetary shock, the present model remains incomplete, since it abstracts from monetary transmission channels other than

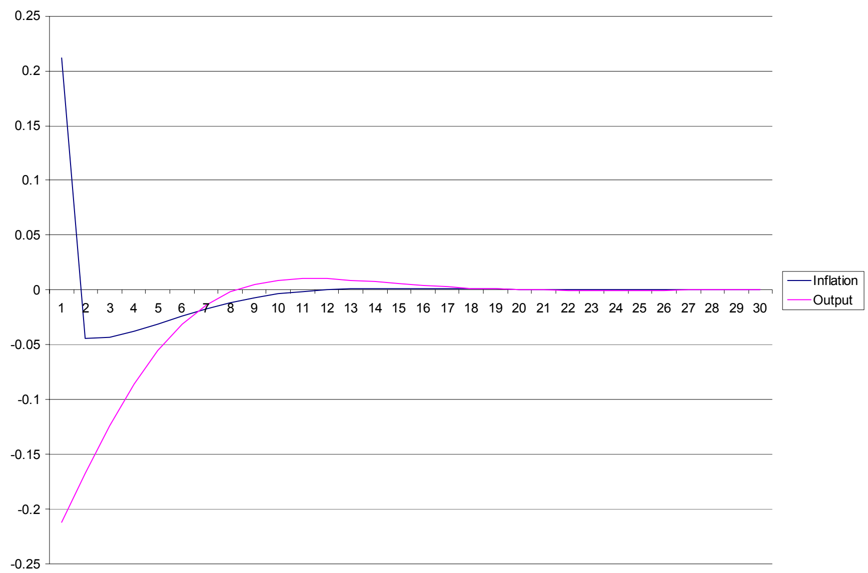

Figure 2: Informational Shocks, $(5, 10)$

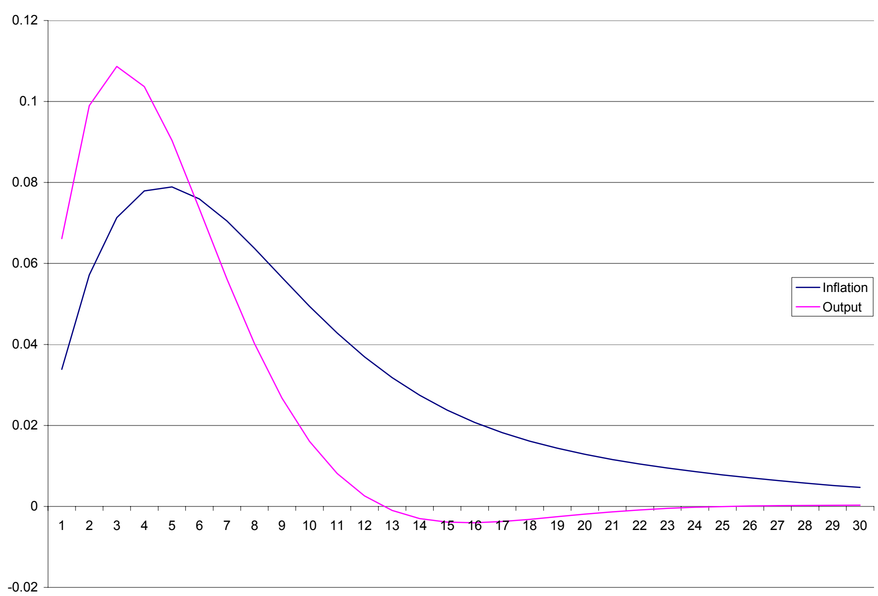

Figure 3: Monetary Shocks, $(3.33, 20)$

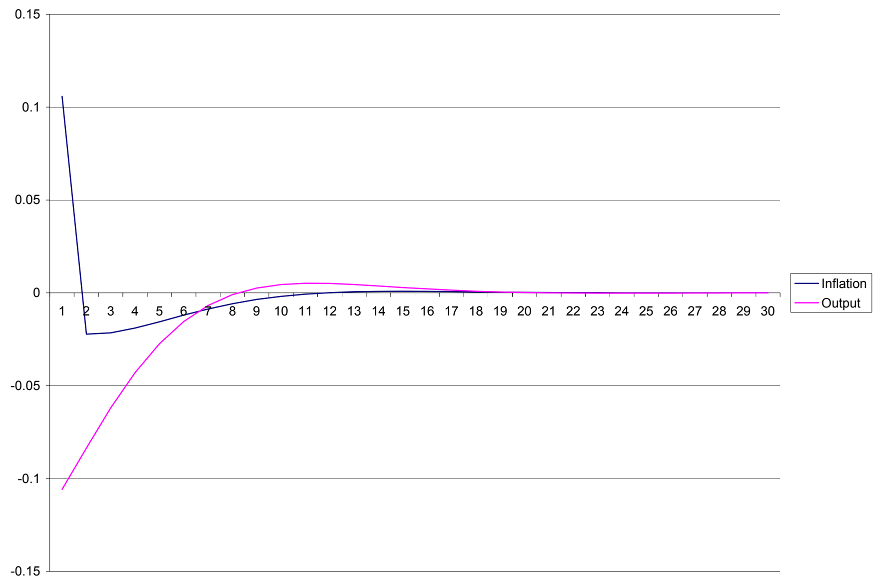

Figure 4: Informational Shocks, $(3.33, 20)$

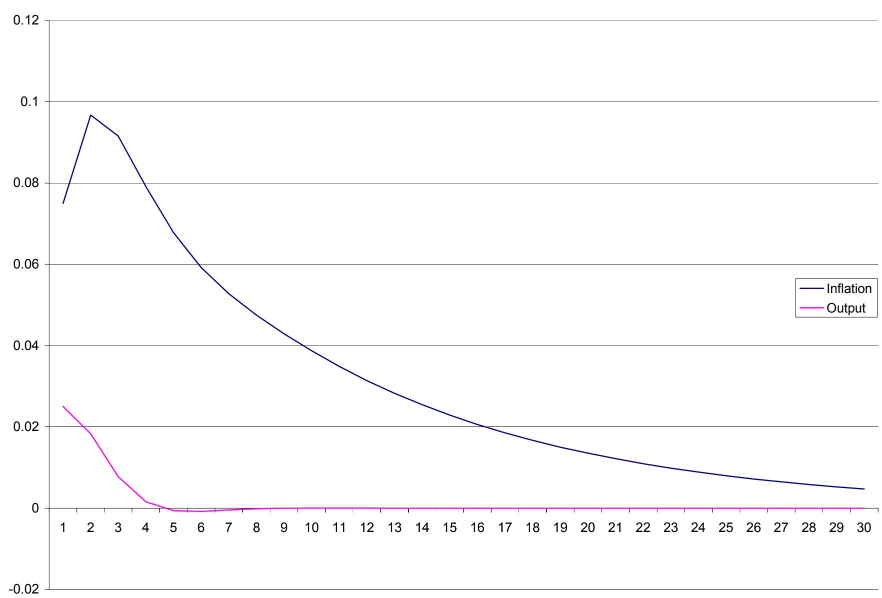

Figure 5: Monetary Shocks, $(5, 1)$

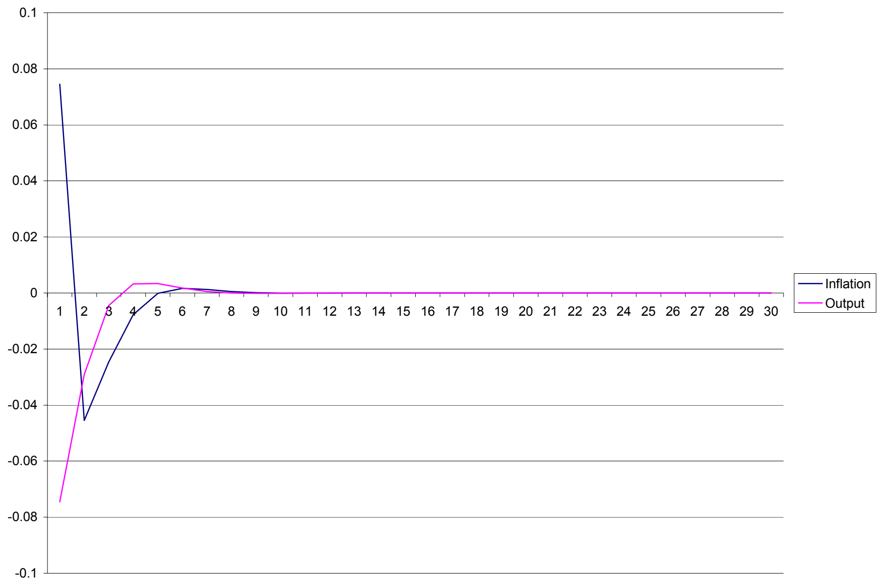

Figure 6: Informational Shocks, $(5, 1)$

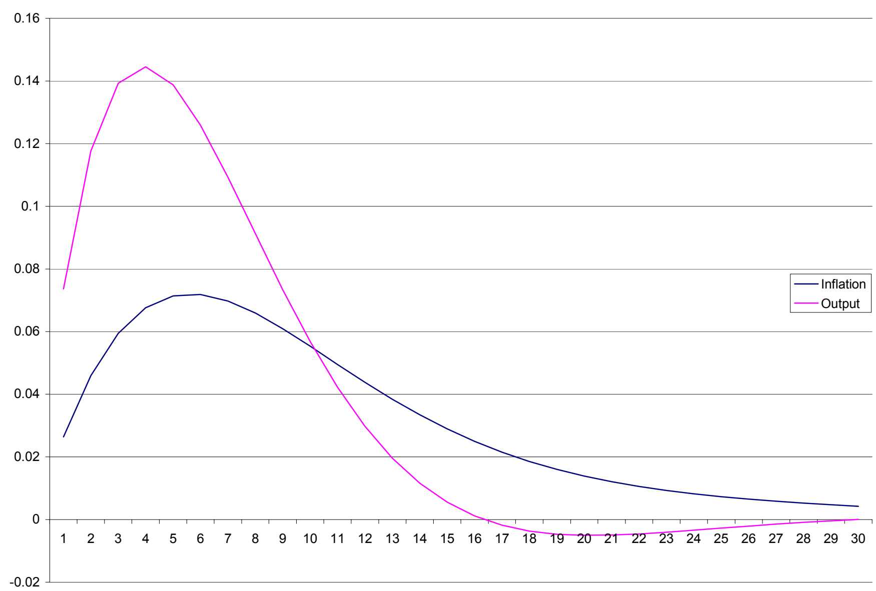

Figure 7: Monetary Shock, $(5, 100)$

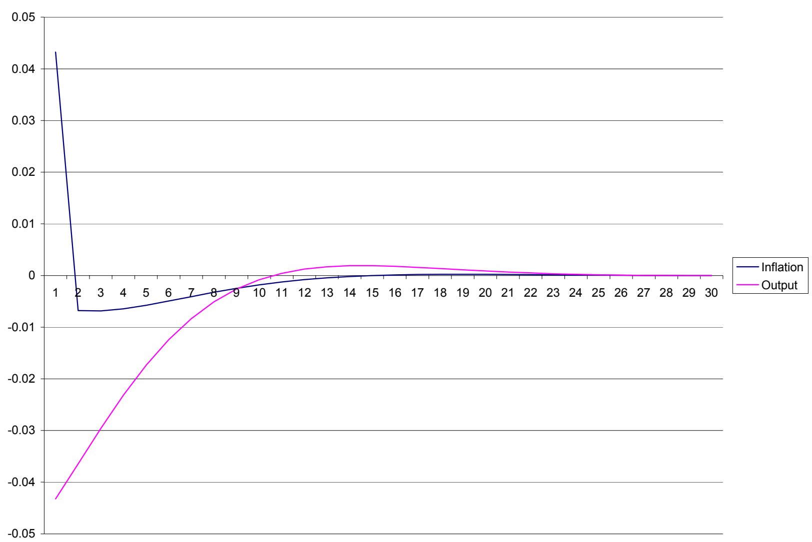

Figure 8: Informational Shock, $(5, 100)$

incomplete information, as well as abstracting from other shocks. Note however, that the delays in price adjustment, and hence the lead of output effects before price effects is entirely based on learning. One should thus expect that to the extent that other shocks are also subject to higher-order uncertainty, the impulse responses they generate will exhibit similar features.

4 Supply Shocks

So far, the analysis focused on the effects of monetary shocks, and incomplete information about the monetary policy process. As noted by Woodford (2001a), restricting the analysis to a unique source of shocks highlights the fact that in contrast with Lucas' original model, the existence of an additional source of noise is not necessary to generate incomplete nominal adjustment; rather the presence of higher-order uncertainty along with strategic complementarities is sufficient to generate output effects; moreover, higher-order uncertainty leads to substantial persistence. The model, however, can easily be augmented to allow for supply shocks, as well as higher-order uncertainty about the latter, which leads to some interesting additional insights.

I therefore augment the model by assuming that the level of potential output, $\overline{y}_t$, follows a linear stochastic process, again allowing for some finite degree of integration. In this case, (1) continues to hold, if we adjust y_t for fluctuations in potential output, i.e. the price-setting equation becomes

$$p_t^i = E_t^i (p_t) + (1-r) E_t^i (y_t - \overline{y}_t) \tag{25}$$

The model can be solved using the same techniques as in section 2. Averaging (25), then substituting forward to express p_t and y_t as functions of the exogenous precesses θ_t and $\overline{y}_t$ yields

$$p_t = (1-r) \sum_{s=0}^{\infty} r^s \left[\overline{E_t}^{(s+1)} (\theta_t) - \overline{E_t}^{(s+1)} (\overline{y}_t) \right] \tag{26}$$

$$y_t = (1-r) \sum_{s=0}^{\infty} r^s \left[\theta_t - \overline{E_t}^{(s+1)} (\theta_t) \right] + (1-r) \sum_{s=0}^{\infty} r^s \overline{E_t}^{(s+1)} (\overline{y}_t) \tag{27}$$

To solve (26) and (27) explicitly for impulse responses to monetary, real and informational shocks, on can again proceed along the same lines as section 2.

To interpret (26) and (27), note that $\theta_t - \overline{y}_t$ is the market-clearing price level that would prevail under common knowledge of the two processes. The realized price level p_t is therefore a *weighted average of average higher-order expectations of the full-information market-clearing price.* y_t can be decomposed into a component due to monetary shocks and a weighted average of higher-order expectations concerning the potential output level. The first component reflects the incomplete nominal adjustment. The second term is new, and is due to fluctations in potential GDP that do not become common knowledge. Because of higher-order uncertainty, real GDP responds only sluggishly to variations in potential GDP. Moreover, the stronger the coordination motive, the stronger the delays in adjustment. The analysis thus suggests that higher-order uncertainty may possibly account for the persistence of shocks other than monetary shocks; such as technology shocks. In the extreme case, where r is close to 1 and factor-specific higher-order uncertainty is important, output potentially remains far from the potential level for a long time.

Since we can separate the effects of θ_t and $\overline{y}_t$, assuming an information structure as above for each of the two components leads to identical impulse response functions as previously.[5] Real

[5] More generally, if the processes for θ_t and $\overline{y}_t$ are correlated (for example when the monetary authority in part responds to its own estimate of the output gap), the techniques of lemma 1 and 2 can be used to derive higher-order expectations about the full-information market-clearing price.

disturbances have the same, but opposite effect on prices as they have on output, however, the output gap $y_t - \overline{y}_t$ responds to real shocks as it does to nominal disturbances. *Most importantly, the factor-specific degree of higher-order uncertainty determines to what extent a disturbance has persistent effects on prices and output.* I conclude this discussion by observing that, although higher-order uncertainty may well explain the *persistence* of shocks, it is unable to account for *amplification*, at least in the short run: Since higher-order expectations are much more sluggish in responding to current information than first-order expectations are, the effects of a supply shock on output are dampened rather than amplified.

5 Some Preliminary Thoughts on Monetary Policy

The previous discussion has highlighted the impact of the information structure on the output-inflation trade-off. In this section, I study the welfare effects of information provision by the Central Bank. A recurring theme in the scientific debate about the optimal conduct of monetary policy is the role of Central Bank transparency, i.e. the desirability of supplying the private sector with precise information about (*i*) the objectives of monetary policy, (*ii*) the macro-economic data on which the central bank bases its decisions, and (*iii*) the actions taken by the central bank. The main argument in favor of monetary transparency is based on the need to *monitor*: The better the information provided, the easier it is to evaluate the Central Bank's behavior ex post and analyze whether the policy objectives have been met. Transparency is thus necessary to monitor whether or not the Central Bank adheres to implicit or explicit rules that govern the Principal Agent relationship between the society and the central bank. This is emphasized in particular in the context of inflation targeting, where an independent central bank retains full authority over its policy actions, and is held accountable for them.[6]

In this section, I discuss the effects of transparent monetary policy in the present model. An important source of information that price-setting firms have access is the central bank, either through direct disclosures, or through its policy actions. Hence, an immediate implication of the previous results is that the information the central bank provides to the public not only has a role for monitoring purposes, but also has a direct influence on coordination among price-setters, and

[6] see Bernanke and Mishkin (1997) and Svensson (1997) for discussions of inflation targeting that emphasize the monitoring role of transparency.

hence the inflation-output trade-off. This *coordination effect* of information provision is the focus of this section. Formally, I embed the present model into a simple model of monetary policy a la Barro and Gordon (1983), taking the aggregate demand process as in part under the central bank's control. The analysis is far from being exhaustive, the main purpose being that of showing that by augmenting the previous model by a formalization of the central bank's objective, we have a natural framework in which to formally study the role the interplay between transparency, monetary policy, and the market's information structure.

To fully understand the role of transparency, it is important to tie this notion to parameters of the information structure. Here, I adopt the view that a higher degree of transparency means a higher degree of "common knowledge"; within the model, this is achieved by a better provision of public information. This view is also taken by Morris and Shin (2001). The motivation behind this use of the term "transparency" lies in the fact that public information coordinates the market's first- and higher-order expectations, whereas private information doesn't (see Hellwig 2000 for results linking the information structure to higher-order uncertainty).

5.1 Transparency as an Implicit Commitment Device

In this section, I study the role of transparency in an environment, where the central bank cannot commit to a particular policy rule. The monitoring role of transparency in the commitment literature was already highlighted; here I show that in an environment characterized by discretion, transparency can act as an implicit commitment device: Even if the central bank is discretionary and tries to use monetary policy to stimulate output, the provision of precise public information decreases the effectiveness of pro-active monetary policy, and hence serves as an implicit commitment device that reduces the inflationary bias. However, improved public information also comes at a cost, since a reduction of the variance of public information raises the macro-economic exposure to informational shocks, at least in environments with a high degree of informational noise; and the higher the pressure put on the central bank to stimulate output, the more it has an incentive to commit to the provision of precise public information.

To illustrate this point formally, consider the previous model, with $T = 1$ for simplicity. Suppose that a discretionary central bank controls the growth rate of nominal GDP with some noise;

i.e. θ_t satisfies

$$\theta_t = \theta_{t-1} + \mu_t + \varepsilon_t \tag{28}$$

where μ_t is under the central bank's control, and ε_t is a monetary policy shock. In each period, the central bank has a target for the growth of θ and for real aggregate output, i.e. it minimizes with respect to $\{\mu_t\}_0^\infty$ the loss function

$$\sum_{t=0}^{\infty} \beta^t L(\mu_t)$$

where $\beta < 1$ is the discount rate, and the per period loss function is

$$L(\mu_t) = E_t (\theta_t - \theta_{t-1})^2 + bE_t (y_t - y^*)^2 ,$$

taking as given the private expectations about the present and future conduct of monetary policy. The output target y^* may be different from the potential output level, which is normalized to 0. The use of a money growth target instead of an inflation target in the loss function is made for convenience, it eliminates any dynamic effects resulting from the choice of μ_t. An inflation target would lead to the same results, but is technically more involved since the effects of discretionary monetary policy on inflation are spread over two periods: immediately through unexpected inflation in the current period, and once the information about the growth of nominal GDP is commonly available (i.e. the following period), through higher anticipated inflation. Here, I abstract from this additional complication.

Price-setters form expectations μ_t^e about the central bank's course of action, and set prices according to the noisy information available about the realization of θ_t. Going along the same lines as before, the output gap then satisfies

$$y_t = \frac{\gamma}{1+\gamma}\sigma \left(\mu_t - \mu_t^e + \varepsilon_t\right) - \frac{\sigma^2}{\sigma_v}\frac{\gamma}{1+\gamma} v_t \tag{29}$$

As a function of μ_t, the loss function can thus be rewritten as

$$\begin{aligned} L(\mu_t) &= \mu_t^2 + \sigma^2 + \frac{b\gamma^2}{(1+\gamma)^2}\left[(\mu_t - \mu_t^e)^2 + \sigma^2 + \frac{\sigma^4}{\sigma_v^2}\right] \\ & \quad - \frac{2b\gamma}{1+\gamma}\left(\mu_t - \mu_t^e\right) y^* + by^{*2} \end{aligned} \tag{30}$$

The first-order condition with respect to μ_t is

$$\mu_t + \frac{b\gamma^2}{(1+\gamma)^2}\left(\mu_t - \mu_t^e\right) - \frac{b\gamma}{1+\gamma} y^* = 0 \tag{31}$$

which, together with the rational expectations hypothesis that $\mu_t^e = \mu_t$, yields

$$\mu_t = \mu_t^e = \frac{b\gamma}{1+\gamma} y^* \tag{32}$$

Substituting into $L(\mu_t)$ yields an expression for the ex ante expected loss:

$$\begin{aligned} EL &= \left[1 + \frac{b\gamma^2}{(1+\gamma)^2}\right] by^{*2} \\ &\quad + \left[1 + \frac{b\gamma^2}{(1+\gamma)^2}\right] \sigma^2 \\ &\quad + \frac{b\gamma^2}{(1+\gamma)^2} \frac{\sigma^4}{\sigma_v^2} \end{aligned} \tag{33}$$

Hence, the inflationary bias is increasing in γ, the degree of higher-order uncertainty about θ_t. The expected loss can be decomposed into three components: The first measures the cost due to the inflationary bias, the second measures the cost due to monetary shocks, and the last component is due to informational noise. We observe that the first two components are decreasing in γ (and hence in σ_v^2), but changes in σ_v^2 have ambiguous effects on the loss due to informational noise: If σ_v^2 is large, the increase of exposure to informational noise that results from a reduction in σ_v^2 dominates the reduction of higher-order uncertainty, while for small values of σ_v^2, the opposite is true. Maximizing EL with respect to σ_v^2 then necessarily leads to a corner solution: Ideally, the central bank would want to set σ_v^2 as close to zero as possible; i.e. provide very precise public information (be very transparent): this eliminates higher-order uncertainty, and therefore the effects of monetary shocks on output as well as the inflationary bias. In practice, it may well be impossible to pursue such an information policy; rather, there is some lower bound on σ_v^2. In that case, it may be optimal to provide no public information at all, to insure the market against informational risk, while at the same time accepting more higher-order uncertainty. At what lower bound on σ_v^2 it becomes optimal to commit to transparent provision of information depends on the output target: The more biased the output target is, the more the central bank has an incentive to provide precise public information.

5.2 Policy Targets and the Information Structure

We can use the insights of this model to discuss the impact of the monetary regime on the information structure, and hence the output-inflation trade-off. The literature on monitoring monetary policy usually advocates that policy should be conducted within a framework that provides well-specified targets. The beneficial effects of such a framework and of policy targets with respect to the information structure are easily understood: The framework, as well as the targets, reduce higher-order uncertainty about the central bank's objectives, and hence about the course of its policy conduct. Formally, specific targets eliminate higher-order uncertainty about the the values of y^* and b in the central banker's objective function, and hence reduce higher-order uncertainty about the resulting policy variable θ_t.

In addition, the targets themselves act to coordinate expectations about the targeted variables; within the model, they act as a public signal about policy. A similar role of coordinating expectations is played by published forecasts. Svensson's interpretation of inflation targets as *inflation forecast* targets (i.e. the central bank should design its policy so that its forecast of inflation is consistent with the target, cf. Svensson (1997)) captures precisely this idea: to the extent that forecasts are unbiased, they act as a public signal about inflation. Monetary regimes typically differ about what variable is targeted, and hence also about the degree of higher-order uncertainty about prices. In the terminology of our model, a regime that targets money growth reduces higher-order uncertainty about θ_t, an inflation target affects higher-order uncertainty about prices. As was observed in (26), the inflation-output trade-off depends on higher-order uncertainty about the "full-information market-clearing price-level"; since the inflation target provides a public signal about the latter, one would conjecture that an inflation target is more beneficial in terms of its informational effect than a money growth target.

5.3 The Signaling Role of Monetary Policy

Finally, the model points to the role that central bank transparency may have in stabilizing output following supply shocks: The analysis in the previous section has highlighted the possibility that in an environment characterized by a high degree of strategic complementarities, adverse supply shocks can have highly persistent output effects, if there is higher-order uncertainty; in other words, even if everyone privately believes that potential output is higher, output remains depressed because of low *higher-order* expectations. In this case, transparency about supply shocks may also be beneficial, since the creation of common expectations about output reduces the persistence of the effects of real shocks.

The provision of public information about supply shocks can come through two channels: First, such information may come from the provision of public forecasts of potential output or the output gap. Second, even if such forecasts are not public, some information becomes available, if the central bank conditions monetary policy on its own estimate of the supply shock. If higher-order uncertainty is small, the "surprise" effect of monetary policy on output is small; nevertheless, the central bank may want to condition its policy on its estimates of potential output, if this increases welfare by coordinating expectations about the potential output level. [7]

6 Concluding Remarks

Building on Woodford (2001), this paper has developed a model of monetary business cycles, in which higher-order uncertainty about the fundamental driving processes, coupled with strategic complementarities between price-setters leads to potentially long adjustment delays for prices after monetary shocks and hence to important short-run effects on output. The main motivation of the analysis was to discuss the effect of the information structure on the inflation-output trade-off; specifically, the model showed how price stickiness becomes more important, the more relevant higher-order uncertainty is (quantifiable as a function of the information structure and of the degree of strategic complementarities). The provision of precise public information thus accelerates nominal adjustment, but potentially comes at the cost of a higher exposure to informational noise. Finally, embedding the nominal adjustment model in a simple version of the monetary policy model a la Barro and Gordon (1983) shows how the provision of precise public information may serve as an implicit commitment device against inflationary biases.

The results in this paper have several positive, normative, and empirical implications. The comparative statics results rely more on the context of strategic complementarities with incomplete information than on the specific environment studied. The conclusions about the role of the information structure therefore should be expected to extend to other contexts of decision-making

[7] This idea mirrors results in Angeletos, Hellwig and Pavan (2002), who study the informational role of policy choices in a global coordination game with multiple equilibria under common knowledge. In their environment, the information conveyed by the policy choice enables the market to coordinate on one of multiple equilibria, and this multiplicity is the root cause of the policy traps discussed in that paper. Here, the preferences of the central bank are aligned with those of the price-setters, and hence inducing better coordination will be beneficial from the central bank's point of view.

with strategic complementarities. Nor are the effects of higher-order uncertainty tied to the nature of the shock; the model suggests that higher-order uncertainty can lead to persistence of *any kind of shock.* Woodford's insight about monetary shocks may therefore be helpful in understanding persistence in other environments as well, for example of technology shocks in an RBC model.

Among the normative aspects of the analysis, I have highlighted the role of transparency as providing commitment against inflationary biases; I should also mention the negative impact that the degree of competition has on higher-order uncertainty: The higher the elasticity of substitution between products is, the more important the strategic complementarities between prices are, and the slower the nominal adjustment after a monetary shock. The positive effects of increasing competition are therefore in part offset by a negative welfare effect due to higher persistence of shocks.

The paper furthermore leads to potentially testable empirical implications: In particular, it points to the effects of the monetary policy regime on the information structure, and raises the question whether there is an empirical link between the way monetary policy is conducted and the inflation-output trade-off. The informational parameters can potentially be estimated from time-series data, which leads to the question whether the changes in information processing over the last 20 years have changed the inflation-output trade-off, or have otherwise altered the transmission channels of monetary policy. There seems to be at least informal evidence about how the underlying parameters have shifted: Morris and Shin (2001), for example suggest that changes in the use of the media have reduced informational noise, but have also raised the public information component in financial markets (lowered α), while the conventional wisdom on product market liberalization suggests an increase in r.[8] Furthermore, evidence in Stock and Watson (2002) suggests that the variance of underlying shocks has decreased; what remains unclear is how these reductions in the variance of shocks compare to the reduction in informational noise, i.e. the ratio $\frac{\sigma^2}{\Sigma}$. How these changes have altered the inflation-output trade-off and the exposure to informational noise remains a priori ambiguous.

Estimating the informational parameters (and possibly testing for their changes) is also of

[8] In his account of Greenspan's activity at the Fed, Woodward suggests based on discussions with the Fed chairman that starting in the mid-90's, increased competition prevented firms from responding to a loosening of monetary policy by raising prices. While I haven't found any similar evidence in the academic literature, the statement seems to be broadly consistent with the view that an increase in r led to an increase in γ during the 90's.

interest with respect to the recent debate on the decline of US output volatility (see Blanchard and Simon, 2001, or Stock and Watson, 2002, for an overview). These papers try to determine to what extent changes in the conduct of monetary policy have contributed to stabilize output growth, and the theoretical contribution of this paper suggests that the monetary policy changes of the 1980's, to the extent that they have altered the information structure, have influenced output volatility not only through the redefinition of policy objectives, but also through their direct effect on the structural parameters.

It should be noted that, as a descriptive model of monetary business cycles, the present model is highly simplified, and relies on information as the unique transmission channel for shocks. The analysis relies on informational assumptions, which, although more complex than Woodford's, are very simplistic. The methodology, however, can easily be adapted to more complex information structures that involve a gradual learning of the process, or a shift from private to public information over time. The model relied on (i) the linearity of best responses, which by forward substitution led to an expression of strategic variables as weighted sums of higher-order expectations about the underlying fundamental processes, and (ii) on the derivation of average expectations out of the information structure. Average expectations can easily be computed for any kind of environment, by first deriving a filtering equation like (8) for the fundamental process, and then using the filtering equation to relate average expectations about the fundamentals to the fundamental process itself; iteration to higher orders then completes the procedure.

Due to its flexibility, the present model of higher-order uncertainty might therefore be useful as a vehicle for studying the role of the information structure in various other dynamic contexts, starting with a more exhaustive analysis of the role of transparency in the conduct of monetary policy. Another empirically appealing extention might be to combine the analysis of incomplete information with sticky prices a la Calvo (1983). As was discussed before, the incomplete information model can replicate the finding of VAR estimations that following a monetary shock, output peaks prior to inflation; however the model cannot account for persistent effects on output and inflation beyond the point at which the initial shock becomes common knowledge. Combining the incomplete information with some forms of price or investment rigidities might therefore lead to a further increase of the persistence of inflation. A combination of incomplete information with rigid price adjustment might also be helpful for a theoretical understanding of the insights drawn from the "new Keynesian" models, where the forward-looking nature of pricing decisions relies on a strong inter-temporal coordination of expectations. Whether such coordination of expectations remains feasible in the presence of informational differences is a yet unresolved question.

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