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The international diversification puzzle is not as bad as you think [1]

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PRELIMINARY AND INCOMPLETE

ABSTRACT

In the data country portfolios are heavily biased toward domestic assets. Standard one-good international macro models predict that, due to the presence of non-diversifiable labor income risk, country portfolios should be heavily biased toward foreign assets; this discrepancy constitutes the international diversification puzzle (Baxter and Jermann, 1997). We show that a simple extension of one-good models help to reconcile theory and data. In particular we analytically solve for the equilibrium country portfolios in a two-country, two-goods model with non-diversifiable labor income and investment. In this set-up, consistently with the data, country portfolios contain a relatively small, but positive, share of foreign assets. The reason why international diversification is low is that terms of trade movements provide considerable insurance against country specific shocks and labor income risk (Cole and Obstfeld 1991, Acemoglu and Ventura, 2002). The reason why international diversification is positive is that foreign assets are crucial to share the financing of investment across countries. Finally in the model a country's portfolio share of foreign assets should depend on its import share over GDP and on its capital income share over GDP. We show how this relation is qualitatively and quantitatively consistent with country portfolios in the cross section of OECD countries in the 1990s.

KEYWORDS: Home bias, international diversification

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1. Introduction

Although there has been rapid growth in international portfolio diversification in recent years, portfolios remain heavily biased towards domestic assets. For example, foreign assets accounted, on average, for only about 14% of the total value of the assets owned by U.S. residents during the 1990*s*. This is puzzling in light of existing macroeconomic models that predict much higher levels of diversification. For example, Baxter and Jermann (1997) argue that due to the presence of non-diversifiable human capital risk, a diversified world portfolio should involve a negative position in domestic marketable assets.

In this paper we show that perfect risk sharing is in fact consistent with relatively low levels of international diversification. Our environment is the two-country extension of the stochastic growth model developed by Backus, Kehoe and Kydland (1992 and 1995, henceforth BKK), which is the workhorse model for quantitative international macroeconomics. While BKK allow for a complete set of Arrow securities, we assume that households only trade shares in domestic and foreign firms. Our innovations relative to Baxter and Jermann are to allow for capital investment dynamics and imperfect substitutability between traded goods. Given particular assumptions on preferences and technologies, we are able to characterize equilibrium portfolio choices analytically.[2] The equilibrium portfolio choice depends on two parameters: the relative preference in consumption for domestically-produced versus imported goods, and capital's share in production. When we calibrate these parameters to replicate appropriate features of the United States economy, our expression implies portfolios comprising around 80% domestic stocks and 20% foreign stocks. An additional interesting property of the model is that two stocks effectively complete markets, in the sense that consumption and leisure in both countries are identical state-by-state to the corresponding

[2] The assumptions required to derive an analytical expression for the portfolio choice are (i) preferences are separable between consumption and leisure and logarithmic in consumption, and (ii) all production technologies are Cobb-Douglas, which implies a unitary elsasticity of substitution between traded goods.

allocations for an economy in which a complete set of Arrow securities is traded.

We use the equilibrium portfolio expression to test our model by considering the extent to which differences in trade shares in a cross-section of countries predict differences in levels of diversification. We find that the theoretical relationship is both qualitatively and quantitatively consistent with the empirical pattern for relatively high-income economies. In particular, the ratio of trade to GDP is a powerful predictor of observed international diversification, while controlling for openness to trade, neither size nor GDP per capita significantly affect diversification.

To better understand the predictions of our model for portfolio choices we compare and contrast our economy to those considered by Lucas (1982), Baxter and Jermann (1997), and Cole and Obstfeld (1991). Lucas (1982) points out that in a symmetric one-good two-country model, perfect risk pooling involves agents of each country owning half the claims to the home endowment and half the claims to the foreign endowment. Baxter and Jermann (1997) extend Lucas' model in one direction by introducing production while retaining the single-good assumption. They show that if returns to capital and labor are highly correlated within a country, then agents can compensate for undiversifiable labor income risk by aggressively diversifying asset holdings. In their examples, fully diversified portfolios typically involve substantial short positions in domestic assets. Cole and Obstfeld (1991) extend Lucas' analysis in a different direction. They retain the focus on an endowment economy, but assume that the two countries receive endowments of different goods that are imperfect substitutes. These goods are then traded, and agents consume bundles comprising both goods. They show that changes in relative endowments induce off-setting changes in the terms of trade. When preferences are log-separable between the two goods, the terms of trade responds one-for-one to changes in relative income, effectively delivering perfect risk-sharing. Thus, in sharp contrast to the results of Lucas or Baxter and Jermann, any level of diversification is consistent with complete risk-pooling, including portfolio autarky.

The key difference in our analysis relative to Baxter and Jermann (1997) is that we follow Cole and Obstfeld in introducing imperfect substitutability between domestic and foreign traded goods. Thus in our model, changes in international relative prices provide some insurance against country-specific shocks and, in the flavor of the Cole and Obstfeld result, the portfolio choice does not have to do all the heavy-lifting when it comes to delivering perfect risk-sharing. In contrast to Cole and Obstfeld, however, the presence of production and particularly investment in our model means that it is not the case that any portfolio automatically delivers perfect risk-sharing. The exact portfolio split determines who finances changes in investment, which induces a unique equilibrium portfolio choice characterized by a strong bias towards domestic assets. Home bias is optimal, since even though most of an extra unit of domestic investment comes directly out of the pocket of domestic shareholders, domestic investment also disproportionately raises demand for domestic goods, inducing a real exchange rate appreciation that equates appropriately measured consumption in the two countries.

The absence of substantial international diversification is not the only feature of the data that has been taken to indicate a lack of complete international risk sharing. Engel (2000) identifies the three "core puzzles" in international macroeconomics as the home-bias-in-equity-portfolios puzzle (why is so little diversification observed?), the Feldstein-Horioka (1980) savings-investment-correlation puzzle (why is the correlation so high?), and the Backus-Kehoe-Kydland (1992) cross-country-consumption-correlations puzzle (why is the correlation so low?). Engel also emphasizes the Backus-Smith (1993) real-exchange-rate-relative-consumption-correlation puzzle (why is domestic consumption relatively high when domestic goods are relatively expensive?). One response to these puzzles would be to argue that international financial markets are far from complete, and that one should therefore discount any priors one might have from models with perfect risk-sharing. By contrast, our approach will be to argue that all of these stylized facts are consistent with perfect international risk sharing.

2. The Model

The modelling framework is the one developed by Backus, Kehoe and Kydland, 1995. There are two countries, each of which is populated by the same measure of identical, infinitely-lived households. Firms in each country use country-specific capital and labor to produce an intermediate good. The intermediate good produced in the domestic country is labeled a, while the good produced in the foreign country is labeled b. These are the only traded goods in the world economy. Within each country goods a and b are combined to produce country-specific final consumption and investment goods. However, the final goods production technologies are asymmetric across countries, in that they are biased towards using a larger fraction of the locally-produced intermediate good. The only source of uncertainty in the model takes the form of country-specific productivity shocks to intermediate goods firms.

We assume that the assets that are traded internationally are shares in the domestic and foreign representative intermediate-goods-producing firms. These firms make investment and employment decisions, and distribute any non-reinvested earnings to shareholders. This is a natural framework for addressing growth in international diversification.

A. Preferences and technologies

In each period t the economy experiences one event $s_t \in S$. We denote by s^t the history of events from date 0 to date t. The probability at date 0 of any particular history s^t is given by $\pi(s^t)$.

Period utility for a household in the domestic country after history s^t is given by[3]

$$(1) \quad U\left(c(s^t), 1 - n(s^t)\right) = \ln c(s^t) + v\left(1 - n(s^t)\right)$$

[3] The equations describing the foreign country are largely identical to those for the domestic country. We use star superscripts to denote foreign variables.

where $c(s^t)$ denotes consumption at date t given history s^t, $n(s^t)$ denotes labor supply, and $v(.)$ captures the utility from leisure. The assumption that utility is log-separable in consumption will be important for some of our later results.

Households supply labor to domestically located perfectly-competitive intermediate-goods-producing firms ($i-$firms). $I-$firms in the domestic country produce good a, while those in the foreign country produce good b.The $i-$firms' production function is Cobb-Douglas in capital and labor:

$$(2) \quad F\left(z(s^t), k(s^{t-1}), n(s^t)\right) = e^{z(s^t)} k(s^{t-1})^{\theta} n(s^t)^{1-\theta}$$

where $z(s^t)$ is an exogenous technology shock. The vector of shocks $\widehat{z}(s^t) = [z(s^t), z^*(s^t)]$ evolves stochastically. The only assumption we make about this process is that it is symmetric.

$I-$firms hold the capital in the economy, and make investment and employment decisions. Let $w(s^t)$ be the wage in terms of the domestically-produced intermediate good. The domestic $i-$firm's maximization problem after history s^t is given by

$$\max_{\{k(s^t), n(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) d(s^t)$$

subject to $k(s^t)$, $n(s^t) \geq 0$, taking as given $k(s^{-1})$, where $Q(s^t)$ is the price the firm uses to value dividends at s^t relative to consumption at date 0, and dividends (in units of the final consumption / investment good) are given by

$$(3) \quad d(s^t) = q_a(s^t)\left[F\left(z(s^t), k(s^{t-1}), n(s^t)\right) - w(s^t)n(s^t)\right] - \left[k(s^t) - (1-\delta)k(s^{t-1})\right].$$

In this expression $q_a(s^t)$ is the price of good a in units of the final good, and δ is the depreciation rate for capital. The first order conditions for the domestic firms' choices for $k(s^t)$ and $n(s^t)$ may be written as

$$(4) \quad -Q(s^t) + \sum_{s^{t+1}} Q(s^{t+1}) \left[q_a(s^{t+1}) \frac{\theta F\left(z(s^{t+1}), k(s^t), n(s^{t+1})\right)}{k(s^t)} + (1-\delta) \right] = 0$$

and

$$(5) \quad (1-\theta) F\left(z(s^t), k(s^{t-1}), n(s^t)\right) - w(s^t) n(s^t) = 0.$$

Analogously, the foreign firms' dividends and first-order conditions are given by

$$(6) \quad d^*(s^t) = q_b^*(s^t) \left[F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right) - w^*(s^t) n^*(s^t) \right] - \left[k^*(s^t) - (1-\delta) k^*(s^{t-1}) \right].$$

$$(7) \quad -Q^*(s^t) + \sum_{s^{t+1}} Q^*(s^{t+1}) \left[q_b^*(s^{t+1}) \frac{\theta F\left(z^*(s^{t+1}), k^*(s^t), n^*(s^{t+1})\right)}{k^*(s^t)} + (1-\delta) \right] = 0$$

and

$$(8) \quad (1-\theta) F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right) - w^*(s^t) n^*(s^t) = 0.$$

After trading in asset markets, households sell their holdings of intermediate goods to domestically located final-goods-producing firms ($f-$firms). The $f-$firms are perfectly competitive and produce final goods using intermediate goods a and b as inputs to a Cobb-Douglas technology:

$$(9) \quad \begin{aligned} G\left(a(s^t), b(s^t)\right) &= a(s^t)^{\omega} b(s^t)^{(1-\omega)} \\ G^*\left(a^*(s^t), b^*(s^t)\right) &= a^*(s^t)^{(1-\omega)} b^*(s^t)^{\omega} \end{aligned}$$

where $\omega > 0.5$ determines the size of the local input bias in the composition of domestically produced final goods. Note that the Cobb-Douglas assumption implies a unitary elasticity of substitution between between domestically-produced goods and imports. In their benchmark calibration, BKK (1995) set this elasticity to 1.5. Heathcote and Perri (2002) estimate the elasticity to be 0.9. The Cobb-Douglas assumption is therefore within the range of existing estimates, and it will be important for some of our later results.

The $f-$firm's static maximization problem in the domestic country after history s^t is given by

$$\max_{a(s^t),b(s^t)} \left\{G(a(s^t),b(s^t)) - q_a(s^t)a(s^t) - q_b(s^t)b(s^t)\right\}$$

subject to $a(s^t)$, $b(s^t) \geq 0$.

The first order conditions for domestic and foreign $f-$firms may be written as

$$\begin{aligned} (10) \quad q_a(s^t)a(s^t) &= \omega G(a(s^t),b(s^t)), \qquad q_b(s^t)b(s^t) = (1-\omega)G(a(s^t),b(s^t)) \\ q_b^*(s^t)b^*(s^t) &= \omega G^*\left(a^*(s^t),b^*(s^t)\right), \ q_a^*(s^t)a^*(s^t) = (1-\omega)G^*\left(a^*(s^t),b^*(s^t)\right) \end{aligned}$$

B. Asset market structure

Let $rx(s^t)$ denote the real exchange rate, defined as the price of foreign relative to domestic consumption. Since the prices of traded intermediate goods in each country are defined relative to local final consumption, applying the law of one price to intermediate goods generates expressions for $rx(s^t)$:

$$(11) \quad rx(s^t) = \frac{q_a(s^t)}{q_a^*(s^t)} \qquad \text{and} \qquad rx(s^t) = \frac{q_b(s^t)}{q_b^*(s^t)}.$$

After receiving dividends each period, individuals can buy and sell shares in the domestic and foreign $i-$firms. The budget constraint for the domestic agent is given by

$$
\begin{aligned}
(12) \qquad & c(s^t) + P(s^t)\left(\lambda(s^t) - \lambda(s^{t-1})\right) + rx(s^t)P^*(s^t)\left(\lambda^f(s^t) - \lambda^f(s^{t-1})\right) \\
= \ & q_a(s^t)w(s^t)n(s^t) + \lambda(s^{t-1})d(s^t) + \lambda^f(s^{t-1})rx(s^t)d^*(s^t) \qquad \forall t \geq 0, s^t
\end{aligned}
$$

Here $P(s^t)$ is the price at s^t of (ex dividend) shares in the domestic firm, in units of domestic consumption. $P^*(s^t)$ is the price of shares in the foreign firm in units of foreign consumption. $\lambda(s^t)$ ($\lambda^{h*}(s^t)$) denotes the fraction of the domestic firm purchased by the domestic (foreign) agent, and $\lambda^f(s^t)$ ($\lambda^*(s^t)$) denotes the fraction of the foreign firm bought by the domestic (foreign) agent. We assume that at the start of period 0, the domestic (foreign) household owns the entire domestic (foreign) firm: thus $\lambda(s^{-1}) = 1$, $\lambda^*(s^{-1}) = 1$, $\lambda^f(s^{-1}) = 0$ and $\lambda^{h*}(s^{-1}) = 0$.

At date 0, domestic households choose $\lambda(s^t)$, $\lambda^f(s^t)$, $c(s^t) \geq 0$ and $n(s^t) \in [0,1]$ for all s^t and for all $t \geq 0$ to maximize

$$
(13) \quad \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t U\left(c(s^t), 1 - n(s^t)\right)
$$

subject to 12.

The domestic households' first-order condition for domestic and foreign stock purchases are, respectively

$$
\begin{aligned}
(14) \qquad U_c(s^t)P(s^t) &= \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t)U_c(s^{t+1})\left(d(s^{t+1}) + P(s^{t+1})\right) \\
U_c(s^t)rx(s^t)P^*(s^t) &= \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t)U_c(s^{t+1})rx(s^{t+1})\left(d^*(s^{t+1}) + P^*(s^{t+1})\right)
\end{aligned}
$$

The domestic household's first-order condition for hours is

$$(15)\quad U_c(s^t)q_a(s^t)w(s^t)+U_n(s^t) \geq 0$$
$$= \text{ if } n(s^t)>0$$

Analogously, the foreign households' first-order condition for domestic and foreign stock purchases and hours are given, respectively, by

$$(16)\quad \begin{aligned} U_c^*(s^t)\frac{P(s^t)}{rx(s^t)} &= \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)U_c^*(s^{t+1})\frac{1}{rx(s^{t+1})}\left(d(s^{t+1})+P(s^{t+1})\right) \\ U_c^*(s^t)P^*(s^t) &= \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)U_c^*(s^{t+1})\left(d^*(s^{t+1})+P^*(s^{t+1})\right) \end{aligned}$$

and

$$(17)\quad U_c^*(s^t)q_b^*(s^t)w^*(s^t)+U_n^*(s^t) \geq 0$$
$$= \text{ if } n^*(s^t)>0.$$

C. State contingent prices

What state-contingent consumption prices $Q(s^t)$ and $Q^*(s^t)$ should firms use in this economy when making state-contingent investment and employment decisions, which determine state-by-state dividend payments? We assume that intermediate-goods-producing firms use a weighted sum of discount factors of existing shareholders to price the cost of foregoing current dividends. Thus

$$\begin{aligned} (18)\quad Q(s^t) &= \upsilon(s^t)\frac{\pi(s^t)\beta^tU_c(s^t)}{U_1\left(c_0,1-n_0\right)}+(1-\upsilon(s^t))\frac{\pi(s^t)\beta^trx(s^0)U_c^*(s^t)}{rx(s^t)U_1\left(c_0^*,1-n_0^*\right)} \\ (19)\quad Q^*(s^t) &= (1-\upsilon^*(s^t))\frac{\pi(s^t)\beta^trx(s^t)U_c(s^t)}{rx(s^0)U_c(s^0)}+\upsilon^*(s^t)\frac{\pi(s^t)\beta^tU_c^*(s^t)}{U_c^*(s^0)} \end{aligned}$$

where, for example, $v(s^t)$ is the relative weight placed on the preferences of domestic shareholders by domestic firms.[4]

D. Definition of equilibrium

An equilibrium is a set of prices $P(s^t)$, $P^*(s^t)$, $r(s^t)$, $r^*(s^t)$, $w(s^t)$, $w^*(s^t)$, $Q(s^t)$, $Q^*(s^t)$, $q_a(s^t)$, $q_a^*(s^t)$, $q_b(s^t)$, $q_b^*(s^t)$, $rx(s^t)$ for all s^t and for all $t \geq 0$ such that when households and firms solve their problems taking these prices as given all markets clear.

Market-clearing for goods a and b requires :

$$
\begin{aligned}
(20) \quad a(s^t) + a^*(s^t) &= F\left(z(s^t), k(s^{t-1}), n(s^t)\right) \\
b(s^t) + b^*(s^t) &= F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right).
\end{aligned}
$$

Market-clearing for final goods requires :

$$
\begin{aligned}
(21) \quad c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) &= G\left(a(s^t), b(s^t)\right) \\
c^*(s^t) + k^*(s^t) - (1-\delta)k^*(s^{t-1}) &= G^*\left(a^*(s^t), b^*(s^t)\right).
\end{aligned}
$$

Stock-market clearing requires :

$$
(22) \quad \lambda(s^t) + \lambda^{h*}(s^t) = 1 \qquad \lambda^f(s^t) + \lambda^*(s^t) = 1.
$$

3. Equilibrium portfolio choices

Suppose that at time zero, productivity is equal to its unconditional mean value in both countries ($z(s^0) = z^*(s^0) = 1$) and that initial capital is equalized across countries, $k(s^{-1}) = k^*(s^{-1}) > 0$. Then there is an equilibrium in this economy with the following properties:

[4] Note that each agent takes $Q(s^t)$ as given, understanding that their individual atomistic portfolio choices will not affect aggregate investment decisions.

$$(23)\quad \lambda(s^t) = \lambda^*(s^t) = 1 - \lambda^f(s^t) = 1 - \lambda^{h*}(s^t) = \lambda = \frac{\omega + \theta - 2\omega\theta}{1 + \theta - 2\omega\theta} \qquad \forall t, s^t$$

$$(24)\quad P(s^t) = k(s^t),\ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t$$

How can we verify that there is in fact an equilibrium with these properties? Consider an allocation specifying state-contingent values for all the variables in the model such that in every date and state the allocation satisfies: (i) the first-order conditions for capital accumulation and labor rental for intermediate-goods firms (equations 4, 5, 7 and 8), (ii) the first order conditions for intermediate-goods purchases for final-goods firms (equation 10), (iii) the first-order conditions for household labor supply (equations 15 and 17), (iv) the market clearing conditions for intermediate and final goods (equations 20 and 21), and (v) the requirement that probabilities over histories are consistent with the process for productivity shocks. For candidate stock prices and a candidate portfolio choice rule that satisfies stock-market clearing, the allocation is an equilibrium if and only if the households' first-order conditions for stock purchases are satisfied (equations 14 and 16).

Our strategy for showing that the portfolio choices satisfy the first-order conditions for stock purchases is to first show that the portfolio decision rules coupled with other equilibrium conditions imply that consumption (valued at the equilibrium exchange rate) is equalized across countries in every date and state. Given this result, we then show that the firms' first order conditions for investment (equations 4 and 7), the definitions for dividends (equation 3 and 6) and the functions for share prices (equation 24) jointly imply that the first-order conditions for stock purchases (equations 14 and 16) are satisfied. Note that given the portfolio choice rule it is immediate that stock markets clear (equation 22 is satisfied).

For notational concision, we temporarily suppress the history-dependent notation. We also drop the arguments of the production technologies; for example we will use F to denote $F\left(z(s^t), k(s^t), n(s^t)\right)$ and G to denote $G\left(a(s^t), b(s^t)\right)$.

It is convenient to let x denote investment, and to define gross domestic product as

$$GDP = q_a F \tag{25}$$

$$GDP^* = q_b^* F^* \tag{26}$$

Let Δv for some variable v denote the difference following a particular history between the value of the variable in the domestic economy and the same variable abroad, where both domestic and foreign variables are measured in units of domestic consumption goods. Thus

$$\begin{aligned}
\Delta GDP &= GDP - rxGDP \\
\Delta G &= G - rxG^* \\
\Delta x &= x - rxx^* \\
\Delta c &= c - rxc^*.
\end{aligned}$$

From equations 20, 11 and 10, domestic GDP is given by

$$\begin{aligned}
GDP &= q_a a + q_a a^* = q_a a + rx q_a^* a^* \\
&= \omega G + rx(1-\omega)G^*
\end{aligned}$$

Similarly, foreign GDP is given by

$$GDP^* = \frac{1}{rx}(1-\omega)G + \omega G^*$$

Using these expressions, it is clear that the difference between domestic and foreign GDP is linearly related to the difference between domestic and foreign absorption:

$$\begin{aligned} \Delta GDP &= \omega G + rx(1-\omega)G^* - [(1-\omega)G + rx\omega G^*] \\ &= (2\omega - 1)\Delta G \\ &= (2\omega - 1)(\Delta c + \Delta x). \end{aligned} \tag{27}$$

Given the portfolio choice rules, the budget constraints of the representative domestic and foreign households for $t \geq 1$ simplify to[5]

$$c = q_a wn + \lambda d + \lambda^f rxd^* \tag{28}$$

and

$$c^* = q_b^* w^* n^* + \lambda^{h*} \frac{1}{rx} d + \lambda^* d^*$$

From equations 3, 5, 6 and 8, and using the definitions of GDP (equation 25) the budget constraints may be rewritten as

$$\begin{aligned} c &= q_a(1-\theta)F + \lambda\left[q_a\theta F - x\right] + \lambda^f rx\left[q_b^*\theta F^* - x^*\right] \\ &= GDP\left[\lambda\theta + (1-\theta)\right] - \lambda x + rxGDP^*\lambda^f\theta - rx\lambda^f x^* \end{aligned} \tag{29}$$

[5] Budget constraints are different at $t = 0$, given the assumption that of an inherited perfect home bias in stock holdings at this date.

and

$$c^* = q_b^*(1-\theta)F^* + \lambda^*\left[q_b^*\theta F^* - x^*\right] + \lambda^{h*}\frac{1}{rx}\left[q_a\theta F - x\right]$$

$$(30) \qquad = GDP^*\left[\lambda^*\theta + (1-\theta)\right] - \lambda^* x^* + \frac{1}{rx}GDP\lambda^{h*}\theta - \frac{1}{rx}\lambda^{h*}x$$

Setting $\lambda = \lambda^*$ and $\lambda^f = \lambda^{h*} = 1-\lambda$ gives

$$(31) \quad \Delta c = (1 - 2(1-\lambda)\theta)\,\Delta GDP + (1-2\lambda)\Delta x$$

Using equation 27 to substitute out ΔGDP and using the candidate portfolio rule 23 for λ gives $\Delta c = 0$.[6] Since the utility function is log-separable between consumption and leisure

$$(32) \quad \Delta c = 0 \Leftrightarrow U_c(s^t)rx(s^t) = U_c^*(s^t)$$

Thus the intermediate-firms discount factors simplify to

$$(33) \quad Q(s^t) = \frac{\pi(s^t)\beta^t U_1\left(c(s^t), 1-n(s^t)\right)}{U_1\left(c_0, 1-n_0\right)} \text{ and } Q^*(s^t) = \frac{rx(s^t)}{rx(s^0)}Q(s^t)$$

Then the firms' first order conditions for new investment (equations 4 and 7) may be rewritten as

$$(34) \quad U_c(s^t) = \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)U_c(s^{t+1})\left[q_a(s^{t+1})\frac{\theta F\left(z(s^{t+1}), k(s^t), n(s^{t+1})\right)}{k(s^t)} + 1 - \delta\right]$$

[6] Note that this reasoning only applies for $t \geq 1$. However, we can also verify that under the candidate portfolio rule $\Delta c = 0$ at date 0, since the assumptions that $z(s^0) = z^*(s^0)$ and $k(s^{-1}) = k^*(s^{-1})$ coupled with the assumption that the shock process is symmetric jointly imply that $rx(s^0) = 1$ and $c(s^0) = c^*\left(s^0\right)$.

and

$$(35)\quad U_c^*(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) U_c^*(s^{t+1}) \left[q_b^*(s^{t+1}) \frac{\theta F\left(z^*(s^{t+1}), k^*(s^t), n^*(s^{t+1})\right)}{k^*(s^t)} + 1 - \delta \right]$$

Multiplying both sides of the first (second) of these two equations by $k(s^t)$ $(k^*(s^t))$ and using the definition of dividends (equations 3 and 6) gives two of the four first-order conditions for stock purchases (equations 14 and 16). The remaining two first-order conditions follow immediately by substituting equation 32 into each of the first two.

4. Intuition for the result

Two equations are key to documenting that our portfolio choice rule is optimal. The first is equation 27, the equation that relates differences in the relative value of absorption $(\Delta c + \Delta x)$ to differences in the relative value of GDP. This equation follows from our assumption that the final-goods firms production technology is Cobb-Douglas, or equivalently, that the elasticity of substitution between the two traded goods is one. This assumption implies that countries devote a constant fraction of total final demand to each of the two traded intermediate goods. Thus changes in relative final demand (for example, changes in relative investment) affect the relative value of intermediate goods output.

This demand effect on relative prices can be illustrated by the following simple examples. Suppose the final good in the model is housing, and that the domestic intermediate good (a) is aluminum while the foreign intermediate good (b) is bricks. When the technologies for producing domestic and foreign houses are the same $(\omega = 0.5)$, changes to the relative foreign versus domestic demand for housing do not impact the relative value of the outputs of bricks and aluminum. Similarly, an increase in the relative quantity of bricks versus aluminum being produced leads to an exactly offsetting change in their relative prices, so that the difference between foreign and domestic GDP (measured in common units) is unaffected. When domestic houses are produced only with aluminum ($\omega = 1$), an increase in the relative demand for domestic houses translates into an equal-sized increase in the relative price of aluminum (assuming no supply response). For intermediate values for ω, the stronger the preference for home-produced goods, the larger the impact on the relative value of aluminum output from an increase in the relative demand for domestic houses. Note that there is no analogue to equation 27 in a one-good model.

The second key equation is the equation that expresses the difference between foreign and domestic consumption as a function of the difference in investment and the difference in GDP (equation 31). This equation is derived by subtracting one budget constraint from the other, using the assumptions that the intermediate-goods firms production is Cobb-Douglas in capital and labor, that stocks are the only assets are traded, and that the optimal portfolio rule implies a constant portfolio split. Note that equation 31 would still hold true in a one good model.

Now for a given portfolio parameter λ we have two independent equations (27 and 31) in three unknowns (ΔGDP, Δc and Δx). Thus changes to relative output and relative investment cannot have independent effects on relative consumption. In particular, we can substitute equation 27 into equation 31 to express the difference in consumption solely as a function of the difference in investment:[7]

$$\Delta c = [1 - 2(1 - \lambda)\theta]\,(2\omega - 1)(\Delta x + \Delta c) + (1 - 2\lambda)\Delta x$$

[7] It makes sense to think of shocks to relative investment, since firms have flexibility in deciding how much to invest and how much to pay out in dividends each period. By contrast households simply consume their income in each period.

which implies that

$$(36) \quad \mu \Delta c = (1 - 2\lambda)\Delta x + (2\omega - 1)\left[1 - 2(1 - \lambda)\theta\right]\Delta x$$

where μ is a constant. We can now decompose the effect of an investment shock Δx on Δc as follows:

The first term in equation 36 is $(1 - 2\lambda)$ which captures the direct impact on relative consumption of the change in investment. The size and sign of the direct effect just depend on how the cost of new investment is split between domestic and foreign shareholders. If $\lambda = 1$ (no diversification) this term is equal to -1, meaning that, for example, a one dollar in domestic investment directly leads to a one dollar reduction in domestic consumption, simply because it directly implies a one dollar decline in dividend income of domestic residents. By contrast, if $\lambda = 0$ then the first term is equal to 1. In this case, each extra dollar of domestic investment implies a dollar reduction in foreign consumption, since foreign consumers are now the only holders of domestic equity.

The second term in equation 36 captures the indirect effect of changes in relative investment on relative consumption, indirect because this effect works through relative prices. This term can be split into two parts. The first, $(2\omega - 1)$, captures the extent to which an increase in domestic absorption (in this case investment) increases the relative value of domestic output. This is exactly the coefficient in equation 27, and captures the fact that an increase in relative demand for domestic final goods has a positive effect on the terms of trade for the domestic economy. As discussed above, this effect is stronger the stronger is the domestic intermediate-good component in domestic final goods production. The second part of the second term $[1 - 2(1 - \lambda)\theta]$ captures the impact of a change in relative output on relative consumption. Notice that the output impact on consumption is negatively related to the diversification level $(1 - \lambda)$ and positively related to labor's share, $(1 - \theta)$.

To see why this is the case, first consider increasing labor's share (reducing θ). For a given level of diversification, the larger is non-diversified labor's share, the larger the positive impact of an improvement in the domestic economy's terms of trade on relative consumption. Similarly, for a given value for θ, the smaller is the diversification level $(1-\lambda)$, the larger is the impact of a change in relative prices (induced initially by a change in relative investment demand) on relative consumption.

Recall that we showed above that when consumption is equalized across countries, the households' first order conditions for stock purchases are satisfied. Thus the equilibrium value for λ is the one for which the direct effect and the indirect effects exactly offset, so that changes in relative investment have no effect on relative consumption.

Why is diversification low?

If the lion's share of income goes to labor ($\theta < 0.5$) and, and if preferences are biased towards domestically-produced goods ($\omega > 0.5$), then the indirect effect of change in relative investment on relative consumption is positive for any λ. Thus the direct effect must be negative, which immediately implies $1-\lambda < 0.5$. In other words, for parameter values in these ranges, an increase in demand for domestic investment improves the domestic economy's terms of trade, which implies that domestic residents can afford to finance the bulk of domestic investment while still equalizing consumption across countries.

Why does diversification depend positively on the trade share?

A larger trade share implies a smaller value for ω. As discussed above, equation 27 then implies a weaker terms of trade response to changes in relative final demand. Thus for any given diversification level, the indirect effect of demand changes on relative consumption that works through relative price changes is going to be smaller. Thus to achieve consumption equalization, direct effect depends only on λ and is given by $2\lambda - 1$, this implies that λ has to be smaller and diversification $(1-\lambda)$ has to be larger. Conversely, with strong home bias in preferences (a small trade share), changes in relative investment induce, via changes in the terms of trade, large increases in the value of income in the country doing the investment. Thus little external investment financing (via diversification) is required.

Why does diversification depend negatively on labor's share?

A larger labor share strengthens the indirect impact of Δx on Δc, so the direct effect also has to be larger in absolute value. This means that diversification is going to be lower. In other words, with a larger labor share the change in relative income induced by the change in the terms of trade, has a larger impact on relative consumption (since a smaller part of this income goes to capital and thus potentially to foreigners). Hence less external financing (the direct effect of diversification) is needed to achieve consumption equalization.

5. Relation to Previous Literature

Lucas (1982), Cantor and Mark (1988) and Baxter and Jermann (1997) consider one-good models. In our set-up this implies that the real exchange rate is always equal to one, and that equation 27 does not apply. Lucas shows that in an endowment economy with common preferences across countries, perfect risk pooling is achieved when agents hold 50 percent of both domestic and foreign shares in each period, where shares are claims to future endowment streams. Since equation 27 does not apply in a one good model, the portfolio choice rule 23 does not apply to Lucas' economy. Nonetheless it is interesting to note that substituting $\omega = 0.5$ into equation 23 we reproduce Lucas' 50-50 portfolio split result.

Cantor and Mark extend Lucas' analysis to a simple environment with production. However, they make several assumptions that ensure that their economy inherits the properties of Lucas'. In particular, (i) domestic and foreign agents have the same log separable preferences over consumption and leisure, (ii) productivity shocks are assumed to be iid through time, (iii) firms must purchase capital and rent labor one period before production takes place, and (iv) there is 100% depreciation. When their two economies are the same size, assumptions (ii) and (iii) ensure that in an efficient allocation capital and labor are always equalized across countries. Thus to deliver perfect risk-sharing, the optimal portfolio choice simply has to ensure an equal division of next period output, which is ensured with Lucas' 50-50 portfolio split.

Baxter and Jermann (1997) consider a one-good model with more conventional timing according to which labor and capital rental rates are both stochastic and both driven by total-factor-productivity shocks to a Cobb-Douglas production technology. They argue that since returns to capital and labor are highly correlated, agents can effectively diversify country-specific labor income risk (that cannot be directly diversified by simply working abroad) by aggressively diversifying claims to capital. The logic for their argument becomes transparent by considering 31, which we reproduce here.

(37) $\Delta c = (1 - 2(1 - \lambda)\theta)\, \Delta GDP + (1 - 2\lambda)\Delta x$

In the Baxter and Jermann one-good case, the real exchange rate is one, so Δc is simply equal $c - c^*$. Baxter and Jermann assume that capital stocks are exogenous. One way to translate this into our model would be to assume firms in both countries target a constant capital stock, in which case $\Delta x = 0$. In this case, to deliver perfect risk-sharing ($\Delta c = 0$) we only need to pick a value for λ such that the coefficient on ΔGDP is zero. The implied value for λ is

$$1 - \lambda = \frac{1}{2\theta}$$

which is exactly the expression described by equation (2) in Baxter and Jermann.

If capital's share θ is set to a third, the value for λ that delivers equal consumption in the two countries is -0.5. Thus, as Baxter and Jermann point out, a diversified portfolio involves a negative position in domestic assets. Note that equation 37 suggests that there will always exist a portfolio that delivers perfect risk sharing as long as Δx is strictly proportional to ΔGDP. Thus, as an alternative to assuming $\Delta x = 0$, we could assume, for example, that firms invest a fixed fraction of output, so that $x(s^t) = \kappa GDP(x^t)$. In this case, in a one-good world, $\Delta x = \kappa \Delta GDP$. Now consumption equalization requires that

$$(38)\quad \Delta c = [(1 - 2(1 - \lambda)\theta) + (1 - 2\lambda)\kappa]\, \Delta GDP = 0$$

which implies

$$\lambda = \frac{2\theta - 1}{2(\theta - \kappa)}.$$

As an example, if the investment rate κ is equal to 0.2 and capital's share is 1/3, the value for λ that delivers consumption equalization is -1.25, implying an even larger short position in domestic assets than the one predicted by Baxter and Jermann. The intuition is simply that now asset income is a less effective hedge, since following an increase in foreign output, foreign investment rises, reducing income from foreign dividends.

Our model enriches the Baxter and Jermann analysis along two dimensions. First, we explicitly endogenize investment. In both examples discussed above, the investment rules are arbitrary, whereas we assume firms make investment and dividend decisions with the interests of their shareholders at heart. Second, we assume that the two countries produce different traded goods that are imperfectly-substitutable when it comes to producing the final consumption-investment good.

The model that is closest to ours is Cole and Obstfeld (1991). They consider a two-country endowment economy, and a version with production in which the two goods may be consumed or used as capital inputs to produce in the next period. Like Cantor and Mark (1988) they assume 100 percent capital depreciation. They show that with when domestic and foreign agents share the same log-separable preferences for consuming the two goods, and (in the production version of the model) when production technologies are Cobb-Douglas in the quantities of the two goods allocated for investment, then a regime of portfolio autarky (100 percent home bias or $\lambda = 1$) delivers the same allocations as a world with a complete set of internationally-traded assets.

It is straightforward to revisit the logic for their results in the context of our model. In particularly, considering an endowment economy effectively sets $\Delta x = 0$, in which case equations 27 and 31 become two independent equations in two unknowns, Δc and ΔGDP. The only possible solution is $\Delta c = \Delta GDP = 0$. Thus for any choice for λ, including the portfolio autarky value $\lambda = 1$ emphasized by Cole and Obstfeld, perfect risk-pooling is achieved. The reason is simply that differences in relative quantities of output are automatically offset one-for-one by differences in the real exchange rate, so $GDP = rxGDP^{*}$. In the production version of the model, Cole and Obstfeld's assumptions of log separable preferences and full depreciation imply that consumption, investment and dividends are all fixed fractions of output, so that $\Delta x = \kappa \Delta GDP$ and, once again, equations 27 and 31 are two independent equations in two unknowns, Δc and ΔGDP. Thus total dividend income in any given period is again independent of the initial portfolio split. In this sense changes in the real exchange rate provide automatic insurance against country-specific income changes.

In contrast to the Cole and Obstfeld result, only one portfolio delivers perfect risk-pooling in our economy. Furthermore, portfolio autarky is only efficient in the case when there is complete specialization in tastes, so that $\omega = 1$. The reason for these differences relative to their results is that with partial depreciation, investment is no longer a fixed fraction of output, and changing the initial portfolio therefore changes the properties of the stream of asset income. However, efficiency can still be achieved for $\omega < 1$ provided the initial portfolio contains an appropriately-weighted mix of both domestic and foreign stock.

Nonetheless, despite these differences, the logic for our results is broadly the same as that in Cole and Obstfeld. In our model, changes in the terms of trade provide automatic insurance against shocks to the relative supply or relative demand for the two traded goods. Were it not for the risk of changes to relative investment, this insurance would automatically deliver perfect risk-pooling, irrespective of portfolio choices. However, in a world with partial depreciation and persistent productivity shocks, efficient investment will not be either constant or a constant fraction of output; rather, as in a standard growth model, positive persistent productivity shocks will be associated with a surge in investment. Thus one way to think about the role of portfolio diversification is to ensure that the cost of funding changes in investment is efficiently split between domestic and foreign residents. As we discussed in the previous section, relatively little diversification is required to achieve this, since an increase in domestic investment demand raises the relative price of domestically produced goods, and thus raises domestic income. Hence it is optimal for domestic residents to finance (by holding most of domestic equity) most of the extra domestic investment.

6. Explaining the cross section of country portfolios

In the theoretical section we solve for the equilibrium level of international diversification in a world with two equal-sized countries. The equilibrium portfolio share of foreign stocks, $\phi = (1 - \lambda)$, is given by

$$\phi = \frac{1-\omega}{1+\theta-2\omega\theta}$$

where $1 - \omega = \eta$ is the share of imports/exports in GDP. This suggests that in the data we should observe a relation between import shares and shares of foreign assets in country portfolios.

In particular, the relation we should observe is

$$\phi = \frac{\eta}{1+\theta-2(1-\eta)\theta} = \frac{\eta}{1-\theta+2\eta\theta}$$

Taking the reciprocal of this relation we obtain

$$(39)\quad \frac{1}{\phi} = 2\theta + (1-\theta)\frac{1}{\eta}$$

which implies that there should be a linear relationship between the reciprocal of the share of foreign assets in country portfolios and the reciprocal of the trade share in GDP. It is easy to explore whether a similar relationship holds in the data.

A. Data issues

We need country-level data on trade shares and on international diversification positions. Trade shares are obtained from the World Bank World Development Indicators: in particular η_i, the trade share for country i, is measured as

$$\eta_i = average\left(\frac{\text{Imports}_i}{GDP_i}, \frac{\text{Exports}_i}{GDP_i}\right)$$

For foreign asset shares we need a measure of total foreign assets divided by a measure of the total value of domestically-owned assets. There are three possible sources: IMF international asset position data, the Kraay, Loayza, Serven, and Ventura (2000) dataset, and the Lane and Milesi-Ferretti (2001) dataset. Coverage of these datasets is similar and consists of 70-80 countries. All these datasets distinguish between bond and stock positions. The Kraay and al. dataset is the only one that also contains capital stock data (constructed by cumulating investment). All these datasets have a time series dimension.

In our first exercise we use the Kraay and al. (2000) dataset and measure the ϕ_i, the international diversification position for country i as

$$\phi_i = average\left(\frac{FA_i}{k + FA_i - FL_i}, \frac{FL_i}{k + FA_i - FL_i}\right)$$

where FA_i is the value of foreign assets (including foreign direct investment, portfolio investment, loans and other investments) owned by domestic residents, FL_i is the value of all the claims that foreigners have on country residents, and k_i is the value of the capital stock in country i. We then compute averages of the constructed ϕ_i and η_i for the years 1990-1997 so as to obtain a cross-section of 53 countries[8] that we use to estimate equation (39).

B. Results

Results are reported in tables 1 and 2 below and figures 1 and 2. Equation (39) is estimated using OLS and median (robust) regression. We perform separate estimations for the entire cross section of countries and for a subgroup of rich countries. Controls for size and for GDP per capita are also used. Estimated coefficients from the data are then compared to the coefficients implied by the model.

In the whole cross section of countries, trade shares are a minor factor in explaining the cross section of country portfolios, while GDP per capita and country size play a bigger role. In particular, rich countries are more diversified while large countries are less diversified. Thus, for poor countries the model significantly overpredicts observed diversification. We suspect that this lack of diversification reflects some features of poorer countries that are not captured in our model, such as the presence of capital controls and under-developed financial markets.

By contrast, among the group of relatively high income economies for whom our model should

[8]We eliminate all countries for which we have fewer than 5 observations.

be most appropriate, we find that trade share is a key factor in explaining the cross section of country portfolios. For these economies the relation between trade share and import share in the data is remarkably close to the one predicted by the model (see figure 2), and neither size nor GDP per capita seem to significantly affect diversification.

7. Conclusion

TO BE COMPLETED

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Table 1. All countries. Independent Variable is $1/(1-\lambda)_i$

	Lin. Reg.		Rob. Reg.		Model $\theta = 0.36$
$\frac{1}{(1-\omega)_i}$	**1.04**	0.18	**0.91**	**0.58**	0.64
	(0.30)	(0.28)	(0.21)	(0.22)	
Constant	**3.21**	-8.95	**2.21**	5.67	0.72
	(1.44)	(9.09)	(1.04)	(7.01)	
Log Ypc		**-2.36**		**-2.16**	
		(0.59)		(0.46)	
Log Pop		**2.22**		**1.08**	
		(0.44)		(0.44)	
Obs	53	53	53	53	
Adj. R^2	0.19	0.56	0.10	0.31	

Note: The numbers in parentheses are standard errors. The coefficients in bold are significant at at least the 5% level.

Table 2. Rich Countries. Independent Variable is $1/(1-\lambda)_i$

	Lin. Reg.		Rob. Reg.		Model $\theta = 0.36$
$\frac{1}{(!-\omega)_i}$	**0.91** (0.14)	**1.10** (0.20)	**1.08** (0.21)	**1.15** (0.23)	0.64
Constant	0.64 (0.62)	13.1 (20.1)	0.06 (0.92)	26.2 (31.1)	0.72
Log Ypc		-0.56 (2.02)		-2.40 (3.16)	
Log Pop		-0.45 (0.36)		-0.14 (0.52)	
Obs	20	20	20	20	
R^2	0.70	0.70	0.31	0.45	

Note: The numbers in parentheses are standard errors. The coefficients in bold are significant at at least the 5% level.

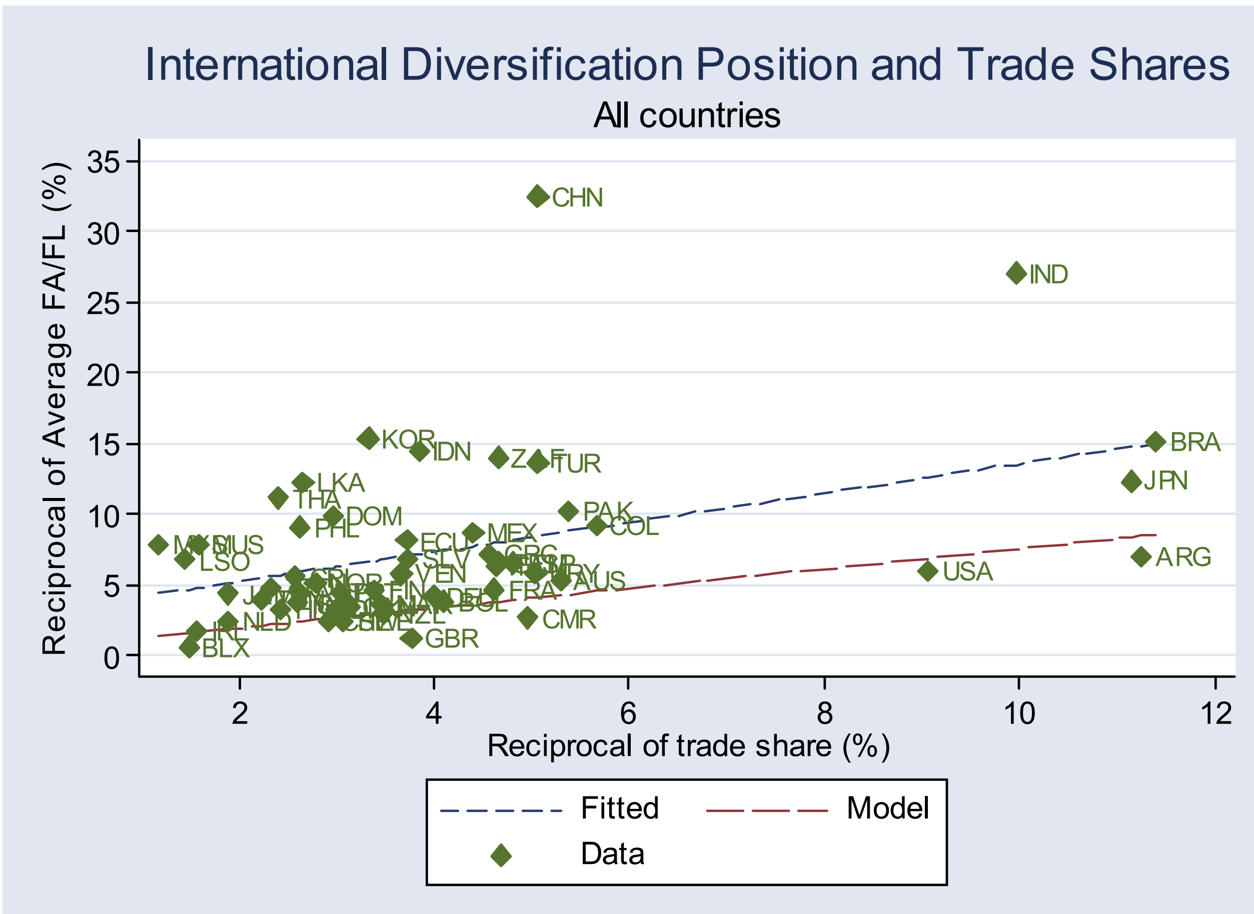
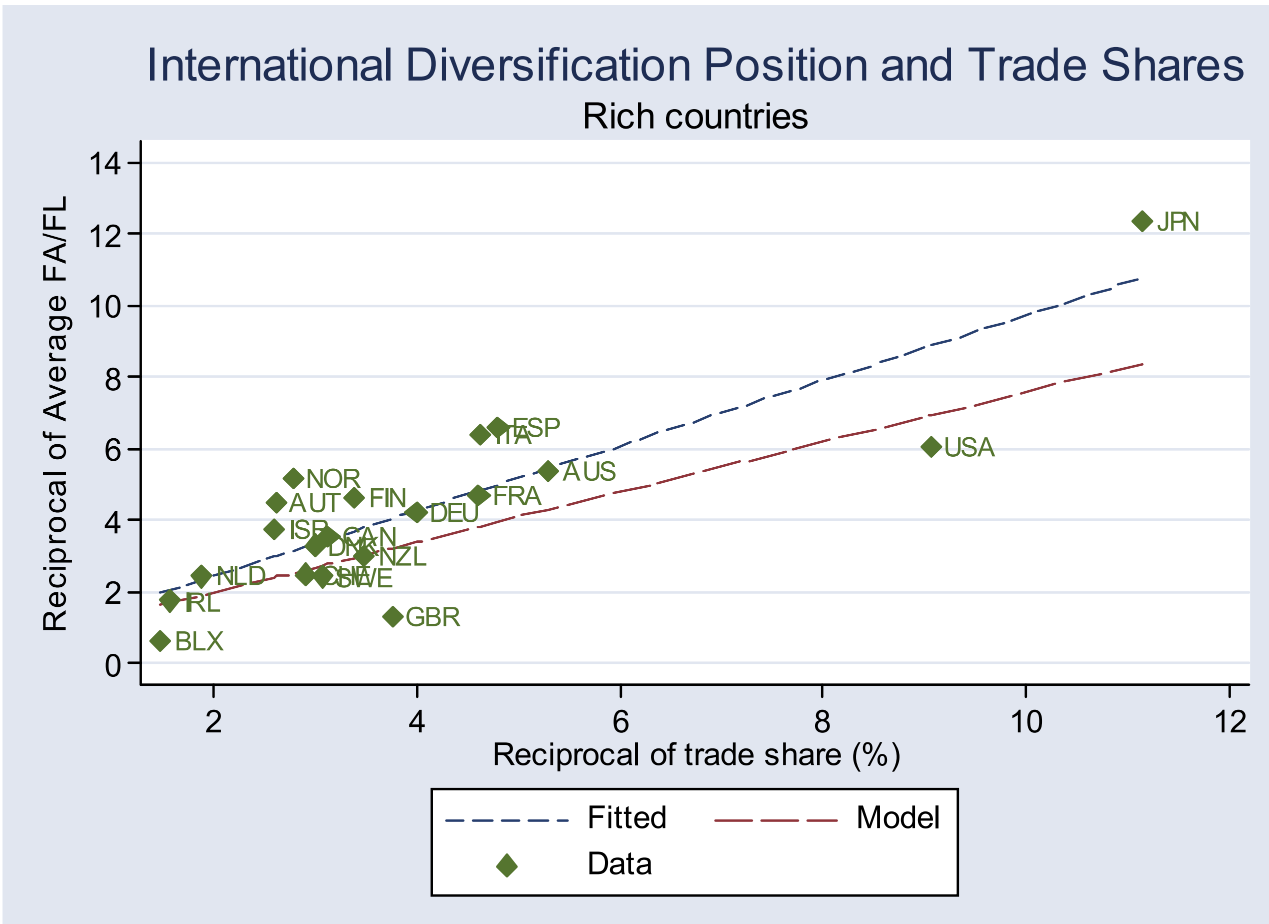

International Diversification Position and Trade Shares
All countries
Reciprocal of Average FA/FL (%)
0
5
10
15
20
25
30
35
2
4
6
8
10
12
Reciprocal of trade share (%)
CHN
IND
KOR
IDN
TUR
BRA
LKA
JPN
THA
PAK
DOM
COL
PHL
MEX
ECU
MUS
LSO
SLV
ARG
VEN
USA
AUS
FRA
BOL
CMR
NLD
BLX
GBR
Fitted
Model
Data

Figure 1:



International Diversification Position and Trade Shares
Rich countries
Reciprocal of Average FA/FL
Reciprocal of trade share (%)
0
2
4
6
8
10
12
14
JPN
USA
ESP
ITA
AUS
FRA
DEU
FIN
NOR
AUT
ISR
CAN
DNK
NZL
NLD
CHE
SWE
IRL
BLX
GBR
Fitted
Model
Data

Figure 2: