Wall Street and Silicon Valley: A Delicate Interaction*

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Abstract

Financial markets look at data on aggregate investment for clues about underlying profitability. At the same time, firms' investment depends on expected equity prices. This generates a two-way feedback between financial market prices and investment. In this paper we study the positive and normative implications of this interaction during episodes of intense technological change, when information about new investment opportunities is highly dispersed. Because high aggregate investment is "good news" for profitability, asset prices increase with aggregate investment. Because firms' incentives to invest in turn increase with asset prices, an endogenous complementarity emerges in investment decisions. We show that this complementarity dampens the impact of fundamental shocks (shifts in underlying profitability) and amplifies the impact of expectational shocks (correlated errors in assessments of profitability). We next show that these effects are symptoms of inefficiency: equilibrium investment reacts too little to fundamentals and too much to noise. We finally discuss policies that improve efficiency without requiring any informational advantage on the government's side.

Keywords: beauty contests, heterogeneous information, complementarity, amplification, volatility, inefficiency.

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1 Introduction

Financial markets follow closely the release of macroeconomic and sectorial data, looking for signals about underlying economic fundamentals. In particular, high current levels of activity tend to forecast high future profitability, leading to an increase in asset prices. At the same time, financial market prices affect the real economy by changing the incentive to invest for individual firms. For a start-up company, higher asset prices raise the value of a potential IPO and facilitate financing from venture capitalists. For firms already quoted on the stock market, higher asset prices raise the value of equity issues and the market valuation of further investments within the firm.

Both directions of causation—from real activity to financial prices and from financial prices to real investment—have been widely explored in existing theoretical and empirical work. The literature on the impact of macroeconomic news on asset prices goes back to Chen, Roll and Ross (1986) and Cutler, Poterba, and Summers (1989); the literature on the impact of asset prices on real investment goes back to Brainard and Tobin (1968), Tobin (1969), Abel and Blanchard (1986), and Barro (1990).

In this paper, we document novel positive and normative implications stemming from the interaction of these two channels when agents do not share the same information. We first show that this interaction generates a feedback mechanism between the real and the financial sector of the economy: high investment drives up aggregate activity; financial markets interpret this as a positive signal about future profitability; asset prices increase; and this adds fuel to the initial increase in investment. We next show that this mechanism can exacerbate non-fundamental movements in real investment and asset prices, and can distort allocative efficiency. This mechanism seems particularly relevant in periods of intense technological change, when information regarding the viability and profitability of new technologies is widely dispersed across the economy.

Preview. We conduct our exercise within a neoclassical economy in which allocations would be first-best efficient if all agents had the same information. A large number of "entrepreneurs" gets the option to invest in a new technology. They have dispersed information about the profitability of this technology and may sell their capital in a competitive financial market before uncertainty is realized. The "traders" who participate in the financial market are also imperfectly informed, but they observe aggregate investment, which provides a summary statistic of the information dispersed among the entrepreneurs. In this environment, movements in real investment and asset prices are driven by two types of shocks: "fundamental shocks," reflecting actual changes in the long-run profitability of investment, and "expectational shocks," reflecting correlated mistakes in individual assessments of this profitability.

The positive contribution of the paper is to study how the interaction between real and financial activity affects the transmission of these shocks in equilibrium. Because high aggregate investment is "good news" for profitability, asset prices increase with aggregate investment. As a result, an endogenous complementarity emerges in investment decisions. An entrepreneur anticipates that the price at which he might sell his capital will be higher the higher the aggregate level of investment. He is thus more willing to invest when he expects others to invest more.

In equilibrium, this complementarity induces entrepreneurs to rely more on common sources of information regarding profitability, and less on idiosyncratic sources of information. This is because common sources of information are relatively better predictors of other entrepreneurs' investment choices, and hence of future financial prices. For the same reason, the entrepreneurs' choices become more anchored to the common prior, and hence less sensitive to changes in the underlying fundamentals. It follows that the feedback between the real economy and financial markets amplifies the impact of common expectational shocks while also dampening the impact of fundamental shocks.

The normative contribution of the paper is to study whether the reaction of the economy to different shocks is optimal from a social perspective. The mere fact that entrepreneurs care about the financial market valuation of their investment does not, on its own, imply any inefficiency. Indeed, as long as information is common across agents, equilibrium asset prices coincide with the common expectation of profitability; whether entrepreneurs try to forecast fundamentals or asset prices is then completely irrelevant for efficiency.

This is not the case, however, when information is dispersed. The sensitivity of asset prices to aggregate investment induces a wedge between private and social returns to investment: while the fundamental valuation of the investment made by a given entrepreneur is independent of the investments made by other entrepreneurs (i.e., there are no production externalities or spillovers), the market valuation is not. By implication, the complementarity that emerges in equilibrium due to the dispersion of information is not warranted from a social perspective. It then follows that the positive effects documented above are also symptoms of inefficiency: equilibrium investment reacts too little to fundamental shocks and too much to expectational shocks.

We conclude by examining policies that can improve efficiency without requiring the government to have any informational advantage vis-a-vis the market. We first consider interventions "during the fact," when uncertainty has not been resolved and information remains widely dispersed. In particular, we consider a tax on financial trades or other policies aimed at stabilizing asset prices. By moderating the reaction of asset prices to aggregate investment, these policies dampen the equilibrium impact of non-fundamental shocks, thus improving efficiency. In so doing, however, these policies also dampen the equilibrium impact of fundamental shocks, which was inefficiently low to start with. It follows that these policies can be welfare improving, but never achieve full efficiency.

We next consider interventions "after the fact," when uncertainty has been resolved. Building on results from Angeletos and Pavan (2007b), we show that full efficiency can be achieved by introducing a tax on capital holdings that is contingent on both realized aggregate investment and realized profitability. Although real and financial decisions are sunk by the time these taxes are collected, the *anticipation* of these contingencies affects the incentives entrepreneurs and traders face "during the fact". By appropriately designing these contingencies, the government can induce agents to respond efficiently to different sources of information, even if it can not directly monitor these sources of information.

Discussion. The US experience in the second half of the 90s has renewed interest in investment and asset-price booms driven by apparent euphoria regarding new technologies (e.g., the Internet), and on the optimal policy response to these episodes. A common view in policy discussions is that entrepreneurs and corporate managers are driven by noise traders and other irrational forces in financial markets, or are irrational themselves. Elements of this view are formalized in Shiller (2000), Cecchetti et. al (2000), Bernanke and Gertler (2001), and Dupor (2005). Similar concerns are currently raised for the investment boom in China. The presumption that the government can detect "irrational exuberance" then leads to the result sought—that the government should intervene.

While we share the view that expectational errors may play an important role in these episodes, we also recognize that these errors may originate from noise in information rather than irrational exuberance. Furthermore, we doubt the government's ability to assess fundamentals better than the market itself. Our approach is thus very different: we identify an informational externality that can justify intervention even by a policy maker with no superior information. At the same time, on the positive side, we show that the interaction between real and financial activity can amplify the impact of noise and that this amplification is stronger when information is more dispersed. This helps explain, without any departure from rationality, why periods of intense technological change are likely to feature significant non-fundamental volatility.

Because the source of both amplification and inefficiency in our model rests on the property that investment is largely driven by expectations about others' choices rather than about fundamentals, our results are reminiscent of Keynes' famous beauty-contest metaphor for financial markets:

> "...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors..." Keynes (1936, p.156).

Implicit in Keynes' argument appears to be a normative judgement that something goes wrong when investment is driven by higher-order expectations. However, Keynes does not explain why this might be the case. More recently, Morris and Shin (2002), Allen, Morris and Shin (2005), and Cespa and Vives (2007) have argued that a mechanism similar to the one articulated by Keynes can increase the sensitivity of equilibrium outcomes to common sources of information. However, these papers also leave unexplained why this would lead to inefficient outcomes.[1] To the best of our knowledge, this is the first paper to provide a complete micro-founded argument for beauty-contest-like inefficiencies in the interaction of real and financial activity.

Other related literature. By focusing on the feedback between investment and financial markets as a potential explanation of "bubbly" episodes, the paper also relates to other lines of work. One line studies rational bubbles in economies with financial frictions or asset shortages (e.g., Ventura, 2003; Caballero, 2006; Caballero, Farhi and Hammour, 2006). The mechanisms studied in these papers also generate significant non-fundamental movements, but they are unrelated to information. The second and more closely related line studies speculative fluctuations in prices and

[1] Morris and Shin (2002) show that, when there is a discrepancy between private and social objectives, public information can have a negative impact on welfare. In our environment, a discrepancy between private and social objectives arises endogenously. The implications for the social value of public information are discussed in Section 5.3.

investment due to heterogeneous priors regarding profitability (e.g., Scheinkman and Xiong, 2003; Gilchrist, Himmelberg, and Huberman, 2005; Panageas, 2005). In these papers, investment and prices are largely driven by expectational shocks regarding others' valuations. This is similar to the role of higher-order expectations in our paper. However, in these papers asset prices continue to reflect the social value of investment, ensuring that no inefficiency emerges.[2] In our paper, instead, the impact of higher-order expectations is also the source of inefficiency.

The paper also relates to the growing macroeconomic literature on heterogeneous information and strategic complementarities (Amato and Shin, 2006; Angeletos and Pavan, 2004, 2007a,b; Baeriswyl and Conrand, 2007; Hellwig, 2005; Lorenzoni, 2006, 2007; Morris and Shin, 2002; Roca, 2006; Mackowiak and Wiederholt, 2006; Woodford, 2002). However, unlike the complementarities that originate in monopolistic price competition, production externalities, or other payoff effects, the complementarity documented in this paper is purely due to an informational externality: it emerges only when information is dispersed and only because aggregate investment is a signal of the underlying fundamentals. This specific source of complementarity is the key to both the positive and the normative results of our paper.[3]

Finally note that the effects documented in this paper are different from other "multiplier effects" previously considered in the literature such as the credit multipliers in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). These mechanisms do not rely on the dispersion of information and do not feature the type of non-fundamental disturbances we are concerned with.

Layout. Section 2 introduces the baseline model. Section 3 characterizes the equilibrium and derives the positive implications of the model. Section 4 characterizes the socially efficient use of information and contrasts it to the equilibrium. Section 5 discusses policy implications. Section 6 considers a number of extensions, relaxing certain simplifying assumptions of the baseline model and introducing alternative sources of non-fundamental volatility. Section 7 concludes. All proofs are in the Appendix.

[2] In Gilchrist, Himmelberg, and Huberman (2005), inefficiency can arise due to the monopoly power of the owners of the firms issuing speculative stocks.

[3] An endogenous complementarity is also present in Goldstein, Ozdenoren and Yuan (2007). In their model, the complementarity originates in the information conveyed to the central bank by the size of the speculative run against the domestic currency.

2 The baseline model

We consider an environment where heterogeneously informed agents choose how much to invest in a "new technology" with uncertain returns. After investment has taken place, but before uncertainty is resolved, agents trade financial claims on the returns of the installed capital. At this point, the observation of aggregate investment partially reveals the information that was dispersed in the population during the investment stage.

Timing, actions, and information. There are four periods, $t \in \{0, 1, 2, 3\}$, and two types of agents: "entrepreneurs" who first get the option to invest in the new technology and "traders" who can later buy claims on the installed capital of the entrepreneurs. Each type is of measure 1/2; we index entrepreneurs by $i \in [0, 1/2]$ and traders by $i \in (1/2, 1]$.

At $t = 0$, nature draws a random variable θ from a Normal distribution with mean $\mu > 0$ and variance $1/\pi_\theta$ (i.e., π_θ is the precision of the prior). This random variable represents the exogenous productivity of the new technology and is unknown to all agents.

At $t = 1$, the "real sector" of the economy operates: each entrepreneur decides how much to invest in the new technology. Let k_i denote the investment of entrepreneur i. The cost of this investment is $k_i^2/2$ and is incurred within the period. When choosing k_i, entrepreneur i has access to two types of information. First, he observes a private signal $x_i = \theta + \xi_i$, where ξ_i is Gaussian noise, independent of θ, independently and identically distributed across agents, with variance $1/\pi_x$ (i.e., π_x is the precision of the private signal). In addition, he observes a common signal $y = \theta + \varepsilon$, where ε is Gaussian noise, independent of θ and of $\{\xi\}_{i \in [0,1/2]}$, with variance $1/\pi_y$ (i.e., π_y is the precision of the common signal).

At $t = 2$, the "financial market" opens: some entrepreneurs sell their installed capital to the traders. In particular, we assume that each entrepreneur is hit by a "liquidity shock" with probability $\lambda \in (0, 1)$. Liquidity shocks are i.i.d. across agents, so λ is also the fraction of entrepreneurs hit by the shock. Entrepreneurs hit by the shock are forced to sell all their capital to the traders. For simplicity, entrepreneurs not hit by the shock are not allowed to trade any claims on installed capital.[4] The financial market is competitive and p denotes the price of one unit of installed capital. The traders, who can acquire the installed capital of the entrepreneurs hit by the liquidity shock,

[4]This stark assumption greatly simplifies the derivation and presentation of our results. In Section 6, we show that all results survive in a setup where entrepreneurs not hit by the shock are allowed to choose how much to trade in the financial market.

do not observe any of the signals $\{x_i\}_{i\in[0,1/2]}$ and y which were revealed to the entrepreneurs at $t = 1$. However, they do observe the aggregate level of investment from period 1, $K = \int_0^1 k_i di$, and they use this observation to update their beliefs about θ.[5]

Finally, at $t = 3$, θ is publicly revealed, each unit of capital gives a payoff of θ to its owner, and this payoff is consumed.

Payoffs. All agents are risk neutral and the discount rate is zero. Payoffs are thus given by $u_i = c_{i1} + c_{i2} + c_{i3}$, where c_{it} denotes agent i's consumption in period t. First consider an entrepreneur. If he is not hit by the liquidity shock his consumption is $(c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, 0, \theta k_i)$. If he is hit by the shock, he sells all his capital at the price p and his consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, pk_i, 0)$. Next consider a trader, and let q_i denote the units of installed capital he acquires in period 2. His consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (0, -pq_i, \theta q_i)$. Summing up, an entrepreneur's payoff is $u_i = -k_i^2/2 + \theta k_i$ if he is not hit by the shock and $u_i = -k_i^2/2 + pk_i$ otherwise, while a trader's payoff is $u_i = (\theta - p)q_i$.

Remarks. The two essential ingredients of the model are (i) that the agents who make the initial investment choices have dispersed private information, so that aggregate investment is a signal of the fundamental, and (ii) that there is some "noise" that prevents aggregate investment from perfectly revealing the fundamental to all agents, so that the dispersion of information does not completely vanish by the time agents trade in the financial market. The information structure used here is only a convenient way to obtain these two properties. In particular, the error ε in the signal y, is a convenient way to introduce correlation in the errors in the entrepreneurs' information; this in turn noises up the inference problem of the traders because, in equilibrium, aggregate investment will move both with θ and with ε. Indeed, the analysis easily extends if we replace y with private signals whose errors are correlated across entrepreneurs, or if we introduce other sources of aggregate noise.[6]

Also notice that the "liquidity shock" need not be taken too literally. Its presence captures the more general idea that when an agent makes an investment decision, be him a start-up entrepreneur or the manager of a quoted company, he cares about the market valuation of his investment at some point in the life of the project. A start-up entrepreneur may worry about the price at which he will

[5] Letting the traders observe the entire cross-sectional distribution of investments does not affect the results. This is because, in equilibrium, such a distribution is Normal with known variance; it follows that the mean investment contains as much information as the entire cross-sectional distribution.

[6] See the variant with shocks to the market valuation of the asset in Section 6.3 and the variant with shocks to the cost of investment in the Appendix.

be able to do a future IPO, a corporate manager may be concerned about the price at which the company will be able to issue new shares. In what follows, we interpret λ broadly as a measure of the sensitivity of the firms' investment decisions to forecasts of future equity prices.

Finally, notice that there are no production spillovers and no direct payoff externalities of any kind: both the initial cost $(-k_i^2/2)$ and the eventual return on capital (θk_i) are independent of the investment decisions of other agents. The strategic complementarity that will be identified in Section 3.1 originates purely in the dispersion of information.

A benchmark with no informational frictions. Before we proceed, it is useful to examine what happens when the dispersion of information vanishes at the time of trading in the financial market. That is, suppose that all the information that is dispersed during period 1 (namely, the signals $\{x_i\}_{i\in[0,1/2]}$ and y) becomes commonly known in period 2. The fundamental θ then also becomes commonly known and the financial market clears if and only if $p = \theta$. It follows that the expected payoff of entrepreneur i in period 1 reduces to $\mathbb{E}[u_i|x_i, y] = \mathbb{E}[\theta|x_i, y]k_i - k_i^2/2$, which in turn implies that equilibrium investment is given by

$$k_i = \mathbb{E}[\theta|x_i, y] = \frac{\pi_\theta}{\pi_\theta + \pi_x + \pi_y}\mu + \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y}x_i + \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y}y.$$

The key result here is that equilibrium investment is driven solely by first-order expectations regarding the fundamental and is independent of the intensity of the entrepreneurs' concern about financial prices (as measured by λ). This result does not require θ to be perfectly known in period 2. Rather, it applies more generally as long as the asymmetry of information about θ vanishes in period 2.[7] This case, which we refer to as the case with "no informational frictions," provides a convenient reference point for the rest of the analysis.

3 Equilibrium

The entrepreneurs' strategy is described by a function $k : \mathbb{R}^2 \to \mathbb{R}$ so that $k(x, y)$ denotes the investment made when the private signal is x and the common signal is y. Aggregate investment

[7] To clarify this point, consider an arbitrary information structure. Let $\mathcal{I}_{i,t}$ denote the information of agent i in period t. Impose that no agent has private information about θ in period 2 : $\mathbb{E}[\theta|\mathcal{I}_{i,2}] = \mathbb{E}[\theta|\mathcal{I}_{j,2}]$ for all (i, j). From market clearing we have $p = \mathbb{E}[\theta|\mathcal{I}_{i,2}]$ for all i. From the law of iterated expectations we then have that $\mathbb{E}[p|\mathcal{I}_{i,1}] = \mathbb{E}[\mathbb{E}[\theta|\mathcal{I}_{i,2}]|\mathcal{I}_{i,1}] = \mathbb{E}[\theta|\mathcal{I}_{i,1}]$ for all i. It follows that every entrepreneur chooses $k_i = \mathbb{E}[\theta|\mathcal{I}_{i,1}]$.

is then a function of θ and y:

$$K(\theta, y) = \int k(x, y)\, d\Phi(x|\theta), \tag{1}$$

where $\Phi(x|\theta)$ denotes the cumulative distribution function of x conditional on θ. Since traders observe aggregate investment and are risk neutral, the unique market-clearing price in period 2 is $p = \mathbb{E}[\theta|K]$, where the latter denotes the expectation of θ conditional on the observed level of K.[8] Since K is determined by (θ, y), it follows that p is also a function of (θ, y). We thus define an equilibrium as follows.

Definition 1 *A (symmetric) equilibrium is an investment strategy $k(x, y)$ and a price function $p(\theta, y)$ that satisfy the following conditions:*

(i) for all (x, y),

$$k(x, y) \in \arg\max_k \mathbb{E}\left[\ (1-\lambda)\,\theta k + \lambda p(\theta, y)\, k - k^2/2 \mid x, y\ \right];$$

(ii) for all (θ, y),

$$p(\theta, y) = \mathbb{E}[\ \theta \mid K(\theta, y)\ ],$$

where $K(\theta, y) = \int k(x, y)\, d\Phi(x|\theta)$.

Condition (i) requires that the entrepreneurs' investment strategy be individually rational, taking as given the equilibrium price function. Condition (ii) requires that the equilibrium price be consistent with rational expectations and individual rationality on the traders' side, taking as given the strategy of the entrepreneurs.

As it is often the case in the literature, tractability requires that we restrict attention to equilibria in which the price function is linear.

Definition 2 *A linear equilibrium is an equilibrium in which $p(\theta, y)$ is linear in (θ, y).*

3.1 Endogenous complementarity

The optimality condition for the entrepreneurs's strategy can be written as

$$k(x, y) = \mathbb{E}[\ (1-\lambda)\,\theta + \lambda p(\theta, y)\ \mid x, y\ ]. \tag{2}$$

[8] Since the price is only a function of K and K is publicly observed, the price itself does not reveal any additional information. Therefore, we can omit any conditioning on p.

The linearity of $p(\theta, y)$ in (θ, y) and of $\mathbb{E}[\theta|x, y]$ in (x, y) then guarantee that the entrepreneurs' strategy is linear in (x, y); that is, there are coefficients $(\beta_0, \beta_1, \beta_2)$ such that

$$k(x, y) = \beta_0 + \beta_1 x + \beta_2 y. \tag{3}$$

By implication, aggregate investment is given by $K(\theta, y) = \beta_0 + (\beta_1 + \beta_2)\theta + \beta_2\varepsilon$. Observing K is thus informationally equivalent to observing a Gaussian signal z with precision π_z, where

$$z \equiv \frac{K - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2}\varepsilon \qquad \text{and} \qquad \pi_z \equiv \left(\frac{\beta_1 + \beta_2}{\beta_2}\right)^2 \pi_y. \tag{4}$$

Standard Gaussian updating then gives the expectation of θ conditional on K as a weighted average of the prior and the signal z:

$$\mathbb{E}[\theta|K] = \frac{\pi_\theta}{\pi_\theta + \pi_z}\mu + \frac{\pi_z}{\pi_\theta + \pi_z}z.$$

Since market clearing in period 2 requires $p = \mathbb{E}[\theta|K]$, we conclude that the equilibrium price is

$$p(\theta, y) = \gamma_0 + \gamma_1 K(\theta, y), \tag{5}$$

where

$$\gamma_0 \equiv \frac{\pi_\theta}{\pi_\theta + \pi_z}\mu - \frac{\pi_z}{\pi_\theta + \pi_z}\frac{\beta_0}{\beta_1 + \beta_2} \qquad \text{and} \qquad \gamma_1 \equiv \frac{\pi_z}{\pi_\theta + \pi_z}\frac{1}{\beta_1 + \beta_2}. \tag{6}$$

These results are summarized in the following lemma.

Lemma 1 *In any linear equilibrium, there are coefficients* $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1)$ *such that*

$$k(x, y) = \beta_0 + \beta_1 x + \beta_2 y \quad \textit{and} \quad p(\theta, y) = \gamma_0 + \gamma_1 K(\theta, y).$$

Moreover, $\gamma_1 > 0$ *if and only if* $\beta_1 + \beta_2 > 0$.

Therefore, provided that higher investment is "good news" for profitability (which is true whenever $Cov(K, \theta) > 0$, or equivalently $\beta_1 + \beta_2 > 0$), the equilibrium price increases with aggregate investment ($\gamma_1 > 0$).[9] This in turn induces strategic complementarity in investment decisions. In-

[9] Throughout the paper, we say that higher investment is good news for profitability if and only if the traders expectations about θ increases with K.

deed, when the entrepreneurs are choosing a higher level of investment, they are sending a positive signal to the financial market, thus increasing the price at $t = 2$. But then each individual entrepreneur's willingness to invest at $t = 1$ is higher when he expects a higher level of investment from other entrepreneurs, which means precisely that investment choices are strategic complements. We formalize these intuitions in the next result, which follows directly from replacing condition (5) into condition (2).

Lemma 2 *In any linear equilibrium, the investment strategy satisfies*

$$k(x, y) = \mathbb{E}[(1-\alpha)\kappa(\theta) + \alpha K(\theta, y) \mid x, y], \tag{7}$$

where $\alpha \equiv \lambda\gamma_1$ *and* $\kappa(\theta) \equiv \frac{(1-\lambda)\theta+\lambda\gamma_0}{1-\lambda\gamma_1}$.

Condition (7) can be interpreted as the best-response condition in the coordination game among the entrepreneurs: it describes the optimal strategy for each entrepreneur as a function of (his expectation) of aggregate investment (i.e., the strategy of other agents). The coefficient α then measures the degree of strategic complementarity in investment decisions: the higher α, the higher the slope of the best response of individual investment to aggregate investment, that is, the higher the incentive of entrepreneurs to align their investment choices.

A similar best-response condition characterizes the class of linear-quadratic games examined in Angeletos and Pavan (2007a), including the special case in Morris and Shin (2002). However, there are two important differences. First, while in those games the degree of strategic complementarity is exogenously given by the payoff structure of the game, here it is endogenously determined as an integral part of the equilibrium. And second, while in those games the degree of complementarity is independent of the information structure, here it actually originates in the dispersion of information.

Indeed, the complementarity in our setup is solely due to the informational content of aggregate investment. How much information aggregate investment conveys about θ determines the coefficient γ_1, which captures the sensitivity of prices to aggregate investment. In turn, the coefficient γ_1 pins down the value of α, which captures the degree of complementarity between in the entrepreneurs' investment decisions. In the absence of informational frictions (the benchmark case examined in the previous section), aggregate investment provides no information to the traders, prices are thus independent of K, and the complementarity in investment choices is absent. When instead information is dispersed, aggregate investment becomes a signal of θ, prices respond to aggregate investment and a complementarity in investment decisions thus emerges. The more informative aggregate investment is about θ, the stronger the complementarity.

Because the complementarity depends on the informational content of aggregate investment, which in turn depends on the strategies of the entrepreneurs, to determine the equilibrium value of α we need to solve a fixed point problem. Before doing so we first derive a result that helps understand how the endogenous complementarity interferes with the entrepreneurs' incentives to use available information.

Lemma 3 *In a linear equilibrium,*

$$\frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x}\frac{1}{1-\alpha}. \tag{8}$$

Therefore, the sensitivity of equilibrium investment to the common signal relative to the private signal is higher the higher the degree of strategic complementarity α.

Let us provide some intuition for this result. When the agents' strategy is $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$, then aggregate investment is $K(\theta, y) = \beta_0 + \beta_1\theta + \beta_2 y$ and an agent's best predictor of aggregate investment is

$$\mathbb{E}[K|x, y] = \beta_0 + \beta_1\mathbb{E}[\theta|x, y] + \beta_2 y.$$

The private signal x helps predict aggregate investment only through $\mathbb{E}[\theta|x, y]$, while the common signal y helps predict aggregate investment both through $\mathbb{E}[\theta|x, y]$ and directly through its effect on the term $\beta_2 y$. Therefore, relative to how much the two signals help predict the fundamental, the common signal y is a relatively better predictor of aggregate investment than the private signal x. But now recall that a higher α means a stronger incentive for the individual entrepreneur to align his investment choice with that of the others. It follows that when α is higher entrepreneurs find it optimal to rely more heavily on the common signal y relative to the private signal x, for it is the former that best helps them align their choice with that of others.

3.2 Equilibrium characterization

As noted earlier, completing the equilibrium characterization requires solving for the equilibrium value of α, which involves a fixed-point problem. On the one hand, how entrepreneurs use their available information depends on α, the complementarity induced by the response of prices to aggregate investment. On the other hand, how sensitive prices are to aggregate investment, and hence the value of α, depends on how informative aggregate investment is about the fundamental, which in turn depends on how entrepreneurs use their available information in the first place. This fixed-point problem captures the two-way feedback between the real and the financial sector. Its solution is provided in the following lemma.

Lemma 4 *There exist functions $F : \mathbb{R} \times (0,1) \times \mathbb{R}^3_+ \to \mathbb{R}$ and $G : \mathbb{R} \times (0,1) \times \mathbb{R}^3_+ \to \mathbb{R}^5$ such that the following are true:*

(i) *In any linear equilibrium, β_2/β_1 solves*

$$\frac{\beta_2}{\beta_1} = F\left(\frac{\beta_2}{\beta_1}; \lambda, \pi_\theta, \pi_x, \pi_y\right) \tag{9}$$

while $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1) = G\left(\beta_2/\beta_1; \lambda, \pi_\theta, \pi_x, \pi_y\right)$.

(ii) *Equation (9) has at least one solution at some $\beta_2/\beta_1 > \pi_y/\pi_x$.*

(iii) *For any $(\pi_\theta, \pi_x, \pi_x)$ there exists a cutoff $\bar{\lambda} = \bar{\lambda}\left(\pi_\theta, \pi_x, \pi_y\right) > 0$ such that if $\lambda < \bar{\lambda}$ then (9) admits a unique solution.*

(iv) *There is an open set S such that, if $(\lambda, \pi_\theta, \pi_x, \pi_x) \in S$, (9) admits multiple solutions.*

The fixed-point problem that leads to the equilibrium characterization is set up in terms of the variable $b = \beta_2/\beta_1$, which represents the relative sensitivity of entrepreneurial investment to the two signals. Given b, we can determine the sensitivity of the price to aggregate investment γ_1. Given γ_1, we can then determine the complementarity α and then the sensitivity b of individual best responses to the two signals. These steps describe the mapping F used in Lemma 4 and provide the intuition for part (i) of the lemma: the fixed points of F identify all the linear equilibria of our economy. Parts (ii)-(iv) then characterize the fixed points of F, leading to the following result.

Proposition 1 *A linear equilibrium always exists and is unique if λ is small enough.*

The possibility of multiple equilibria for high values of λ is interesting for several reasons. First, it illustrates the potential strength of the two-way feedback between real and financial activity. Second, this multiplicity originates solely from an informational externality rather than from the more familiar direct payoff interdependences. Finally, this multiplicity can induce additional non-fundamental volatility in both real investment and financial prices if we allow for sunspots.

However, for the rest of the paper, we leave aside the possibility of multiple equilibria and focus on the case where λ is small enough that the equilibrium is unique. The next proposition shows that in this case aggregate investment necessarily increases with θ. Together with Lemmas 1 and 2, this ensures that the equilibrium price increases with K and that $\alpha > 0$. The proposition then further establishes that α is increasing in λ, which guarantees that the complementarity in investment decisions is stronger when entrepreneurs are more sensitive to financial market prices.

Proposition 2 *Whenever the equilibrium is unique, the following are true:*

(i) *individual investment increases with both signals, i.e.* $\beta_1, \beta_2 > 0$*, and, by implication, aggregate investment is positively correlated with the fundamental,* $Cov(K, \theta) > 0$*.*

(ii) *The equilibrium price increases with aggregate investment,* $\gamma_1 > 0$*.*

(iii) *The equilibrium degree of complementarity* α *is positive and increasing in* λ*.*

3.3 Sensitivity to fundamental and expectational shocks

To further appreciate the positive implications of informational frictions—and the complementarity thereof—it is useful to rewrite aggregate investment as

$$K = \beta_0 + (\beta_1 + \beta_2)\,\theta + \beta_2 \varepsilon.$$

Aggregate investment thus depends on two types of shocks: *fundamental shocks*, captured by θ, and *expectational shocks*, captured by ε. How entrepreneurs use available information affects how investment respond to these shocks: the sensitivity to fundamentals is governed by the sum $\beta_1 + \beta_2$, while the sensitivity to expectational shocks is governed by β_2.

We showed earlier (Lemma 3) that a stronger complementarity α is associated with a higher sensitivity to the common signal relative to the private, β_2/β_1. Because $Cov(K, \theta) > 0$ suffices for the equilibrium price to increase with K and hence for $\alpha > 0$, it also suffices for $\beta_2/(\beta_1 + \beta_2) > \pi_y/(\pi_x + \pi_y)$.[10] In contrast, when there are no informational frictions (the benchmark case examined in the previous section), then the equilibrium price is independent of K, $\alpha = 0$, and $\beta_2/(\beta_1 + \beta_2) = \pi_y/(\pi_x + \pi_y)$. The following is an immediate implication.

[10] Recall that when $\alpha = 0$, then $\beta_2/(\beta_1 + \beta_2) = \pi_y/(\pi_x + \pi_y)$.

Corollary 1 ***(i)*** *In any equilibrium in which high investment is "good news" for θ, the impact of expectational shocks relative to fundamental shocks is higher than in the absence of informational frictions.* ***(ii)*** *If λ is small enough the equilibrium is unique and high investment is "good news" for θ.*

This result is the key positive prediction of the paper: it says that informational frictions amplify non-fundamental volatility, that is, they reduce the R-square of a regression of aggregate investment on expected profits. Importantly, because α increases with λ (see part (iii) of Proposition 2), this amplification effect is stronger the more entrepreneurs care about asset prices. The next proposition reinforces these findings by examining the *absolute* impact of the two shocks.

Proposition 3 *There exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda}]$, there is a unique linear equilibrium and the following comparative statics hold:*

(i) *higher λ reduces $\beta_1 + \beta_2$, thus dampening the impact of fundamental shocks;*

(ii) *higher λ increases β_2, thus amplifying the impact of expectational shocks.*

The key intuition for these results is again the role of the complementarity for the equilibrium use of information. To see this, suppose for a moment that $\gamma_0 = 0$ and $\gamma_1 = 1$, meaning that $p(\theta, y) = K(\theta, y)$ in all states. The entrepreneurs' best response then reduces to

$$k(x, y) = \mathbb{E}\left[\ (1 - \lambda)\theta + \lambda K(\theta, y)\ \mid x, y\ \right], \tag{10}$$

so that the degree of complementarity now coincides with λ. One can then easily show that the unique solution to (10) is $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$, with

$$\beta_0 = \frac{\pi_\theta}{\pi_\theta + \pi_x(1-\lambda) + \pi_y}\mu, \quad \beta_1 = \frac{\pi_x(1-\lambda)}{\pi_\theta + \pi_x(1-\lambda) + \pi_y}, \quad \text{and} \quad \beta_2 = \frac{\pi_y}{\pi_\theta + \pi_x(1-\lambda) + \pi_y}.$$

It is then immediate that a higher λ increases the sensitivity to the prior (captured by β_0) and the sensitivity to the common signal (captured by β_2), while it decreases the sensitivity to the private signal (captured by β_1). We have already given the intuition for the result that a stronger complementarity amplifies the reliance on the common signal and dampens the reliance on the private signal. That it also increases the reliance on the prior is for exactly the same reason as for the common signal: the prior is a relative good predictor of others' investment choices.

However note that the average return on investment coincides with the mean of θ, no matter λ; this is because the average price must equal the mean of θ, for otherwise the traders would make on average non-zero profits, which would be a contradiction. But then the average investment must also be equal to the average of θ, that is, $\beta_0 + (\beta_1 + \beta_2)\,\mu$ must equal μ. It is then immediate that, because a higher λ increases β_0, it also reduces the sum $\beta_1 + \beta_2$. In other words, investment is less sensitivity to changes in fundamentals simply because the complementarity strengthens the anchoring effect of the prior.

These intuitions are exact only when $\gamma_0 = 0$ and $\gamma_1 = 1$, so that the equilibrium price coincides with aggregate investment. In turn, one can show that this is the case only when the prior is completely uninformative. More generally, the price is an increasing function of aggregate investment but with $\gamma_0 \neq 0$ and $\gamma_1 \neq 1$. This explains why Proposition 3 has been established only for a subset of the parameter space, namely for λ small enough. Nevertheless, extensive simulations suggest that, for any $\lambda > 0$, any equilibrium in which high investment is "good news" for profitability (i.e. $Cov(K, \theta) > 0$) features

$$\beta_1 + \beta_2 < \frac{\pi_x + \pi_y}{\pi_\theta + \pi_x + \pi_y} \qquad \text{and} \qquad \beta_2 > \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y},$$

which means that the impact of fundamental shocks is lower and that of expectational shocks is higher than in the absence of informational frictions.

4 Constrained efficiency

The analysis so far has focused on the positive properties of the equilibrium. We now study its normative properties by examining whether there is an allocation that, given the underlying information structure, leads to higher welfare.

The question of interest here is whether society can do better, relative to equilibrium, by having the agents use their available information in a different way—not whether society can do better by giving the agents more information. We thus adopt the same constrained efficiency concept as in Angeletos and Pavan (2007a,b): we consider the allocation that maximizes ex-ante welfare subject to the sole constraint that the choice of each agent must depend only on the information available to him. In other words, we let the planner dictate how agents use their available information, but we do not let the planner transfer information from one agent to another. In so doing, we momentarily disregard incentive constraints; in the next section we will then identify tax systems that implement the efficient allocation as an equilibrium.

Note that the payments made in the financial market, pk_i and pq_i, represent pure transfers between the entrepreneurs and the traders and therefore do not affect ex-ante utility.[11] We can thus focus on the investment strategy and define the efficient allocation as follows.

Definition 3 *The efficient allocation is a strategy $k(x,y)$ that maximizes ex-ante utility*

$$\mathbb{E}u = \int \left\{ \int \tfrac{1}{2} \left[(1-\lambda)\,\theta k\,(x,y) - \tfrac{1}{2} k\,(x,y)^2 \right] d\Phi(x|\theta) + \tfrac{1}{2} \left[\theta \lambda K\,(\theta,y) \right] \right\} d\Psi(\theta,y) \tag{11}$$

with $K\,(\theta,y) = \int k\,(x,y)\,d\Phi(x|\theta)$.

Condition (11) gives ex-ante utility for an arbitrary strategy. The first term in square brackets is the payoff of an entrepreneur with information (x,y); the second term in square brackets is the payoff of a trader when aggregate investment is $K(\theta,y)$; finally Ψ denotes the cumulative distribution function of the joint distribution of (θ,y). Note that the transfer of capital from the entrepreneurs that are hit by the liquidity shock to the traders does not affect the return to investment. It follows that (11) can be rewritten compactly as[12]

$$\mathbb{E}u = \tfrac{1}{2}\mathbb{E}[V\,(k\,(x,y)\,,\theta)\,] = \tfrac{1}{2}\mathbb{E}[\mathbb{E}[V\,(k\,(x,y)\,,\theta)\,|x,y]],$$

where

$$V\,(k,\theta) \equiv \theta k - \tfrac{1}{2}k^2.$$

From the society's viewpoint, λ is irrelevant and it is as if the entrepreneurs' payoffs are $V(k,\theta)$. It is then obvious that a strategy $k(x,y)$ is efficient if and only if, for almost all x and y, $k(x,y)$ maximizes $\mathbb{E}[V(k,\theta)|x,y]$, which gives the following result.

Proposition 4 *The efficient investment strategy is given by*

$$k\,(x,y) = \mathbb{E}\,[\theta|x,y] = \delta_0\mu + \delta_1 x + \delta_1 y,$$

[11] By ex-ante, we mean before the realization of any random variable, including those that determine whether an agent will be a trader or an investor.

[12] It suffices to substitute the expression for $K\,(\theta,y)$ in (11).

where

$$\delta_0 \equiv \frac{\pi_\theta}{\pi_\theta + \pi_x + \pi_y}, \quad \delta_1 \equiv \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y}, \quad \delta_2 \equiv \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y}.$$

Note that the efficient strategy would have coincided with the equilibrium strategy if it were not for informational frictions. It follows that our earlier positive results admit a normative interpretation.

Corollary 2 ***(i)*** *In the presence of informational frictions, the efficient strategy is never an equilibrium.* ***(ii)*** *In any equilibrium in which high investment is "good news" for θ, the impact of expectational shocks relative to fundamental shocks is inefficiently high.*

This corollary summarizes the key normative prediction of the paper. To understand part (i), note that if all entrepreneurs $j \neq i$ were to follow the efficient strategy, then aggregate investment would be a positive signal of θ and the price would thus increase with K; but then entrepreneur i would perceive a complementarity between his investment choice and those of other entrepreneurs and would respond by raising his reliance on the prior and on the common signal and by reducing his reliance on the private signal. Part (ii) then follows directly from Corollary 1 along with the property that the efficient strategy coincides with the equilibrium in the absence of informational frictions.

While part (ii) establishes the inefficiency in terms of the *relative* impact of expectational shocks, Proposition 3 and numerical simulations give the result in absolute terms as well: equilibrium investment reacts too little to fundamental shocks and too much to expectational shocks.

5 Policy implications

Having identified a potential source of inefficiency, we now analyze the effect of different policies. We first consider interventions "during the fact," by which we mean intervention in the financial market at stage $t = 2$, while uncertainty about θ has not yet been resolved. We next consider policies "after the fact," by which we mean policies contingent on information that becomes public at stage $t = 3$, after uncertainty about θ has been largely resolved. In both cases, we impose that the government has no informational advantage vis-a-vis the private sector, which is our preferred benchmark for policy analysis. At the end of this section, however, we also consider situations where the government can directly affect the information available to the agents.

5.1 Intervention "during the fact": price stabilization

We start by considering policies aimed at reducing asset price volatility. In particular, suppose the government imposes a proportional tax τ on financial trades (purchases of capital) at date 2. This tax can depend on the price, which is public information. For simplicity, it takes the following linear form:

$$\tau(p) = \tau_0 + \tau_1 p, \tag{12}$$

where (τ_0, τ_1) are scalars. Tax revenues are rebated as a lump sum.

The equilibrium price in the financial market is now given by $p = \mathbb{E}[\theta|K] - (\tau_0 + \tau_1 p)$; equivalently,

$$p = \frac{1}{1+\tau_1}\left(\mathbb{E}[\theta|K] - \tau_0\right) = \frac{1}{1+\tau_1}\left(\gamma_0 + \gamma_1 K - \tau_0\right), \tag{13}$$

where γ_0 and γ_1 are given, as before, by (6). When the tax is pro-cyclical (i.e., $\tau_1 > 0$), its effect is to dampen the response of asset prices to the traders' expectation of θ, and thereby their response to the news contained in aggregate investment. In equilibrium, this tends to reduce the degree of complementarity in investment choices. To see this more clearly, note that the degree of complementarity is now given by

$$\alpha = \frac{\lambda\gamma_1}{1+\tau_1}.$$

If γ_1 were exogenous, it would be immediate that α decreases with τ_1. But if α falls, we know from Lemma 3 that β_2/β_1, the relative weight on common sources of information, must also fall. This in turn means that aggregate investment becomes more informative of θ, so that γ_1 increases, counteracting the direct effect of τ_1 on α. However, one can show that, at least as long as the equilibrium is unique, the direct effect dominates, guaranteeing that the *equilibrium* degree of complementarity decreases with τ_1.[13]

We conclude that a higher τ_1, by reducing the degree of complementarity, necessarily reduces the relative impact of expectational shocks. However, by reducing the overall sensitivity of prices to all sources of variation in investment, a higher τ_1 also reduces the impact of fundamental shocks. As argued in the previous section, in the absence of policy intervention, investment is excessively sensitive to expectational shocks and insufficiently sensitive to fundamental shocks. It follows that the welfare consequences of the tax are ambiguous: while reducing the impact of expectational

[13] This follows from an argument similar to the one that establishes that α is monotonic in λ.

shocks improves efficiency, reducing the impact of fundamental shocks has the opposite effect.

These intuitions are illustrated in Figure 1 where for each value of τ_1, the value of τ_0 is chosen optimally to maximize welfare. The top panel depicts the difference in welfare under the stabilization policy considered here and under the constrained efficient allocation; the bottom panels depict the sensitivity to expectational shocks β_2 and to fundamental shocks $(\beta_1 + \beta_2)$.[14] The figure is drawn for a baseline set of parameters: $\pi_\theta = \pi_x = \pi_y = 1$ and $\lambda = 0.5$). However, its qualitative features are robust across a wide set of parametrizations. In particular, we have randomly drawn 10,000 parameter vectors $(\lambda, \pi_\theta, \pi_x, \pi_y)$ from $(0, 1) \times \mathbb{R}^3_+$. For each such vector, we have found that the optimal τ_1 is positive and it induces a lower β_2 and a lower $\beta_1 + \beta_2$ as compared to the equilibrium without policy, reflecting the trade off discussed above.

[Insert Figure 1 here]

While these numerical results suggest that the optimal policy always involves some price stabilization (i.e., $\tau_1 > 0$), we have not been able to establish this result formally. However, it is easy to rule out full price stabilization (i.e., $\tau_1 \to \infty$). In this limit, prices cease to react to aggregate investment, the strategic complementarity disappears, and equilibrium investment reduces to $k(x, y) = (1 - \lambda)\,\mathbb{E}[\theta|x, y]$. By implication, the relative sensitivity of investment to expectational shocks $\beta_2/(\beta_1 + \beta_2)$ is at its efficient level, but its overall sensitivity to the fundamental is λ times lower than at the efficient level. At this point, a marginal increase in the relative sensitivity implies only a second-order welfare loss, while a marginal increase in the overall sensitivity implies a first-order welfare gain. It follows that it is never optimal to fully stabilize the price.

Proposition 5 *A tax that stabilizes prices can increase welfare; however, a tax that completely eliminates price volatility is never optimal.*

5.2 Intervention after the fact: corrective taxation

Suppose now that the government imposes a proportional tax τ on asset holdings in period 3. The tax is now paid by the entrepreneurs not hit by the liquidity shock and by the traders who acquired capital in period 2. The advantage of introducing a tax in period 3 is that the tax rate τ can now

[14] Note that τ_0 affects the unconditional average of $k(x, y)$, but has no effect on the sensitivity of investment to the signals x and y, i.e., on β_1 and β_2. We henceforth concentrate our attention on τ_1.

be made contingent on all information which is publicly available in that period, including K and θ. We focus on linear tax schemes of the form

$$\tau(\theta, K) = \tau_0 + \tau_1\theta + \tau_2 K, \tag{14}$$

where (τ_0, τ_1, τ_2) are scalars, and we assume that tax revenues are rebated in a lump-sum fashion to the agents. The following result shows that these simple tax schemes can implement the constrained efficient allocation.[15]

Proposition 6 *There exists a unique linear tax scheme that implements the efficient allocation as an equilibrium. The optimal tax satisfies $\tau_0 > 0$, $\tau_1 < 0$, and $\tau_2 > 0$.*

The intuition behind this result is that the government can control the degree of strategic complementarity perceived by the agents by appropriately designing the contingency of the marginal tax rate τ on aggregate investment: the higher the elasticity τ_2 of the marginal tax rate with respect to K, the lower the degree of complementarity in investment choices, α, and the lower the sensitivity of equilibrium investment to common noise relative to idiosyncratic noise. This effect is analogous to that of the stabilization policies discussed above. However, now the government has an extra instrument available, the elasticity τ_1 of the tax to the realized fundamental. Through τ_2 the government can thus induce the optimal relative sensitivity to expectational shocks $\beta_2/(\beta_1 + \beta_2)$ while, at the same time, adjust τ_1 to obtain the optimal absolute sensitivities to each shock.[16]

The contingency of the tax rate on θ and K is used in order to affect the entrepreneurs' investment decisions at date 1. By announcing in advance its response to information that will become publicly available only in the future, the government is able to manipulate the agents' use of information, even if it cannot monitor directly the agents' sources of information.[17]

Although this result does not require any informational advantage on the government's side, it assumes that the government observes perfectly the fundamental θ and the agents' capital holdings at the time taxes are collected. However, the result easily extends to situations where these quantities are observed with measurement error. In particular, suppose that in stage 3 the government

[15] See Angeletos and Pavan (2007b) for more general results closely related to this proposition.

[16] The optimal τ_0 is then chosen to induce the optimal level of unconditional average investment. Similar tax schemes implement the efficient investment strategy in all the extensions considered in Section 6.

[17] These intuitions draw from Angeletos and Pavan (2007b), who consider optimal policy in a general class of economies with dispersed information on commonly-relevant fundamentals. See also Lorenzoni (2007) for monetary policy in a business-cycle model with dispersed information on underlying productivity.

only observes $\tilde{\theta} = \theta + \epsilon$ and $\tilde{s}_i = s_i + \eta_i$ for each i, where s_i is the capital holding of agent i, while ϵ and η_i are measurement errors, possibly correlated with one another, but independent of θ and of the agents' information in period-2. Then let $\tilde{K}$ be the cross-sectional mean of $\tilde{s}_i$ (i.e., the government's measure of aggregate investment) and consider a proportional tax on $\tilde{s}_i$ of the following form: $\tau(\tilde{\theta}, \tilde{K}) = \tau_0 + \tau_1\tilde{\theta} + \tau_2\tilde{K}$. It is then easy to check that there continues to exist a unique set of coefficients (τ_0, τ_1, τ_2) that implement the efficient allocation as an equilibrium and that these coefficients continue to satisfy $\tau_1 < 0 < \tau_2$.

5.3 Optimal release of information

We now turn to policies that affect the information available to the agents. This seems relevant given the role of the government in collecting (and releasing) macroeconomic data.

To capture this role, suppose that in stage 2 traders can only observe average investment with noise, that is, they observe

$$\tilde{K} = K + \eta,$$

where η is aggregate measurement error, which is a random variable, independent of all other shocks, with mean zero and variance $1/\pi_\eta$. The equilibrium derivations for this case are straightforward extensions of the baseline case.

Suppose now that the government can affect the precision π_η of the macroeconomic data available to financial traders. By changing π_η the government determines the weight that traders assign to $\tilde{K}$ when estimating future profitability. This is another channel by which the government is able to affect the degree of strategic complementarity in investment decisions. Indeed, the choice of π_η is formally equivalent to the choice of the tax elasticity τ_1 in the setup with a tax on financial transactions (Section 5.1). For each value of the tax elasticity τ_1 there is a value of π_η which achieves the same level of social welfare, and vice versa. To see this, note that in the setup with noisy macroeconomic data the observation of $\tilde{K}$ is informationally equivalent to the observation of a signal

$$z \equiv \frac{\tilde{K} - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2}\varepsilon + \frac{1}{\beta_1 + \beta_2}\eta,$$

with precision

$$\pi_z = \left(\left(\frac{\beta_2}{\beta_1 + \beta_2} \right)^2 \pi_y^{-1} + \left(\frac{1}{\beta_1 + \beta_2} \right)^2 \pi_\eta^{-1} \right)^{-1}.$$

The equilibrium price is then given by $p(\theta, y, \eta) = \lambda_0 + \lambda_1(K + \eta)$ and hence the degree of strategic complementarity remains equal to $\alpha \equiv \lambda\gamma_1$, as in the baseline model, with γ_1 given by (6). By changing the value of π_η, the government can then directly manipulate γ_1, and thus the degree of strategic complementarity, with no other effects on welfare.[18]

Therefore, the choice of π_η is subject to the same trade-offs emphasized in the choice of τ_1: decreasing π_η reduces the relative response of investment to expectational shocks, but, at the same time, it also reduces its response to fundamental shocks.

Finally, one could consider policies which affect directly the agents' information regarding the fundamental θ. In particular, the government can collect some information about θ in period 1 and decide whether to disclose this information to the entrepreneurs, or to both the entrepreneurs and the traders. In the first case, this policy corresponds to an increase in the precision of the entrepreneurs' common signal π_y. Although entrepreneurs have a more precise estimate of the fundamental, this information is not shared with the traders. Therefore, an increase in π_y can magnify the feed-back effects between investment and asset prices, with possible negative consequences on social welfare.

In the second case, instead, the policy corresponds to an increase in the precision of the common prior π_θ. This policy is socially beneficial for two reasons: first it improves the quality of the information available to the entrepreneurs and hence it permits them to better align their decisions to the fundamental. Second it reduces the reliance of financial markets on the endogenous signal K in their estimate of the fundamental. The second effect tends to reduce the degree of strategic complementarity in investment decisions, by reducing γ_1, and therefore further boosts welfare.

6 Extensions

Our analysis has identified a mechanism through which the dispersion of information induces complementarity, amplification, and inefficiency, all at once. In the baseline model, we have made a number of assumptions to illustrate this mechanism in the simplest possible way. In particular, we have assumed that the traders' demand for installed capital is perfectly elastic, that entrepreneurs who are not hit by the liquidity shock do not trade in the financial market, and that the traders' valuation of the asset coincides with that of the entrepreneurs. In this section, we relax each of

[18] In particular, it is possible to show that the same equilibrium values of (β_1, β_2) which can be achieved by choosing a tax elasticity $\tau_1 \in [0, \infty)$, can be achieved by choosing the appropriate value of $\pi_\eta \in [0, \infty)$.

these assumptions.

We first extend the model to allow for the traders' demand for capital to be downward sloping. This extension is interesting because it introduces a potential source of strategic substitutability in the entrepreneurs' investment decisions: holding constant the information conveyed by aggregate investment, when other entrepreneurs invest more, the equilibrium price of installed capital goes down and thus lowers the incentive to invest of each individual entrepreneur.

In a second extension, we allow entrepreneurs not hit by the liquidity shock to participate in the financial market. This extension is interesting for two reasons: first, it allows for some of the entrepreneurs' information to be aggregated in the financial market; second, it introduces a non-trivial allocative role for prices. Although some interesting differences arise, the key positive and normative predictions of the paper (Corollaries 1 and 2) remain valid in both extensions: as long as the dispersion of information does not completely vanish in the financial market, the signaling effect of aggregate investment continues to be the source of amplification and inefficiency in the response of the equilibrium to common sources of noise.

Finally, we consider a variant that introduces shocks to the financial market valuation for the installed capital and to the entrepreneurs' information about these shocks. This variant brings the paper closer to the recent literature on "mispricing" and "bubbly" asset prices. It also helps clarify that the details of the information structure we assumed in the baseline model are not essential: any source of common noise in the information that aggregate investment conveys about the underlying fundamentals opens the door to amplification and inefficiency.

6.1 Sources of strategic substitutability

We modify the benchmark model as follows. The net payoff of trader i, who buys q_i units of capital at the price p, is now given by

$$u_i = (\theta - p)\, q_i - \frac{1}{2\phi} q_i^2, \tag{15}$$

where ϕ is a positive scalar. The difference with the benchmark model is the presence of the last term in (15), which represents a transaction cost associated to the purchase of q_i units of capital. A convex transaction cost function ensures a finite price elasticity for the traders' demand, which is now given by $q(p, K) = \phi\,(\mathbb{E}[\theta|K] - p)$. The parameter ϕ captures the price elasticity of this demand function and our benchmark model corresponds to the special case where the demand is infinitely elastic, i.e. $\phi \to \infty$.[19]

As in the benchmark model, in any linear equilibrium the traders' expectation of θ is given by $\mathbb{E}[\theta|K] = \gamma_0 + \gamma_1 K$, for some coefficients γ_0 and γ_1. However, unlike in the benchmark model, the equilibrium price does not coincide with $\mathbb{E}[\theta|K]$. Market clearing now requires that $q(p, K) = \lambda K$, so the equilibrium price is

$$p = \mathbb{E}[\theta|K] - \tfrac{\lambda}{\phi}K = \gamma_0 + \left(\gamma_1 - \tfrac{\lambda}{\phi}\right) K. \tag{16}$$

It follows that aggregate investment has two opposing effects on the price of installed capital, p. On the one hand, it raises the traders' expectation of θ, thereby pushing the price up. On the other hand, it raises the supply of capital, thereby pulling the price down. The strength of these two effects determines whether investment choices are strategic complements or substitutes.

Proposition 7 ***(i)*** *In any linear equilibrium, the investment strategy satisfies*

$$k\,(x,y) = \mathbb{E}\left[\ (1-\alpha)\,\kappa\,(\theta) + \alpha K\,(\theta,y)\ \mid x,y\ \right], \tag{17}$$

with $\alpha \equiv \lambda\gamma_1 - \lambda^2/\phi$ *and* $\kappa\,(\theta) \equiv \frac{(1-\lambda)\theta+\lambda\gamma_0}{1-\lambda\gamma_1+\lambda^2/\phi}$.

(ii) λ *small enough suffices for the equilibrium to be unique, for investment to increase with* θ*, and for* γ_1 *to be positive.*

Substituting the equilibrium price derived in (16) into (2), we obtain the following expression for each entrepreneur best response

$$k\,(x,y) = \mathbb{E}\left[\ (1-\lambda)\,\theta + \lambda\gamma_0 + (\lambda\gamma_1 - \lambda^2/\phi)K\,(\theta,y)\ \Big|\ x,y\ \right], \tag{18}$$

which immediately gives (17). The degree of complementarity α is now the sum of two terms. The first term $\lambda\gamma_1$ captures the, by now familiar, informational effect of investment on asset prices documented in the benchmark model. The second term, $-\lambda^2/\phi$, captures the simple supply-side effect that emerges once the demand for the asset is finitely elastic. If either the information contained in aggregate investment is sufficiently poor (low γ_1) or the price elasticity of demand is sufficiently low (low ϕ), investment choices become strategic substitutes ($\alpha < 0$). However, the

[19] This way of introducing a finitely elastic demand is more tractable than the one originating from risk aversion.

question of interest here is not whether investment choices are strategic complements or substitutes, but how the positive and normative properties of the equilibrium are affected by the dispersion of information. In this respect, the implications that emerge in this extension are essentially the same as in the benchmark model.

First consider the positive properties of the equilibrium. Lemma 3 immediately extends to the modified model: equilibrium investment satisfies $k(x,y) = \beta_0 + \beta_1 x + \beta_2 y$ and its sensitivity to common noise relative to fundamentals is given by

$$\frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x}\frac{1}{1-\alpha}, \tag{19}$$

which is increasing in α. Provided that aggregate investment is good news for θ, i.e. $\gamma_1 > 0$, Proposition 7 guarantees that $\alpha > -\lambda^2/\phi$. In contrast, when there are no informational frictions, the equilibrium price is simply $p = \theta - (\lambda^2/\phi)K$, so that $\alpha = -\lambda^2/\phi$. It follows that the dispersion of information increases the value of α and hence amplifies the impact of common expectational shocks relative to that of fundamental shocks, even if α happens to be negative. We conclude that Corollary 1, which summarizes the key positive predictions of the model, continues to hold.

We next turn to the normative properties. Because of the convexity of the transaction costs, it is necessary for efficiency that all traders take the same position in the financial market: $q_i = \lambda K$ for all $i \in (1/2, 1]$. Ex-ante utility then takes the form

$$\mathbb{E}u = \int \left\{ \tfrac{1}{2} \int \left[(1-\lambda)\,\theta k(x,y) - \tfrac{1}{2} k(x,y)^2 \right] d\Phi(x|\theta) + \tfrac{1}{2} \left[\theta\lambda K(\theta,y) - \tfrac{1}{2\phi} [\lambda K(\theta,y)]^2 \right] \right\} d\Psi(\theta,y) \tag{20}$$

and the efficient investment strategy is the function $k(x,y)$ that maximizes (20).

Proposition 8 *The efficient investment strategy is the unique linear solution to*

$$k\,(x,y) = \mathbb{E}\left[(1-\alpha^*)\,\kappa^*\,(\theta) + \alpha^* K\,(\theta,y) \mid \, x, y \;\right], \tag{21}$$

where $\alpha^* \equiv -\lambda^2/\phi < 0$, $\kappa^*\,(\theta) \equiv \theta/\left(1+\lambda^2/\phi\right)$, *and* $K(\theta,y) = \int k(x,y) d\Phi(x|\theta)$.

To understand this result, note that the social return to investment is given by $(1-\lambda)\,\theta + \lambda\,(\theta - \lambda K/\phi) = \theta - \lambda^2 K/\phi$. The new term, relative to the benchmark model, is $-\lambda^2 K/\phi$ and it reflects the cost associated with transferring λK units of the asset from the entrepreneurs to the traders. If information were complete, efficiency would require that each agent equates his marginal cost of investing to the social return to investment, which would give $k = \theta - \lambda^2 K/\phi$.[20] The analogue under incomplete information is that each agent equates the marginal cost to the expected social return:

$$k(x,y) = \mathbb{E}\left[\ \theta - \left(\lambda^2/\phi\right) K(\theta, y)\ \middle|\ x, y\ \right]. \tag{22}$$

Rearranging this condition gives (21).

The key finding here is that the presence of a downward sloping demand for capital makes both the private and the social return to investment a decreasing function of K, adding the term $-\lambda^2/\phi K$ to both expressions (18) and (22). The reason why this term is equally present in both expressions is that this is just a pecuniary externality which is fully internalized by the equilibrium price. As a result, it is only the informational effect of investment that generates a discrepancy between the equilibrium and the efficient allocation.

As in the benchmark model, this discrepancy manifests itself in the response of equilibrium to expectational and fundamental shocks. Indeed, while equilibrium investment satisfies (19), efficient investment satisfies $k_i = \beta_0^* + \beta_1^* x_i + \beta_2^* y$ with

$$\frac{\beta_2^*}{\beta_1^*} = \frac{\pi_y}{\pi_x} \frac{1}{1-\alpha^*}.$$

It follows that the relative sensitivity of the equilibrium strategy to common noise is inefficiently high if and only if $\alpha > \alpha^*$. Because the latter in turn is true if and only if high investment is good news for θ, we conclude that Corollary 2, which is the key normative prediction of the model, continues to hold.

Clearly, there are also other reasons why strategic substitutability can arise, even with a perfectly elastic demand for capital. For example, the production technology that employs the capital K could display decreasing returns to capital, due to the presence of other factors such as labor or land. If the supply of these factors is not infinitely elastic, higher aggregate investment will decrease the private return to investment by raising wages or the rental rate on land. The analysis presented here can easily be extended to these cases.

[20] Note that under full information the optimal level of investment would be equal to $\kappa^*(\theta)$.

6.2 Information aggregation through prices

The analysis so far has imposed that the entrepreneurs who are not hit by the liquidity shock can not access the financial market. Apart from being unrealistic, this assumption rules out the possibility that the price in the financial market aggregates (at least partly) the information that is dispersed among the entrepreneurs. To address this possibility, in this section we extend the analysis by allowing entrepreneurs not hit by the liquidity shock to participate in the financial market.

To guarantee downward sloping demands, we assume that traders and entrepreneurs alike incur a transaction cost for trading in the financial market, of the same type as in the previous section.[21] Thus, the payoff of an entrepreneur i who is not hit by a liquidity shock, has invested k_i units in the first period and trades q_i units in the second period is

$$u_i = -\frac{1}{2}k_i^2 - pq_i - \frac{1}{2\phi}q_i^2 + \theta\left(k_i + q_i\right),$$

while the payoff of a trader i is (15), as in the previous section.

Because the observation of K in the second period perfectly reveals θ to every entrepreneur,[22] their demand for the asset in the second period reduces to $q_E = \phi\left(\theta - p\right)$. The demand of the traders, on the other hand, is given by $q_T = \phi\left(\mathbb{E}\left[\theta|K,p\right] - p\right)$. Note that traders now form their expectation of θ based on K and on the information revealed by the equilibrium price p.[23] Because the aggregate demand for the asset is $\frac{1}{2}\left(1-\lambda\right)q_E + \frac{1}{2}q_T$ and the aggregate supply is $\frac{1}{2}\lambda K$, market clearing implies

$$p = \tfrac{1}{2-\lambda}\mathbb{E}\left[\theta|K,p\right] + \tfrac{1-\lambda}{2-\lambda}\theta - \tfrac{1}{\phi(2-\lambda)}\lambda K.$$

Now the joint observation of K and p perfectly reveals θ to the traders. The asymmetry of information thus vanishes and the equilibrium price satisfies $p = \theta - \frac{1}{\phi(2-\lambda)}\lambda K$. As in the previous section, this is just the social return to investment, adjusted for the fact that $\lambda K/2$ is now equally distributed among the traders and the entrepreneurs not hit by the liquidity shock. Since the

[21] We assume that the entrepreneurs hit by the liquidity shock do not pay the transaction cost for the units of the asset that they *have* to sell in the second period; this is a completely inconsequential simplification.

[22] This presumes that investors use their private information when they decide how much to invest ($\beta_1 \neq 0$), so that $K\left(\theta, y\right)$ varies with θ for given y.

[23] In the benchmark model, as well as in the extension of the previous section, we did not take into account the information revealed by the equilibrium price, because all agents doing voluntary trades in the financial market had symmetric information.

equilibrium price coincides with the social return to investment it follows that the equilibrium is efficient.

This result is no different from what we established at the beginning of the analysis: if the dispersion of information vanishes at the time of financial trades, the equilibrium investment is driven merely by first-order expectations of θ and is efficient.

The aforementioned result hinges on the fact that the equilibrium price perfectly reveals θ. In the subsequent analysis we therefore introduce an additional source of noise which prevents prices from being perfectly revealing.

Assume that the cost of trading for the entrepreneurs is subject to a shock ω, that is revealed to them at the time they trade but which is not observed by the traders. In particular, the payoff of an entrepreneur not hit by the liquidity shock is now given by

$$u_i = -\frac{1}{2}k_i^2 - pq_i - \omega q_i - \frac{1}{2\phi}q_i^2 + \theta\left(k_i + q_i\right),$$

where ω is assumed to be independent of all other random variables, with $\mathbb{E}[\omega] = 0$ and $Var[\omega] = \sigma_\omega^2 \equiv \pi_\omega^{-1}$.

In what follows, we look at linear rational expectations equilibria; we continue to denote the investment strategy by $k\left(x, y\right)$ and we denote by $p\left(\theta, y, \omega\right)$ the equilibrium price. Because the observation of aggregate investment in the second period continues to reveal θ to the entrepreneurs (but not to the traders), asset demands can be written as $q_E = \phi\left(\theta - \omega - p\right)$ for entrepreneurs and $q_T = \phi\left(\mathbb{E}\left[\theta|K, p\right] - p\right)$ for the traders. Market clearing then implies that the equilibrium price is

$$p = \tfrac{1}{2-\lambda}\mathbb{E}\left[\theta|K, p\right] + \tfrac{1-\lambda}{2-\lambda}\left(\theta - \omega\right) - \tfrac{1}{\phi(2-\lambda)}\lambda K. \tag{23}$$

Once again, the price is a weighted average of the traders' and of the entrepreneurs' valuation of the asset, net of trading costs. However, because the shock ω is not known to the traders, the price no longer perfectly reveals θ, ensuring that the informational effect of K on the traders' expectation of the fundamental reemerges. This effect is captured by the first term in the right-hand-side of (23). At the same time, the supply-side effect of K is also present and is captured by the last term in (23).

While the supply-side effect induces strategic substitutability, the informational effect induces complementarity. In any linear rational expectations equilibrium, $\mathbb{E}\left[\theta|K, p\right]$ is the projection of θ

on (K,p) and, by (23), the price p can be expressed as a linear combination of (K,θ,ω). It follows that, for any linear equilibrium, there exist coefficients $(\gamma_0,\gamma_1,\gamma_2,\gamma_3)$ such that

$$\mathbb{E}\left[\theta|K,p\right]=\gamma_0+\gamma_1K+\gamma_2\theta+\gamma_3\omega. \tag{24}$$

Using (24), (23), and the fact that the private return to investment is the expectation of $(1-\lambda)\,\theta+\lambda p$, we reach the following characterization result.

Proposition 9 ***(i)*** *In any linear equilibrium, the investment strategy satisfies*

$$k\left(x,y\right)=\mathbb{E}[(1-\alpha)\,\kappa(\theta)+\alpha K\left(\theta,y\right)\ \mid x,y],$$

where $\alpha=\frac{\lambda}{2-\lambda}\gamma_1-\frac{\lambda^2}{\phi(2-\lambda)}$, $\kappa(\theta)=\frac{\lambda\gamma_0+[2(1-\lambda)+\lambda\gamma_2]\theta}{2-\lambda-\lambda\gamma_1+\lambda^2/\phi}$ *and*

(ii) *λ small enough suffices for the equilibrium to be unique, for investment to increase with θ, and for γ_1 to be positive.*

As in the previous section, α combines an informational effect (captured by $\frac{\lambda}{2-\lambda}\gamma_1$) with a supply-side effect (captured by $-\frac{\lambda^2}{\phi(2-\lambda)}$). The supply-side effect always contributes to strategic substitutability, while the informational effect contributes to strategic complementarity if and only if high investment is "good news" about θ (i.e. $\gamma_1>0$). Once again, the overall effect is ambiguous, but the role of informational frictions remains the same as before: Corollary 1 continues to hold.

We now turn to the characterization of the efficient allocation for this economy. The efficiency concept we use is the same as in the preceding sections; however, now we need to allow the planner to mimic the information aggregation that the market achieves through prices. We thus proceed as follows.

First, we define an *allocation* as a collection of strategies $k(x,y)$, $q_E\left(x,y,K,p,\omega\right)$ and $q_T\left(K,p\right)$, along with a shadow-price function $p\left(\theta,y,\omega\right)$ with the following interpretation: in the first period, an entrepreneur who has observed the exogenous signals (x,y) invests $k\left(x,y\right)$; in the second period, all agents observe the realizations of aggregate investment $K=K\left(\theta,y\right)$ and the shadow price $p=p\left(\theta,y,\omega\right)$; the quantity of the asset held by an entrepreneur is then given by $q_E\left(x,y,K,p,\omega\right)$, while the quantity held by a trader is given by $q_T\left(K,p\right)$.

Next, we say that the allocation is *feasible* if and only if, for all (θ, y, ω),

$$\lambda K(\theta, y) = (1-\lambda)\int q_E(x, y, K(\theta, y), \omega, p(\theta, y, \omega))d\Phi(x|\theta) + q_T(K(\theta, y), p(\theta, y, \omega)). \tag{25}$$

As with equilibrium, this constraint plays two roles: first, it guarantees that the second-period resource constraint is not violated; second, it defines the technology that is used to generate the endogenous public signal (equivalently, the extent to which information can be aggregated through the shadow price).

Finally, for any given $k(x, y)$, $q_E(x, y, K, p, \omega)$ and $q_T(K, p)$, ex ante utility can be computed as $\mathbb{E}u = W(k, q_E, q_T)$, where

$$\begin{aligned} W(k, q_E, q_T) \equiv\; & \frac{1}{2}\int\int\{-\frac{1}{2}k(x,y)^2 + \lambda p(\theta, y, \omega)k(x, y) + (1-\lambda)\theta k(x, y) + \\ & + (1-\lambda) R\left(\theta - \omega, q^E(x, y, K(\theta, y), \omega, p(\theta, y, \omega))\right)\}d\Phi(x|\theta)\, d\Psi(\theta, y, \omega) \\ & + \frac{1}{2}\int R\left(\theta, q^T(K(\theta, y), p(\theta, y, \omega))\right)\}d\Psi(\theta, y, \omega), \end{aligned}$$

and where $R(v, q) \equiv vq - q^2/(2\phi)$. We then define an efficient allocation as follows.

Definition 4 *An efficient allocation is a collection of strategies $k(x, y)$, $q_E(x, y, K, p, \omega)$ and $q_T(K, p)$, along with a shadow price function $p(\theta, y, \omega)$, that jointly maximize ex-ante utility, $\mathbb{E}u = W(k, q_E, q_T)$, subject to the constraint* (25).

Because utility is transferable, the shadow price does not directly affect payoffs; its sole function is to provide an endogenous public signal upon which the allocation of the asset in the period 2 can be conditioned. The next lemma then characterizes the efficient allocation of the asset.

Lemma 5 *The efficient allocation in the second period satisfies*

$$q_E^* = \frac{\lambda K}{2-\lambda} - \frac{\phi\omega}{2-\lambda} \quad \textit{and} \quad q_T^* = \frac{\lambda K}{2-\lambda} + \frac{\phi(1-\lambda)\omega}{2-\lambda} \tag{26}$$

To understand this result, suppose for a moment that information were complete in the second period. For any given K, efficiency in the second period would require that all entrepreneurs hold the same q_E and that (q_T, q_E) maximize

$$\left\{\theta q_T - \frac{1}{2\phi}q_T^2\right\} + (1-\lambda)\left\{\theta q_E - \omega q_E - \frac{1}{2\phi}q_E^2\right\}$$

subject to $(1-\lambda)\, q_E + q_T = \lambda K$. Clearly, the solution to this problem is (26). In our environment, information is incomplete but the same allocation can be induced through the following shadow-price and demand functions: $p(\theta, y, \omega) = -\frac{\phi(1-\lambda)\omega}{2-\lambda}$, $q_E(x, y, K, p, \omega) = \frac{\lambda K}{2-\lambda} - \frac{\phi\omega}{2-\lambda}$, and $q_T(K, p) = \frac{\lambda K}{2-\lambda} - p$.[24]

We now turn to the characterization of the efficient investment decisions. Using Lemma 5, ex ante utility reduces to

$$\mathbb{E}u = \frac{1}{2}\mathbb{E}\left\{-\frac{1}{2}k^2 + \theta k - \frac{1}{2\phi(2-\lambda)}(\lambda K)^2\right\} + \frac{(1-\lambda)}{2(2-\lambda)}\phi\sigma_\omega^2. \tag{27}$$

Except for two minor differences—the smaller weight to $(\lambda K)^2$ which adjusts the cost associated with absorbing the fixed supply λK in the second period for the fact that now this quantity is split across a larger pool of agents, and the last term in (27), which captures how the volatility of ω affects the allocation of capital across entrepreneurs and traders in the second period—ex-ante utility has the same structure as in the previous section.

Proposition 10 *The efficient investment strategy is the unique linear solution to*

$$k(x, y) = \mathbb{E}\left[(1-\alpha^*)\,\kappa^*(\theta) + \alpha^* K(\theta, y) \mid x, y\right], \tag{28}$$

where $\alpha^* \equiv -\frac{\lambda^2}{\phi(2-\lambda)}$, $\kappa^*(\theta) \equiv \frac{1}{1-\alpha^*}\theta$, *and* $K(\theta, y) = \int k(x, y) d\Phi(x|\theta)$.

Comparing the efficient strategy with the equilibrium we have that, once again, when high investment is good news for θ, i.e. when $\gamma_1 > 0$, then $\alpha > \alpha^*$ and hence the key normative predictions of the paper, as summarized by Corollary 2, continue to hold.[25]

6.3 Financial market shocks

In the specifications considered so far, entrepreneurs and traders share the same fundamental valuation for the installed capital. We now develop a variant of the model where entrepreneurs and traders have different valuations. In this variant, non-fundamental volatility can originate

[24] Note that the proposed shadow price is also the unique market-clearing price given the proposed demand functions. The efficient trades can thus be implemented by inducing these demand functions through a tax system and then letting the agents trade freely in the market.

[25] Note that γ_1, which captures the informational impact of K on the traders' expectation of θ, now combines the information that is directly revealed by K with the information revealed through the equilibrium price.

from correlated errors in the entrepreneurs' expectations regarding the traders' valuations; our mechanism then amplifies the impact of these shocks. This variant thus helps connect our model to the recent work on speculative trading à la Harrison and Kreps (1978).[26]

We consider the following modification of the baseline model. The traders' valuation of the asset differs from the valuation of the entrepreneurs. The traders' utility in period $t = 3$ is given by $(\theta + \omega) k_i$, where ω is a random variable, independent of θ and of any other exogenous random variable in the economy, Normally distributed with mean zero and variance σ_ω^2. This random variable is a private-value component in the traders' valuation. It can be interpreted as changes in the hedging motive of the traders, as changes in their discount factor, or as heterogeneous valuations due à la Harrison and Kreps (1978). For our purposes, what matters is that the presence of ω in the traders' utility is taken as given by the social planner; that is, the planner respects the preference orderings revealed by the agents' trading decisions. We thus choose a neutral label for ω and simply call it a "financial market shock."

We also modify the entrepreneurs' information set, to allow for information regarding ω to affect investment decisions. In particular, the entrepreneurs observe a common signal $w = \omega + \zeta$, where ζ is common noise, independent of any other exogenous random variable in the economy, with variance σ_ζ^2. The signal w is observed by all entrepreneurs but not by the traders; as in the baseline model, this is a shortcut for introducing correlated errors in the entrepreneurs' expectations regarding the financial-market shock. Finally, to focus on common expectational shocks about the financial shock ω rather than about the productivity shock θ, we remove the common signal y : the entrepreneurs observe only private signals about θ, $x_i = \theta + \xi_i$, where ξ_i is idiosyncratic noise as in the baseline model.

In this environment, the asset price in period 2 is given by

$$p = \mathbb{E}[\theta | K, \omega] + \omega.$$

It follows that equilibrium investment choices depend not only the entrepreneurs' expectations of θ, but also on their expectations of ω : there exists coefficients $(\beta_0, \beta_1, \beta_2)$ such that individual investment is given by $k(x, w) = \beta_0 + \beta_1 x + \beta_2 w$ and, by implication, aggregate investment is given by $K(\theta, w) = \beta_0 + \beta_1 \theta + \beta_2 w$. Following similar steps as in the baseline model, we can now show

[26] See Scheinkman and Xiong (2003), Gilchrist, Himmelberg, and Huberman (2005), and Panageas (2005).

the following.

Proposition 11 ***(i)*** *In any equilibrium, there exist a scalar $\alpha > 0$ and a function $\kappa : \mathbb{R}^2 \to \mathbb{R}$ such that*

$$k(x, w) = \mathbb{E}\left[\ (1-\alpha)\,\kappa(\theta, \omega) + \alpha K(\theta, w)\ \mid x, w\right].$$

(ii) *λ small enough suffices for the equilibrium to be unique and for investment to increase with both θ and w.*

(iii) *The efficient investment satisfies*

$$k(x, w) = \mathbb{E}\left[\ \theta + \lambda\omega \mid x, w\right].$$

(iv) *In any equilibrium in which investment increases with both θ and w, investment underreacts to θ and overreacts to w.*

In this economy, our mechanism implies that entrepreneurs pay too much attention to their signals regarding future demand in the financial market. The reason for this result is that when entrepreneurs have high (correlated) expectations regarding the financial market shock, they increase investment, which, in turns, increases the financial traders' beliefs regarding the fundamental. Through this channel, an increase in investment that was purely driven by expectations regarding future financial prices is amplified.

Absent our informational mechanism, the response of aggregate investment to θ, ω, and v would be efficient. Since ω can be interpreted as the difference between the traders' and the entrepreneurs' fundamental valuations of the asset, this case is reminiscent of the efficiency results obtained in richer models of "bubbles" based on heterogeneous priors; in particular, Panageas (2006) derives a similar efficiency result for a model that introduces heterogeneous valuations à la Harrison and Kreps (1978) in a q-theory model of investment. The interesting novelty here is that inefficiency arises once we introduce dispersed information. Traders are uncertain whether high investment is driven by good fundamentals or by the entrepreneurs' expectations of speculative valuations. This uncertainty sets in motion our feedback effect between financial prices and investment, leading to an inefficient outcome.

6.4 Other extensions

Throughout all the preceding extensions, we have maintained the assumption that traders cannot directly invest in the new technology in the first period. Clearly, nothing changes if we relax this assumption.[27] Hence, one could drop altogether the distinction between entrepreneurs and traders and simply talk about differentially informed agents who first make real investment choices and then trade financial claims on the installed capital.

Also, the assumption that a fraction λ of the agents is hit by a liquidity shock and is forced to sell their capital in the financial market was a modeling device that ensured that the private return to first-period investment depends on (anticipated) second-period financial prices while ensuring tractability once coupled with the simple linear-quadratic structure of payoffs. If one were to drop the assumption of risk neutrality, or assume that the second-period transaction costs depend on gross positions, or introduce short-sale constraints in the financial market, then the profits an agent could do in the financial market would depend on how much capital he enters the market with; this in turn would ensure that private returns to first-period investment depend on expectations of future financial prices, even in the absence of liquidity shocks.

Finally, the assumption that profitability is perfectly correlated across agents is a simplification; what is essential is only that there is a common component, about which agents have dispersed information. For example, we could let the productivity of the new technology for entrepreneur i be $\tilde{\theta}_i = \theta + v_i$, where θ is the common component and v_i is an idiosyncratic component; we could then also let the entrepreneur's signals be $\tilde{\theta}_i$ plus noise instead of θ plus noise. Finally, we could introduce common and idiosyncratic shocks to the cost of investment. In this case, unobservable common shocks to the cost of investment would also act as a source of noise in the information that aggregate investment conveys about θ.

7 Conclusion

This paper examined the interaction between real and financial decisions in an economy in which information about underlying profitability is dispersed. By conveying a positive signal about prof-

[27] For example, consider the benchmark model and suppose that each trader j chooses first-period real investment k_j at cost $k_j^2/2$ and then trades an additional q_j units in the second-period financial market. Neither the equilibrium price in the financial market nor the entrepreneurs' choices in the first period are affected; all what happens is that aggregate investment now includes the investment of the traders, which is simply given by $k_T = \mathbb{E}\theta$ and, clearly, does not affect the information structure in the second period.

itability, higher aggregate investment stimulates higher asset prices, which in turn raise the incentives to invest. This creates an endogenous complementarity, making investment decisions sensitive to higher-order expectations. In turn, this can dampen the impact of fundamental shocks and amplify the impact of common expectational shocks. Importantly, all these effects are symptoms of inefficiency.

These effects are likely to be stronger during periods of intense technological change, when the dispersion of information is particularly high. Our analysis therefore predicts that such periods come hand-in-hand with episodes of higher-than-usual non-fundamental volatility and comovement in investment and asset prices. At some level, this seems consistent with the recent experiences surrounding the internet revolution or the explosion of investment opportunities in China. This is crucial. What looks like irrational exuberance could be the amplified, but rational, response to noise in information; and while both scenarios feature similar symptoms of inefficiency, they lead to very different conclusions regarding the optimal policy response.

Finally, note that the relevance of our results is clearly not limited to episodes of intense technological change. Rather, our mechanism represents also a likely source of non-fundamental volatility and inefficiency over the business cycle. Extending the analysis to richer business-cycle frameworks so as to quantify these effects is an important direction for future research.

Appendix

Proof of Lemma 3. The derivations of β_1 and β_2 are in the proof of the Lemma 4, below. Rearranging (31), gives

$$\beta_2 = \frac{1}{1-\lambda\gamma_1}\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_2.$$

Using (30), $\alpha = \lambda\gamma_1$, and the fact that $\delta_2/\delta_1 = \pi_y/\pi_x$, gives the desired expression. ■

Proof of Lemma 4. The proof proceeds in several steps. We start by proving part (i). We continue with some auxiliary results regarding the function F which are used in the last steps. We conclude by establishing parts (ii), (iii) and (iv). Throughout, to simplify notation, we suppress the dependence of F and G on $(\pi_\theta, \pi_x, \pi_y, \lambda)$ and let $\pi \equiv \pi_\theta + \pi_x + \pi_y$, $\delta_0 \equiv \pi_\theta/\pi$, $\delta_1 \equiv \pi_x/\pi$, and $\delta_2 \equiv \pi_y/\pi$.

Part (i). Substituting $K(\theta, y) = \beta_0 + \beta_1\theta + \beta_2 y$ into (7) and using $\mathbb{E}[\theta|x,y] = \delta_0\mu + \delta_1 x + \delta_2 y$ gives

$$\begin{aligned}
k(x,y) &= (1-\lambda)\,\mathbb{E}[\theta|x,y] + \lambda\gamma_0 + \lambda\gamma_1\left(\beta_0 + \beta_1\mathbb{E}[\theta|x,y] + \beta_2 y\right) \\
&= \left(1-\lambda+\lambda\gamma_1\beta_1\right)\mathbb{E}[\theta|x,y] + \lambda\gamma_0 + \lambda\gamma_1\beta_0 + \lambda\gamma_1\beta_2 y \\
&= \left[\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_0\mu + \lambda\gamma_0 + \lambda\gamma_1\beta_0\right] + \\
&\quad + \left[\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_1\right]x + \left[\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_2 + \lambda\gamma_1\beta_2\right]y
\end{aligned}$$

Because in equilibrium the above must coincide with $\beta_0 + \beta_1 x + \beta_2 y$ for all x and y, the following conditions must hold

$$\beta_0 = \left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_0\mu + \lambda\gamma_0 + \lambda\gamma_1\beta_0, \tag{29}$$

$$\beta_1 = \left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_1, \tag{30}$$

$$\beta_2 = \left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_2 + \lambda\gamma_1\beta_2. \tag{31}$$

It is immediate that any equilibrium must satisfy $\beta_1 \neq 0$. Then let $b \equiv \beta_2/\beta_1$. From (4) and (6),

$$\gamma_1\beta_1 = h(b) \equiv \frac{\delta_2(1+b)}{\delta_0 b^2 + \delta_2(1+b)^2} \tag{32}$$

while from (30) and (31),

$$b = \frac{\delta_2}{\delta_1} + \frac{\lambda\gamma_1\beta_1 b}{(1-\lambda+\lambda\gamma_1\beta_1)\,\delta_1}. \tag{33}$$

Substituting (32) into (33) gives $b = F(b)$, where

$$F(b) \equiv \frac{\delta_2}{\delta_1}\left\{1 + \frac{\lambda(1+b)\,b}{(1-\lambda)(\delta_0+\delta_2)\,b^2 + (2-\lambda)\delta_2 b + \delta_2}\right\}. \tag{34}$$

Note that the domain of F is the set of all $b \in \mathbb{R}$ such that $1-\lambda+\lambda\gamma_1\beta_1 \neq 0$. Using (32), the latter is given by

$$\mathbb{B} \equiv \{b \in \mathbb{R} : (1-\lambda)(\delta_0+\delta_2)b^2 + (2-\lambda)\delta_2 b + \delta_2 \neq 0\}.$$

It follows that, in any linear equilibrium, b is necessarily a fixed point of F, while the coefficients $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1)$ are given by the following conditions:

$$\beta_1 = [1-\lambda+\lambda h(b)]\delta_1 \tag{35}$$

$$\beta_2 = b\beta_1 = b[1-\lambda+\lambda h(b)]\delta_1 \tag{36}$$

$$\gamma_1 = \frac{\gamma_1\beta_1}{\beta_1} = \frac{h(b)}{[1-\lambda+\lambda h(b)]\delta_1} \tag{37}$$

$$\beta_0 = (1-\lambda+\lambda h(b))\,\delta_0\mu + \frac{\lambda\delta_0\mu}{\delta_0 + \delta_2\left(\frac{1+b}{b}\right)^2} \tag{38}$$

$$\gamma_0 = \frac{\delta_0}{\delta_0 + \delta_2(1+\frac{1}{b})^2}\mu - \gamma_1\beta_0 \tag{39}$$

Conditions (35)-(39) uniquely define the function G.

Auxiliary results. Let $g(b) \equiv (1-\lambda)(\delta_0+\delta_2)b^2 + (2-\lambda)\delta_2 b + \delta_2$; the domain of F is $\mathbb{B} = \{b \in \mathbb{R} : g(b) \neq 0\}$ and whose complement is $\mathbb{B}_c = \{b \in \mathbb{R} : g(b) = 0\}$. Note that the discriminant of $g(b)$ is $\Delta \equiv (\delta_2\lambda)^2 - 4\delta_0\delta_2(1-\lambda)$. If $\Delta < 0$, then $\mathbb{B}_c = \varnothing$; if $\Delta = 0$, then $\mathbb{B}_c = \left\{-\frac{(2-\lambda)\delta_2}{2(1-\lambda)(\delta_0+\delta_2)}\right\}$; finally, if $\Delta > 0$, then $\mathbb{B}_c = \left\{-\frac{(2-\lambda)\delta_2+\sqrt{\Delta}}{2(1-\lambda)(\delta_0+\delta_2)}, -\frac{(2-\lambda)\delta_2-\sqrt{\Delta}}{2(1-\lambda)(\delta_0+\delta_2)}\right\}$. Because there are values for $(\delta_0, \delta_2, \lambda)$ that make Δ negative, zero, or positive, all three cases are possible in general. However, because Δ is continuous in λ and $\Delta = -4\delta_0\delta_2 < 0$ when $\lambda = 0$, λ small enough suffices for $\mathbb{B}_c = \varnothing$. Moreover, because $g(b) \geq \delta_2 > 0$ for any $b \geq 0$, $\mathbb{R}_+ \subset \mathbb{B}$ always.

The function F is continuously differentiable over its entire domain, with

$$\lim_{b\to-\infty} F(b) = \lim_{b\to+\infty} F(b) = F_\infty \equiv \frac{\delta_2}{\delta_1}\left\{1 + \frac{\lambda}{(1-\lambda)(\delta_0+\delta_2)}\right\},$$

$$F(-1) = F(0) = \frac{\delta_2}{\delta_1},$$

$$F(\delta_2/\delta_1) > \delta_2/\delta_1,$$

and

$$F'(b) = \lambda \frac{\delta_2}{\delta_1} \frac{\phi_1(b)}{g(b)^2}$$

with $\phi_1(b) \equiv [\delta_2 - (1-\lambda)\delta_0]b^2 + 2\delta_2 b + \delta_2$.

First, consider the case $\delta_2 = (1-\lambda)\delta_0$.Then $\phi_1(b) = 0$ admits a unique solution at $b = -1/2$. Because $\Delta < 0$, the function F is defined over the entire real line, it is decreasing for $b < -1/2$ and increasing for $b > -1/2$.

Next, consider the case $\delta_2 \neq (1-\lambda)\delta_0$. Then $\phi_1(b) = 0$ admits exactly two solutions, at $b = b_1$ and at $b = b_2$, where

$$b_1 \equiv \frac{-\delta_2 - \sqrt{(1-\lambda)\delta_0}}{\delta_2 - (1-\lambda)\delta_0} \text{ and } b_2 \equiv \frac{-\delta_2 + \sqrt{(1-\lambda)\delta_0\delta_2}}{\delta_2 - (1-\lambda)\delta_0}.$$

The function F then reaches a local maximum at b_1 and a local minimum at b_2.

Part (ii). By the preceding results we have that F is continuous over $\mathbb{R}_+$, with $F(\delta_2/\delta_1) > \delta_2/\delta_1$ and $\lim_{b\to\infty} F(b) < \infty$. It follows that the equation $F(b) = b$ admits at least one solution at $b > \delta_2/\delta_1$, which proves part (ii).

Part (iii). Fix any $(\delta_1, \delta_2) \in (0,1)^2$. If $\delta_2 = (1-\lambda)\delta_0$, where $\delta_0 = 1 - (\delta_1 + \delta_2)$, then let $\underline{F} \equiv F(-1/2)$ and $\bar{F} \equiv F_\infty$. If, instead, $\delta_2 \neq (1-\lambda)\delta_0$, then let $\underline{F} \equiv \min\{F_\infty, F(b_2)\}$ and $\bar{F} \equiv \max\{F_\infty, F(b_1)\}$. It is easy to check that both $\underline{F}$ and $\bar{F}$ converge to δ_2/δ_1 as $\lambda \to 0$. Since F is continuous over its entire domain, $\mathbb{B}$, and λ small enough suffices for $\mathbb{B} = \mathbb{R}$, we have that λ small enough also suffices for F to be bounded in $[\underline{F}, \bar{F}]$. But then F has to converge uniformly to δ_2/δ_1 as $\lambda \to 0$. It follows that for any $\varepsilon > 0$ there exists $\hat{\lambda} = \hat{\lambda}(\varepsilon) > 0$ such that, whenever $\lambda < \hat{\lambda}$, $\mathbb{B} = \mathbb{R}$ and F has no fixed point outside the interval $[\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon]$.

Now note that, for any given b, ϕ_1 and g are continuous in λ, with

$$\frac{\phi_1(b)}{g(b)^2} \to \frac{(1+2b)\delta_2 + b^2(\delta_2 - \delta_0)}{[(1+2b)\delta_2 + b^2(\delta_0 + \delta_2)]^2} \equiv l(b)$$

as $\lambda \to 0$. Since $l(b)$ is continuous in b and $l(\delta_2/\delta_1)$ is bounded, for $\varepsilon > 0$ small enough $l(b)$ is bounded for all $b \in [\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon]$. It follows that, for any $\eta \in (0,1)$, there exist $\tilde{\varepsilon} = \tilde{\varepsilon}(\eta) > 0$

and $\tilde{\lambda} = \tilde{\lambda}(\eta)$ such that $-1 < -\eta < F'(b) < \eta < 1$ for all $b \in [\delta_2/\delta_1 - \tilde{\varepsilon}, \delta_2/\delta_1 + \tilde{\varepsilon}]$ and all $\lambda \in [0, \tilde{\lambda}]$.

Combining these results with the continuity of F, we have that there exist $\bar{\varepsilon} > 0$ and $\bar{\lambda} > 0$ such that, for any $\varepsilon \in [0, \bar{\varepsilon}]$ and any $\lambda \in [0, \bar{\lambda}]$, the following are true: for $b \notin [\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon]$, $F(b) \neq b$; for $b \in [\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon]$, F is continuous and differentiable in b, with $F'(b) < 1$. It follows that, for $\lambda < \bar{\lambda}$, F has at most one fixed point. Together with the fact, from part (i), that F necessarily has at least one fixed point, this proves part (ii).

Part (iv). It is easy to check that $(\delta_1, \delta_2, \lambda) = (.2, .1, .75)$ implies $\mathbb{B} = \mathbb{R}$ (so that F is continuous in over the entire real line) and $F(b_2) < b_2 < 0$. These properties, together with the properties that $F(0) > 0$ and $\lim_{b \to \infty} F(b) > 0 > -\infty$, ensure that, in addition to a fixed point in $(\delta_2/\delta_1, +\infty)$, F admits at least one fixed point in $(-\infty, b_2)$ and one in $(b_2, 0)$. Indeed, as illustrated in the left panel of Figure 1, in this example F admits exactly three fixed point, which are "strict" in the sense that $F(b) - b$ changes sign around them. But since F is continuous in $(b, \delta_1, \delta_2, \lambda)$ in an open neighborhood of $(\delta_1, \delta_2, \lambda) = (.2, .1, .75)$, there necessarily exists an open set $S \subset (0,1)^3$ such that F admits three fixed points whenever $(\delta_1, \delta_2, \lambda) \in S$. ■

Proof of Proposition 2. Let $b(\lambda)$ denote the unique solution to $F(b;\lambda) = b$. Parts (i) and (ii) follow from conditions (35), (36) and (37) observing that $b > 0$ suffices for $h(b) > 0$. For part (iii), note that

$$\frac{\partial F(b;\lambda)}{\partial \lambda} = \frac{\delta_2}{\delta_1} \frac{b(1+b)\left(\delta_2(1+2b) + (\delta_0 + \delta_1) b^2\right)}{g(b)^2},$$

so that $b \geq 0$ suffices for $F(b;\lambda)$ to increase with λ. The result then follows from this property together with the fact that $b(\lambda) > \delta_2/\delta_1$ and $\frac{\partial F(b;\lambda)}{\partial b} < 1$ at $b = b(\lambda)$. ■

Proof of Proposition 3. Take any $\lambda < \bar{\lambda}$. Let $b(\lambda)$ denote the unique fixed point to $F(b;\lambda) = b$ and denote by $\beta_0(\lambda), \beta_1(\lambda), \beta_2(\lambda), \gamma_0(\lambda)$ and $\gamma_1(\lambda)$ the corresponding equilibrium coefficients, as given by (35)-(39). Note that all these functions are continuous.

Part (i). Using conditions (35)–(39), the sensitivity of investment to the realization of θ is given by

$$\beta_1(\lambda) + \beta_2(\lambda) = W(\lambda)(\delta_1 + \delta_2),$$

where $W(\lambda) \equiv w(b(\lambda), \lambda)$, with $w(b, \lambda) \equiv (1+b)(1 - \lambda + \lambda h(b))\frac{\delta_1}{\delta_1 + \delta_2}$ and $h(b)$ defined as in (32). We can compute $b'(\lambda)$ and $b''(\lambda)$ using the Implicit Function Theorem on $F(b,\lambda) - b$. We can then use this to compute $W'(\lambda)$ and $W''(\lambda)$. After some tedious algebra (which is available upon request), we find that $W'(0) = 0$ and $W''(0) = -\frac{2\delta_0\delta_1\delta_2}{(\delta_1(\delta_1+\delta_2)+\delta_2)^2} < 0$. Together with the fact that $b(0) = \delta_2/\delta_1$ and hence $W(0) = 1$, this ensures that there exists $\hat{\lambda} \in (0, \bar{\lambda}]$ such that, for all $\lambda \in (0, \hat{\lambda})$, $W(\lambda) < W(0) = 1$ and $W'(\lambda) < 0$; that is, $\beta_1 + \beta_2$ is lower than $\delta_1 + \delta_2$, its value in the frictionless benchmark, and is decreasing in λ.

Part (ii). From condition (35), we have that $\beta_1(\lambda) = [1 - \lambda + \lambda h(b(\lambda))]\delta_1$ and hence $\beta_1'(\lambda) = \delta_1[-1 + h(b(\lambda)) + \lambda h'(b(\lambda)b'(\lambda))]$. Since $b(0) = \delta_2/\delta_1$ and $h(\delta_2/\delta_1) = \frac{\delta_1(\delta_1+\delta_2)}{\delta_1(\delta_1+\delta_2)+\delta_2} < 1$, we have that $\beta_1'(0) = \delta_1[-1 + h(\delta_2/\delta_1)] < 0$, which together with the result from part (i) that $\beta_1'(0) + \beta_2'(0) = 0$ gives $\beta_2'(0) > 0$. The result then follows from the local continuity of $\beta_2'(\lambda)$ in λ. ■

Proof of Proposition ??. The first statement is proved by the numerical example presented above. To prove the second statement, suppose prices are fully stabilized at $p = \bar{p}$. Substituting in the entrepreneurs best response (2) gives a vector of coefficients $(\beta_0, \beta_1, \beta_2)$ equal to

$$\begin{aligned}\beta_0 &= (1-\lambda)\,\delta_0\mu + \lambda\bar{p},\\ \beta_1 &= (1-\lambda)\,\delta_1,\\ \beta_2 &= (1-\lambda)\,\delta_2.\end{aligned}$$

After some manipulation the expression (11) for social welfare can be rewritten as

$$\begin{aligned}\mathbb{E}u &= \int\left[-\frac{1}{2}k(x,y)^2 + \theta k(x,y)\right]d\Phi(x|y)\,d\Psi(\theta,y) =\\ &= -\frac{1}{2}\beta_0^2 + \beta_0\mu - \frac{1}{2}\beta_1^2\sigma_x^2 - \frac{1}{2}(\beta_1+\beta_2)^2\sigma_\theta^2 - \frac{1}{2}\beta_2^2\sigma_\epsilon^2 + (\beta_1+\beta_2)\,\sigma_\theta^2.\end{aligned}$$

Notice that the price $\bar{p}$ only affects the first two terms of this expression, through its effect on β_0. Suppose that $\bar{p}$ is chosen so as to maximize $-(1/2)\,\beta_0^2 + \beta_0\mu$, and let us concentrate on the remaining terms, defining

$$W(\beta_1,\beta_2) = -\frac{1}{2}\beta_1^2\sigma_x^2 - \frac{1}{2}(\beta_1+\beta_2)^2\sigma_\theta^2 - \frac{1}{2}\beta_2^2\sigma_\epsilon^2 + (\beta_1+\beta_2)\,\sigma_\theta^2.$$

We will show later that for any $h \in (0,1)$ we can find a τ_1 such that, for any value of τ_0, the price stabilization policy (τ_0, τ_1) implements a competitive equilibrium with $\lambda\gamma_1/(1+\tau_1) = h$. Under such a policy the values of $(\beta_0, \beta_1, \beta_2)$ are equal to

$$\beta_0 = (1 - \lambda + \lambda h \beta_1)\, \delta_0 \mu + \lambda (\gamma_0 - \tau_0) + \lambda h \beta_0,$$

$$\beta_1 = (1 - \lambda + h \beta_1)\, \delta_1, \tag{40}$$

$$\beta_2 = (1 - \lambda + h \beta_1)\, \delta_2 + h \beta_2. \tag{41}$$

Notice that the case of full stabilization corresponds to the limit case $h = 0$. Notice also for any value of h, τ_0 can be chosen so as to maximize $-(1/2)\,\beta_0^2 + \beta_0 \mu$. Then, to prove our statement it is sufficient to prove that

$$\frac{\partial W}{\partial \beta_1} \frac{d\beta_1}{dh} + \frac{\partial W}{\partial \beta_2} \frac{d\beta_2}{dh} > 0,$$

at $h = 0$. To prove this claim notice that

$$\frac{\partial W}{\partial \beta_1} = -\beta_1 \sigma_x^2 - (\beta_1 + \beta_2)\, \sigma_\theta^2 + \sigma_\theta^2,$$

$$\frac{\partial W}{\partial \beta_2} = -(\beta_1 + \beta_2)\, \sigma_\theta^2 - \beta_2 \sigma_\epsilon^2 + \sigma_\theta^2.$$

Substituting $\beta_1 = (1 - \lambda)\, \delta_1$ and $\beta_2 = (1 - \lambda)\, \delta_2$, after some algebra, gives

$$\left. \frac{\partial W}{\partial \beta_1} \right|_{\beta_1 = (1-\lambda)\delta_1} = \left. \frac{\partial W}{\partial \beta_2} \right|_{\beta_2 = (1-\lambda)\delta_2} = \lambda \sigma_\theta^2.$$

Therefore, we need to prove that $d\beta_1/dh + d\beta_2/dh > 0$, at $h = 0$. This can be easily shown by differentiation of (40) and (41). ■

Lemma 6 ***Proof.*** *Finally, it remains to show that for each $h \in (0, 1)$ there is a τ_1 that achieves $\lambda \gamma_1 / (1 + \tau_1) = h$ in equilibrium. The equilibrium values of $(\beta_0, \beta_1, \beta_2)$ have been derived above. The value of γ_1 can then be derived using (4) and (6). The desired value of τ_1 is then simply obtained from $\tau_1 = \lambda \gamma_1 / h - 1$.* ■

Proof of Proposition 6. Rewrite the tax rule as

$$\tau = -\phi_0 - (\phi_1 - 1)\theta - \phi_2 K,$$

where $\phi_0 \equiv -\tau_0$, $\phi_1 \equiv 1-\tau_1$, and $\phi_2 \equiv -\tau_2$. Then, the equilibrium price is given by

$$\begin{aligned} p &= \mathbb{E}[\theta-\tau|K] = \phi_0 + \phi_1\mathbb{E}[\theta|K] + \phi_2 K \\ &= \phi_0 + \phi_1\gamma_0 + (\phi_1\gamma_1+\phi_2) K \end{aligned}$$

where we have used $\mathbb{E}[\theta|K] = \gamma_0 + \gamma_1 K$. It follows that the FOC for an entrepreneur is

$$\begin{aligned} k_i &= \mathbb{E}[(1-\lambda)(\theta-\tau) + \lambda p|x,y] = \\ &= \mathbb{E}[(1-\lambda)(\phi_0+\phi_1\theta+\phi_2 K) + \lambda(\phi_0+\phi_1\gamma_0+(\phi_1\gamma_1+\phi_2)K)|x,y] \\ &= (1-\lambda)\phi_0 + \lambda(\phi_0+\phi_1\gamma_0) + (1-\lambda)\phi_1\mathbb{E}[\theta|x,y] + [(1-\lambda)\phi_2 + \lambda(\phi_1\gamma_1+\phi_2)]\mathbb{E}[K|x,y] \end{aligned}$$

For the equilibrium with the above tax rule to implement the efficient allocation, it is necessary and sufficient that the above coincides with $k_i = \mathbb{E}[\theta|x,y]$, which gives

$$\begin{aligned} (1-\lambda)\phi_2 + \lambda(\phi_1\gamma_1+\phi_2) &= 0 \\ (1-\lambda)\phi_1 &= 1 \\ (1-\lambda)\phi_0 + \lambda(\phi_0+\phi_1\gamma_0) &= 0 \end{aligned}$$

with (γ_0,γ_1) determined as in (6) with $(\beta_0,\beta_1,\beta_2) = (\delta_0\mu,\delta_1,\delta_2)$. Equivalently,

$$\begin{aligned} \tau_0 &= -\phi_0 = \frac{\lambda}{1-\lambda}\gamma_0 > 0 \\ \tau_1 &= 1-\phi_1 = -\frac{\lambda}{1-\lambda} < 0 \\ \tau_2 &= -\phi_2 = \frac{\lambda}{1-\lambda}\gamma_1 > 0 \end{aligned}$$

which completes the argument. ■

Proof of Proposition 7. *Part (i).* The entrepreneurs' best response is (2) as in the benchmark model. As argued in the text, market clearing imposes that $p(\theta,y) = \gamma_0 + (\gamma_1 - \lambda/\phi)K(\theta,y)$. Given the definitions of α and $\kappa(\theta)$, (17) follows immediately.

Part (ii). Substituting (4) into (6) gives

$$\gamma_1 = \frac{\pi_z}{\pi_\theta + \pi_z} \frac{1}{\beta_1 + \beta_2} = \frac{(\beta_1 + \beta_2)\, \pi_y}{\beta_2^2 \pi_\theta + (\beta_1 + \beta_2)^2\, \pi_y} = \frac{(\beta_1 + \beta_2)\, \delta_2}{\beta_2^2 \delta_0 + (\beta_1 + \beta_2)^2\, \delta_2}.$$

In the limit, as $\lambda \to 0$, we have $\beta_0 \to \delta_0$, $\beta_1 \to \delta_1$, $\beta_2 \to \delta_2$, and hence $\gamma_1 \to \frac{(\delta_1+\delta_1)\delta_2}{\delta_2^2\delta_0+(\delta_1+\delta_1)^2\delta_2} > 0$. By continuity, then, there exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda})$, $\gamma_1 > \frac{\lambda}{\phi}$ and therefore $\alpha = \lambda\,(\gamma_1 - \lambda/\phi) > 0$. ■

Proof of Proposition 8. Let $V\,(k, K, \theta) \equiv -\frac{1}{2}k^2 + \theta k - \frac{\lambda^2}{2\phi}K^2$. The result then follows from Proposition 3 in Angeletos and Pavan (2007a) by noting that $\kappa^*\,(\theta) \equiv \arg\max_K V\,(K, K, \theta) = \frac{1}{1+\lambda^2/\phi}\theta$ and

$$\alpha^* \equiv 1 - \frac{V_{kk} + 2V_{kK} + V_{KK}}{V_{kk}} = V_{KK} = -\lambda^2/\phi.$$

■

Proof of Proposition 9. From (2), in any equilibrium in which p is linear in (θ, y, ω), there are coefficients $(\beta_0, \beta_1, \beta_2)$ such that $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$. From (23) and (24), the equilibrium price is then

$$p(\theta, y, \omega) = P(K(\theta, y), \theta, \omega) \equiv \eta_0 + \eta_1 K(\theta, y) + \eta_2 \theta + \eta_3 \omega. \qquad (42)$$

for some $(\eta_0, \eta_1, \eta_2, \eta_3)$.

Now consider the optimality of the traders' strategies. As in the benchmark model, the information that $K(\theta, y)$ reveals about θ is the same as that of a signal

$$z \equiv \frac{K(\theta, y) - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2}\varepsilon$$

whose precision is $\pi_z \equiv \left(\frac{\beta_1+\beta_2}{\beta_2}\right)^2 \pi_y$,while the information that $p(\theta, y, \omega)$ reveals about θ given $K(\theta, y)$ is the same as that of a signal

$$s = \frac{1}{\eta_2}[p(\theta, y, \omega) - \eta_0 - \eta_1 K(\theta, y)] = \theta + \frac{\eta_3}{\eta_2}\omega$$

whose precision is $\pi_s = \left(\frac{\eta_2}{\eta_3}\right)^2 \pi_\omega = \phi^2(1 - \lambda)^2 \pi_\omega$. A trader who observes K and p thus believes that θ is normally distributed with mean

$$\begin{aligned}\mathbb{E}\left[\theta \mid K(\theta,y),p(\theta,y,\omega)\right] &= \frac{\pi_\theta}{\pi_\theta+\pi_z+\pi_s}\mu_\theta + \frac{\pi_z}{\pi_\theta+\pi_z+\pi_s}z + \frac{\pi_s}{\pi_\theta+\pi_z+\pi_s}s \\ &= \gamma_0 + \gamma_1 K(\theta,y) + \gamma_2\theta + \gamma_3\omega\end{aligned}$$

where

$$\gamma_0 = \frac{\pi_\theta}{\pi_\theta+\pi_z+\pi_s}\mu_\theta - \frac{\pi_z}{\pi_\theta+\pi_z+\pi_s}\frac{\beta_0}{\beta_1+\beta_2} \tag{43}$$

$$\gamma_1 = \frac{\pi_z}{\pi_\theta+\pi_z+\pi_s}\frac{1}{\beta_1+\beta_2} \tag{44}$$

$$\gamma_2 = \frac{\pi_s}{\pi_\theta+\pi_z+\pi_s} \tag{45}$$

$$\gamma_3 = \frac{\pi_s}{\pi_\theta+\pi_z+\pi_s}\frac{\eta_3}{\eta_2} \tag{46}$$

Combining (23) with (42) we then have that

$$\eta_0 = \frac{\gamma_0}{2-\lambda} \tag{47}$$

$$\eta_1 = \frac{1}{2-\lambda}\left(\gamma_1 - \frac{\lambda}{\phi}\right) \tag{48}$$

$$\eta_2 = \frac{1}{2-\lambda}\left(\gamma_2 + 1 - \lambda\right) \tag{49}$$

$$\eta_3 = \frac{1}{2-\lambda}\left(\gamma_3 - 1 + \lambda\right) \tag{50}$$

Lastly, consider the optimality of the entrepreneurs' investment strategies. From condition (2), the strategy $k\left(x,y\right) = \beta_0 + \beta_1 x + \beta_2$ is individually rational if and only if $(\beta_{0,}\beta_1,\beta_2)$ satisfy $\beta_0+\beta_1 x+\beta_2 y = \left(1-\lambda\right)\mathbb{E}\left[\theta|x,y\right]+\lambda\mathbb{E}\left[p(\theta,y,\omega)|x,y\right]$. That is, $(\beta_{0,}\beta_1,\beta_2)$ must satisfy the following conditions

$$\beta_0 = [1-\lambda+\lambda\eta_1\beta_1+\lambda\eta_2]\delta_0\mu_\theta + \lambda\eta_0 + \lambda\eta_1\beta_0 \tag{51}$$

$$\beta_1 = \left(1-\lambda+\lambda\eta_1\beta_1+\lambda\eta_2\right)\delta_1 \tag{52}$$

$$\beta_2 = \left(1-\lambda+\lambda\eta_1\beta_1+\lambda\eta_2\right)\delta_2 + \lambda\eta_1\beta_2 \tag{53}$$

A linear equilibrium is a thus a solution to (43)-(53).

The existence of a linear equilibrium and its uniqueness for λ small enough can be established following steps similar to those in the benchmark model. Here we prove that λ small enough suffices for $\gamma_1 > 0$, and even for $\alpha > 0$.

Substituting $\pi_z \equiv \left(\frac{\beta_1+\beta_2}{\beta_2}\right)^2 \pi_y$ and $\pi_s = \phi^2(1-\lambda)^2\pi_\omega$ into (44) gives

$$\begin{aligned}
\gamma_1 &= \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s}\frac{1}{\beta_1+\beta_2} \\
&= \frac{(\beta_1+\beta_2)\,\pi_y}{\beta_2^2\pi_\theta + (\beta_1+\beta_2)^2\,\pi_y + \beta_2^2\phi^2(1-\lambda)^2\pi_\omega} \\
&= \frac{(\beta_1+\beta_2)\,\delta_2}{\beta_2^2\delta_0 + (\beta_1+\beta_2)^2\,\delta_2 + \frac{\beta_2^2\phi^2(1-\lambda)^2\pi_\omega}{\pi_\theta+\pi_y+\pi_x}}.
\end{aligned}$$

In the limit, as $\lambda \to 0$, we have $\beta_0 \to \delta_0$, $\beta_1 \to \delta_1$, $\beta_2 \to \delta_2$, and $\gamma_1 \to \frac{(\delta_1+\delta_1)\delta_2}{\delta_2^2\delta_0+(\delta_1+\delta_1)^2\delta_2+\frac{\delta_2^2\phi^2(1-\lambda)^2\pi_\omega}{\pi_\theta+\pi_y+\pi_x}} > 0$. By continuity, then, there exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0,\hat{\lambda})$, $\gamma_1 > \frac{\lambda}{\phi}$ and therefore $\alpha = \frac{\lambda}{2-\lambda}\left(\gamma_1 - \frac{\lambda}{\phi}\right) > 0$. ■

Proof of Proposition 10. Let

$$V(k,K,\theta) \equiv \theta k - \frac{1}{2}k^2 - \frac{\lambda^2}{2\phi(2-\lambda)}K^2.$$

The result then follows for the same argument as in the proof of Proposition 8. ■

Proof of Proposition 11. *Part (i).* In any (linear) equilibrium, there are coefficients $(\beta_0, \beta_1, \beta_2)$ such that the investment strategy can be written as

$$k(x,w) = \beta_0 + \beta_1 x + \beta_2 w,$$

implying that aggregate investment satisfies $K = \beta_0+\beta_1\theta+\beta_2\omega+\beta_2\zeta$. For the traders, who know ω but neither ζ nor θ, observing K is then equivalent to observing a Gaussian signal z with precision π_z, where

$$z \equiv \frac{K-\beta_0-\beta_2\omega}{\beta_1} = \theta + \frac{\beta_2}{\beta_1}\zeta \quad \text{and} \quad \pi_z \equiv \left(\frac{\beta_1}{\beta_2}\right)^2\pi_\zeta,$$

with $\pi_\zeta \equiv \sigma_\zeta^{-2}$. It follows that the equilibrium price satisfies

$$p(\theta,\omega,w) = \gamma_0 + \gamma_1 K(\theta,w) + (1-\gamma_1\beta_2)\,\omega, \tag{54}$$

where

$$\gamma_1 = \frac{\pi_z}{\beta_1 (\pi_\theta + \pi_z)} = \frac{\pi_\zeta}{\beta_1 \left(\left(\frac{\beta_2}{\beta_1} \right)^2 \pi_\theta + \pi_\zeta \right)}. \tag{55}$$

Substituting (54) into the optimality condition for the entrepreneurs gives

$$\begin{aligned} k(x,w) &= \lambda \mathbb{E} \left[p(\theta, \omega, w) | x, w \right] + (1-\lambda) \mathbb{E} \left[\theta | x, w \right] \\ &= \lambda\gamma_0 + (1-\lambda) E\left[\theta|x,w\right] + \lambda (1-\gamma_1\beta_2) \mathbb{E}\left[\omega|x,w\right] + \lambda\gamma_1 \mathbb{E}\left[K(\theta,w)|x,w\right], \end{aligned} \tag{56}$$

which in turn reduces to the condition stated in part (i) of the proposition if we let

$$\alpha \equiv \lambda\gamma_1 \quad \text{and} \quad \kappa(\theta,\omega) \equiv \frac{(1-\lambda)\theta + \lambda\gamma_0 + \lambda(1-\gamma_1\beta_2)\omega}{1-\lambda\gamma_1}.$$

Finally, that $\alpha > 0$ is shown in the next part.

Part (ii). Substituting $K(\theta, w) = \beta_0 + \beta_1\theta + \beta_2 w$ into (56) gives

$$k(x,w) = \lambda(\gamma_0 + \gamma_1\beta_0) + (1-\lambda+\lambda\gamma_1\beta_1) \mathbb{E}\left[\theta|x,w\right] + \lambda(1-\gamma_1\beta_2)\mathbb{E}\left[\omega|x,w\right] + \lambda\gamma_1\beta_2 w$$

Using the facts that $\mathbb{E}[\theta|x,w] = \mathbb{E}[\theta|x] = \delta_0 + \delta_1 x$ and $\mathbb{E}[\omega|x,w] = \mathbb{E}[\omega|w] = \eta_1 w$, where $\delta_0 \equiv \sigma_\theta^{-2}/(\sigma_\theta^{-2}+\sigma_x^{-2})\mu_\theta$, $\delta_1 \equiv \sigma_x^{-2}/(\sigma_\theta^{-2}+\sigma_x^{-2})$, and $\eta \equiv \sigma_\zeta^{-2}/(\sigma_\omega^{-2}+\sigma_\zeta^{-2})$, the above reduces to

$$\begin{aligned} k(x,w) &= \lambda(\gamma_0+\gamma_1\beta_0) + (1-\lambda+\lambda\gamma_1\beta_1)\delta_0 + (1-\lambda+\lambda\gamma_1\beta_1)\delta_1 x \\ &\quad + \lambda(\delta_2 + (1-\delta_2)\gamma_1\beta_2) w \end{aligned}$$

For this to coincide with $k(x,w) = \beta_0 + \beta_1 x + \beta_2 w$, it is necessary and sufficient that the coefficients $(\beta_0, \beta_1, \beta_2)$ solve the following system:

$$\beta_0 = \lambda(\gamma_0 + \gamma_1\beta_0) + (1-\lambda+\lambda\gamma_1\beta_1)\delta_0 \tag{57}$$

$$\beta_1 = (1-\lambda+\lambda\gamma_1\beta_1)\delta_1, \tag{58}$$

$$\beta_2 = \lambda(\delta_2 + (1-\delta_2)\gamma_1\beta_2). \tag{59}$$

By (56),

$$\gamma_1\beta_1 = \frac{\pi_\zeta}{\left(\frac{\beta_2}{\beta_1}\right)^2 \pi_\theta + \pi_\zeta} \in (0,1)\,, \tag{60}$$

which together with (58) guarantees that $\beta_1 \in (0, \delta_1)$. From (58) and (59) we then get

$$\frac{\beta_2}{\beta_1} = \lambda \frac{\delta_2 \left(\pi_\theta \left(\frac{\beta_2}{\beta_1}\right)^2 + \pi_\zeta\right) + (1-\delta_2)\,\pi_\zeta \left(\frac{\beta_2}{\beta_1}\right)}{\delta_1 \left((1-\lambda)\left(\pi_\theta \left(\frac{\beta_2}{\beta_1}\right)^2 + \pi_\zeta\right) + \lambda \pi_\zeta\right)}$$

or equivalently

$$\frac{\beta_2}{\lambda\beta_1} = F\left(\frac{\beta_2}{\lambda\beta_1}; \lambda\right)$$

where

$$F(b;\lambda) \equiv \frac{\delta_2}{\delta_1}\left\{1 + \lambda \frac{\pi_\theta b^2 + (\pi_\zeta + \pi_\omega)\, b}{(1-\lambda)\,\pi_\theta b^2 + \lambda^2 \pi_\zeta}\right\}.$$

It is then easy to show that, for λ small enough, F has a unique fixed point and this fixed point is in a neighborhood of

$$\frac{\beta_2}{\lambda\beta_1} = \frac{\delta_2}{\delta_1}.$$

Along with the fact that β_1 is always positive, this guarantees that β_2 is also positive for λ small enough.

Part (iii). The social planner's problem can be set up as in the baseline model, giving the optimality condition stated in part (iii) of the proposition.

Part (iv). From part (iii) the efficient strategy is given by

$$k(x,w) = \beta_0^* + \beta_1^* x + \beta_2^* w$$

with

$$\beta_0^* = \delta_0, \quad \beta_1^* = \delta_1 \quad \text{and} \quad \beta_2^* = \lambda\delta_2.$$

We have already shown, in the proof of part (ii), that $\beta_1 < \delta_1 = \beta_1^*$, which means that investment underreacts to θ. Next, note that $\beta_1 > 0$ implies $\gamma_1 > 0$. From (59) it then follows that, in any equilibrium in which $\beta_2 > 0$, it is also the case that

$$\beta_2 = \lambda\delta_2 + \lambda(1-\delta_2)\,\gamma_1\beta_2 > \lambda\delta_2 = \beta_2^*,$$

which means that investment overreacts to w. ■

Alternate source of noise. Consider the following variant of the benchmark model. The private signals, $x_i = \theta + \xi_i$, are the same as before, but the common signal y is absent. Instead, the cost of investing for entrepreneur i is now $k_i^2/2 - w_i k_i$, where w_i is a random variable reflecting a shock to the marginal cost of investment. We assume that $w_i = \omega + \zeta_i$, where the common shock ω is normally distributed with mean 0 and variance σ_ω^2, while the idiosyncratic shocks ζ_i are independent of ω, normally distributed with mean 0 and variance σ_ζ^2, and independent across entrepreneurs. Each entrepreneur observes w_i, but does not observe ω.

This environment differs from the benchmark model in two ways. First, there is no piece of information that is commonly observed by the entrepreneurs but not by the traders: entrepreneurs have only private information. Second, the problem is no longer one of pure common values: the w_i shocks introduce a private-value component. At the same time, the role of the aggregate shock ω is similar to the role of the expectational shock ε in the benchmark model: it is an unobserved random variable that is uncorrelated with θ and that moves aggregate investment.

In this environment, the price continues to satisfy $p = \mathbb{E}[\theta|K]$, but now an entrepreneur's investment depends, not only on his private signal x, but also on his private cost w. Following similar steps as in Proposition 11 (proof available upon request), we can show the following.

Proposition 12 ***(i)*** *In any equilibrium, there exist a scalar $\alpha > 0$ and a function $\kappa : \mathbb{R}^2 \to \mathbb{R}$ such that*

$$k(x, w) = \mathbb{E}[\ (1-\alpha)\kappa(\theta, w) + \alpha K(\theta, \omega)\ \mid x, w\ ]. \tag{61}$$

(ii) *λ small enough suffices for the equilibrium to be unique and for investment to increase with both θ and w.*

(iii) *The efficient investment satisfies*

$$k(x, w) = \mathbb{E}[\ \theta + w \mid x, w\ ]. \tag{62}$$

(iv) *In any equilibrium in which investment increases with both θ and w, investment underreacts to θ and overreacts to w.*

As in the benchmark model, whenever high investment is "good news" for θ, the equilibrium price increases with aggregate investment, inducing complementarity in investment choices. In fact, we can now guarantee that this is the case in *every* equilibrium. Once again, this complementarity is unwarranted from a social perspective, and now it acts to amplify the response of investment to the common cost shock ω.

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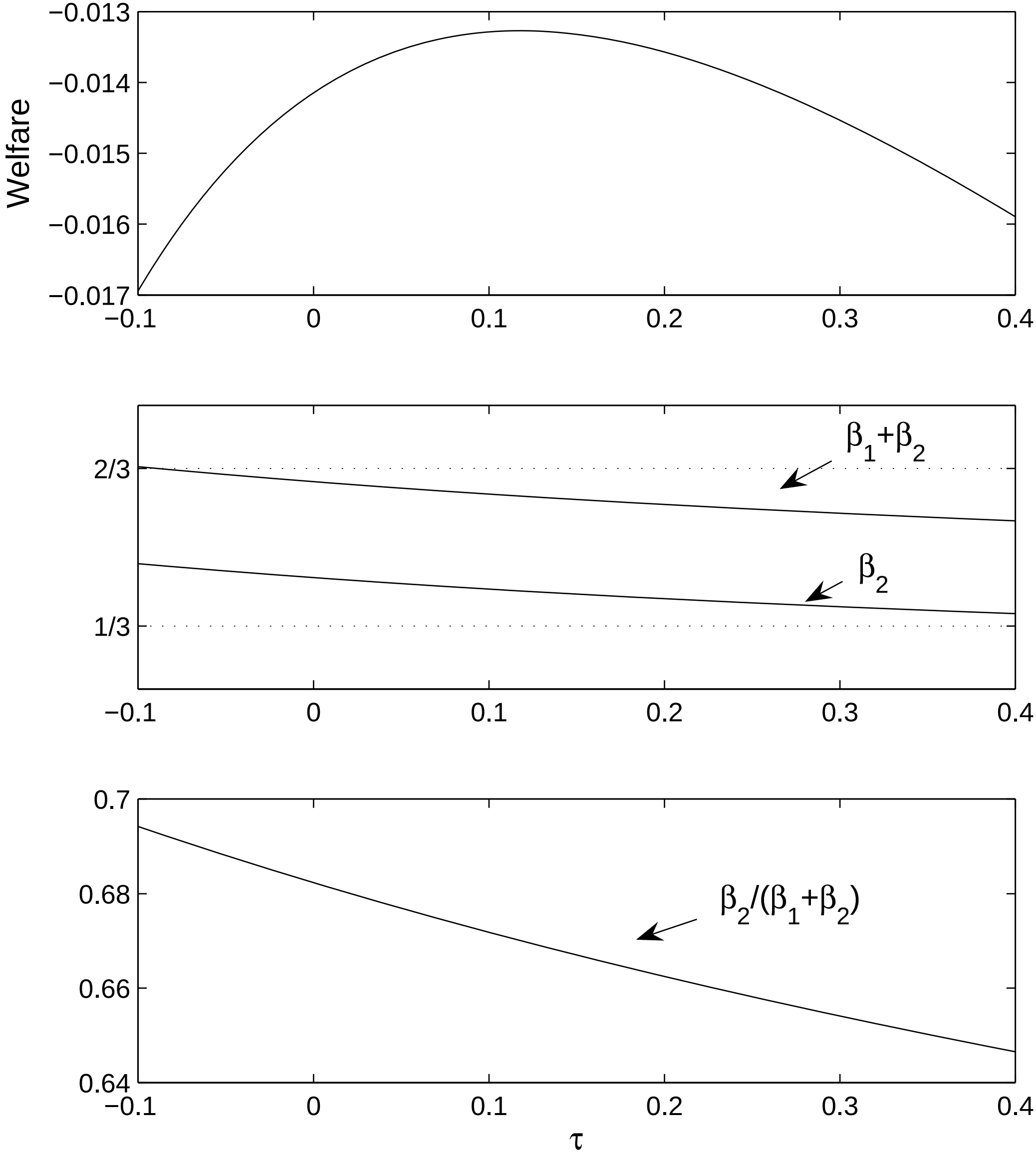

Welfare
−0.013
−0.014
−0.015
−0.016
−0.017
−0.1
0
0.1
0.2
0.3
0.4
$\beta_1+\beta_2$
β_2
2/3
1/3
0.7
0.68
0.66
0.64
$\beta_2/(\beta_1+\beta_2)$
τ