Fund managers and defaultable debt*

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July 2008

Preliminary

Abstract

We propose a general equilibrium model of defaultable debt where investors hire fund managers to invest their capital either in risky bonds or in a riskless asset. The risky bonds are issued by entrepreneurs who need to finance a risky project and can decide to default ex-post. There is only a small fraction of managers who have private information about the productivity of the risky project and, hence, can predict default. Looking at the past performance, investors update their beliefs on the information of their managers and make hiring and firing decisions. This leads to career concerns which affect the investment decision of uninformed managers, generating a "reputational premium". When the default probability is high enough, uninformed managers prefer to invest in the riskless asset to reduce the probability of being fired. On the contrary, if the probability of default is low enough, investing in the risky bonds has a reputational advantage and the premium is negative. As the economic and financial conditions change, the reputational premium can switch sign, magnifying the reaction of prices and capital flows.

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1 Introduction

In the last few years, before the subprime turmoil in August 2007, market observers seemed to be concerned about a growing "overenthusiasm" for risky investments, including high-yield corporate bonds, mortgage-backed assets and emerging market bonds. One observer notices:

> Bonds issued by Ecuador, which is politically very unstable, are among the riskiest bets in the emerging markets. It is hard to predict what will happen there next month, let alone in 10 years time. Yet buyers appear to be ready and willing to line up for a sale by the government of up to Dollars 750m in 10-years bonds, the first international bond offer since the country defaulted in 1999. The issue, [...] is the latest example that the prolonged love affair with emerging market debt is far from over. (December 9, 2005, Financial Times).

A similar observation related to leveraged buy-out deals follows:

> The head of one of the biggest commercial lenders in the US describes the amount of leverage on some buy-out deals as "nutty". Much of the wildest lending is being done by hedge funds awash with cash, he says. "Some funds believe they have to invest the money even if it's not a smart investment. They think the people that gave them the money expect them to invest it. But it's madness." (March 14, 2005, Financial Times)

Figure 1 shows the pattern of the yield spreads of some emerging market bonds, the AAA and the B-graded corporate bonds, and the BBB graded commercial mortgage-backed assets, between October 1994 and February 2008. The figure shows at least two periods in which all spreads shrunk to very low levels, close to the AAA corporate spreads: in 1996-1997 and then again from 2005 to the summer of 2007.[1] Observers describe these periods as periods of overenthusiasm which typically occur right before the emergence of a crisis (e.g. Kamin and von Kleist, 1999, IMF, 1999b, Duffie et al., 2003). The figure also shows three episodes of high turbulence in which the spreads of many high-risk bonds jump up and capital tends to flow out of these markets, a phenomenon dubbed as flight-to-liquidity or flight-to-quality.

We propose a stylized dynamic general equilibrium model able to rationalize both types of episodes. In our paper, investors rationally allocate their capital to fund managers, who may

[1] As a columnist of the Wall Street Journal observes, the 5-year credit default swap spreads for Brazil, Peru, Columbia were at the record-tight levels of 0.70, 0.65 and 0.80 percentage point at the time when, for example, the Boston Scientific Corp, an investment grade company traded at 0.78 percentage point. (April 24, 2007, Tight spreads are emerging, WSJ).

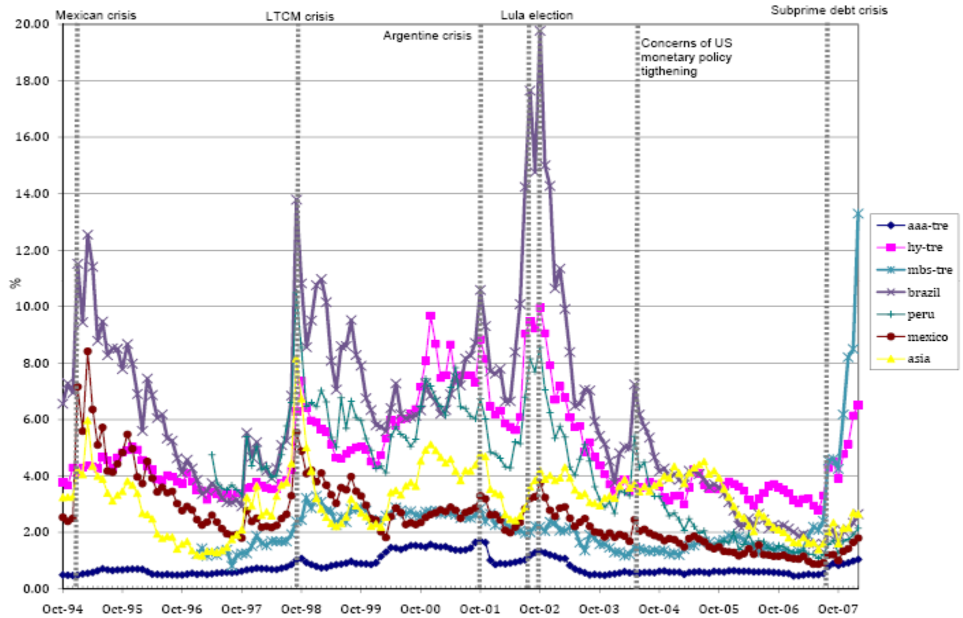

Figure 1: The JPMorgan EMBI+ spread for Asia, Brazil, Mexico, Peru, the yield spread of AAA corporate bonds and B-graded corporate bonds and the yield spread of BBB graded commercial mortgage-backed assets between October 1994 and February 2008. Source: Datastream, St. Louis Fed.

have different degrees of information about risky investments. The core of our model builds on the career concerns of the uninformed managers, which affect their investment decisions. This leads to rational "overinvestment" in risky investments when the expected default probability is low enough, and "underinvestment" when it is high enough, generating "excess volatility" of prices and capital flows.

Our economy is populated by three types of agents: investors, fund managers, and entrepreneurs. Investors delegate their portfolio decision to risk-neutral fund managers. Fund managers can invest either in risk-less assets or in risky bonds issued by entrepreneurs running a risky project. After observing the realization of the project's productivity, the entrepreneurs can choose to default on their debt. As shown in Figure 2, the model is structured on two sets of interactions: investors/managers and managers/borrowers.

On the one hand, the interaction between investors and managers shapes the managers' career concerns. There is a small portion of informed managers who have private information about the productivity of the risky project. Using this information, they can formulate a more precise estimate of the default probability of the risky bond than the uninformed managers. At the end of each period, based on the managers' performance, investors update their beliefs and decide whether to keep their manager or to hire a new one. The firing decision of the investors distorts the investment decision of uninformed managers who would like to be perceived as informed managers.

On the other hand, the interaction between managers and borrowers determines the price of the risky bond, the probability of default and the value of debt in the economy. The investment choice of the managers determines the required bond price for a given probability of default. The entrepreneurs issue bonds to cover their consumption and the fixed cost of the risky project. At the end of the period, they observe the productivity of the project and decide whether to pay back the outstanding debt or to default and suffer a loss. For a given price, their default rule determines the ex-ante probability of default on the bond. Hence, the equilibrium bond price and default probability are jointly determined by the conditions of both the financial market and the fundamentals of the risky project.

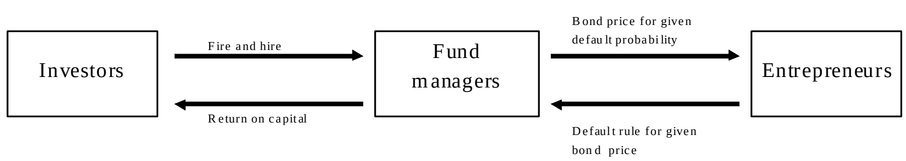

Figure 2: The structure of the model

The focus of our paper is to study the effect of the agency problem between investors and managers, coming from the first interaction, on the equilibrium bond price and default frequency, coming from the second interaction.

Our main result is that managers' career concerns amplify the effect of both financial and fundamental shocks on the bond price, the default probability, and the level of borrowing. This amplification effect arises in general equilibrium as the outcome of two reinforcing mechanisms. First, on the real side, when borrowing is more expensive, entrepreneurs respond by borrowing less, but, given their smoothing desire, the value that they have to repay is higher and, hence, they default with larger probability. Second, on the financial side, career concerns impose a reputational premium on the spread of risky bonds that depends on the default probability.

Uninformed fund managers try to time the market in order to behave as if they were informed and knew in advance if there would be default or not. Default will hurt the reputation of uninformed managers who invest in the risky bond, and no default will hurt the reputation of uninformed managers who invest in the risk-less bond. Thus, when the probability of default is high, the reputational premium is positive to compensate for the foregone reputation. When instead the default probability is low, the risky bond will trade with a negative reputational premium. The real side of the model implies that a larger return on bond leads to a larger probability of default. The financial side implies that a larger probability of default leads to a larger return on bond, also because of a larger reputational premium. These two mechanisms reinforce each other in equilibrium and generate excess volatility in bond prices: bond spreads are particularly low in good times and high in bad times.

A natural application of our model is to think of the entrepreneurs as an emerging economy. In this context, our results are in line with the empirical evidence of excess volatility in emerging market bond spreads and capital flows in Neumeyer and Perri (2005) and Uribe and Yue (2006). However, our result more generally applies to any type of credit market characterized by substantial fluctuations in the fundamentals of the underlying risk and by a crucial role of delegated portfolio management.

On the empirical side, our results are also broadly consistent with the puzzle that a large proportion of the variation in prices of both corporate and emerging market bonds cannot be explained by the variation of fundamentals and that a large part of this unexplained component is common across bonds (see Collin-Dufresne at al., 2000, Gruber et al., 2001, Westphalen, 2001). Furthermore, the recent papers of Singleton and Pan (2007) and Longstaff et al (2007) show that US financial market conditions have a large role in explaining the variation of emerging market spreads compared to emerging market fundamentals. Our model argues that fund managers' career concerns generate an important channel through which financial markets can affect the pricing of debt.

Literature review. To our knowledge, this is the first paper to address the asset pricing consequences of the interaction between the real economy and the agency problem between fund managers and investors. Our work is related to several areas of macroeconomics and finance.

First, our paper is related to reputational herding models,[2] where, as in our paper, decision makers with career concerns make inefficient decisions to convince their clients that they are informed. However, there are two main points of departure from our work. On the one hand, this literature traditionally concentrates on partial equilibrium models while our focus is on the interaction of career concerns and asset prices. On the other hand, these papers present mechanisms in which each decision maker herds on others' decision because going against the average action is a bad signal about his ability. In our model, at the equilibrium prices, fund managers choose the inefficient action regardless of other managers' decision. That is, there are no strategic complementarities. The closest paper to ours is Rajan (1994), who shows that reputational herding might motivate bank executives to overextend credit in good times by amplifying real shocks. In contrast to our model, Rajan (1994) predicts that in bad times banks provide the right amount of credit while we argue that in bad times managers underinvest in the risky bonds.

Second, there is a growing literature which analyzes the effect of delegated portfolio management on asset prices, including Shleifer and Vishny (1997), Allen and Gorton (1993), Cuoco and Kaniel (2007), Vayanos (2003), Dasgupta and Prat (2005, 2007), Dasgupta, Prat and Verardo (2008) He and Krishnamurthy (2007). This literature is silent about the real effects of the agency frictions in financial markets. Unlike our work, most of this literature takes managers' distorted incentives as given. A notable exception is Dasgupta and Prat (2006, 2007) and Prat and Verardo (2008) who introduce reputational herding into a Glosten-Milgrom type of sequential trading model. They show that reputational concerns can lead to excessive trading, slow revelation of information and (if the market maker has market power) biased prices. This series of work is a predecessor to our model in the sense that they are the first to use the term reputational premium and to point out that the potential trade-off between reputation and trading profits might lead managers to choose bets with negative net present value. However, our context is different as we are interested in the way reputational effects amplify the price response of financial and real shocks depending on the state of the economy. We also emphasize that reputational effects have systematic price effects even in a standard, competitive, asset pricing model.

Our paper is also related to a large literature on the propagation and amplification of fundamental shocks due to the interaction between asset values and collateralized lending.

[2] See Scharfstein and Stein (1990), Rajan (1994), Zweibel (1995) and Ottaviani and Sorensen (2006).

Seminal papers in this area are Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) on the macro-side, and Gromb and Vayanos (2002) on the finance side[3]. The main difference with our mechanism is that these papers have typically an asymmetric distortion, given that collateral constraints build into the model an external finance premium, usually generating underinvestment. In our model, instead, we microfound the financial distortion and we generate a premium that can be either positive or negative.

Finally, our application on emerging markets is also related to the vast literature on sovereign debt, reversal of capital flows and financial crisis in emerging economies.[4] However, this literature abstracts away from the effects of intermediation in financial markets.

The rest of the paper is organized as follows. In Section 2, we describe the model. In Section 3, we define and characterize an equilibrium. In Section 4, we analyze the limit equilibrium when the mass of informed managers is infinitesimal. In Section 5, we analyze an economy where productivity is persistent and we propose some numerical exercises. Finally, Section 6 concludes. The appendix includes all the proofs which are not in the text.

2 An example

In this section, we introduce a simple example to show the main mechanism of the model: reputational concerns can distort fund managers' investment decisions and, hence, affect equilibrium prices and capital flows.

Assume that a large group of risk-neutral fund managers have to decide whether to invest a unit of capital in a risky asset or in a riskless asset. The risky asset has price p and defaults with probability q. It pays 1 if there is no default and 0 otherwise. The riskless asset pays the safe return $R < 1/p$. Moreover, assume that a manager obtains a *reputational bonus* W if he times the market, that is, if he invests in the risky bond when there is no default or in the riskless asset otherwise. The riskless asset is in infinite supply, while the supply of the risky bond is fixed and smaller than the total capital invested by the managers.

It is straightforward to see that the bond market clears if and only if managers are indifferent between investing in the risky bond and the riskless asset. Hence, the equilibrium price of the

[3] See also Aghion, Banerjee and Piketty (1999), Rampini (2003), Krishnamurthy (2003), Gai, Kondor and Vause (2005), Guerrieri and Lorenzoni (2007) on the macro side and Danielsson, Shin and Zigrand (2004), Morris and Shin (2004), Bernardo and Welch (2004) and Kondor (2007a) on the finance side.

[4] Atkenson (1991), Cole and Kehoe (2000), Aguiar and Gopinath (2006), Caballero and Krishnamurthy (2003), Calvo and Mendoza (2000), Benczur and Ilut (2005), Arellano (2006), Uribe and Yue (2006), Kovrijnykh and Szentes (2007).

risky bond has to satisfy the following indifference condition

$$(1-q)\left(\frac{1}{p}+W\right)=R+qW. \tag{1}$$

The left-hand side of equation (1) represents the expected payoff of a manager who invests in the risky bond. With probability $1-q$ there is no default and the manager gets a return $1/p$ and the bonus W. If instead there is default, the manager gets zero revenues and no bonus. Similarly, the right-hand side of equation (1) represents the expected payoff of a manager who invests in the riskless asset. He gets always a return R, but the bonus only if there is default.[5]

In order to characterize the price distortion generated by the reputational bonus W, we define the *reputational premium* Π as the difference between the expected return and the risk free rate

$$\Pi \equiv \frac{1-q}{p} - R.$$

The indifference condition (1) immediately implies that $\Pi = 0$ when there is no reputational bonus. In this case, fund managers care only about the expected returns of the bond and the reputational premium is zero. When instead $W > 0$, the reputational premium can be negative or positive. In particular, if $q > 1/2$, the payoff of the risky bond is skewed to the left as the probability of default is larger than the probability of no default. In this case, investing in the riskless asset has a reputational advantage over the risky bond as this ensures the bonus payment with larger probability. If the expected return of the two assets were equal, all managers would prefer the riskless one, because of this reputational advantage. Thus, in equilibrium there must be a positive premium on the risky bond to induce managers to hold it. Similarly, if $q < 1/2$ the payoff of the risky bond is skewed to the right. In this case, the risky bond has a reputational advantage and the reputational premium is negative.

In the rest of the paper we build a dynamic equilibrium model of delegated portfolio management, where investors rationally learn about the ability of the fund managers based on their past performance. Managers' reputational concerns generate a reputational premium similar to the one described in this example, with W becoming an equilibrium object. Moreover, we introduce entrepreneurs who issue the risky bond, hence endogenizing the default probability q

[5] The equlibrium price is consistent with the assumption that $1/p > R$ if

$$\frac{qR}{1-q} + \left(\frac{q}{1-q} - 1\right) W > 0.$$

This is always true if W is sufficiently small.

and the supply of the risky bond. We show that small shocks to the financial market or to the fundamentals of the risky project may lead to large changes in asset prices and capital flows, due to the presence of the reputational premium.

3 The model

3.1 The entrepreneurs

Time is discrete and there are overlapping generations of entrepreneurs who live for two periods. A generation is represented by a continuum of measure 1 of entrepreneurs. In each period a new generation is born. Consider an entrepreneur born at time t. When she is young, she can choose to pay a cost $k > 0$ to invest in a risky project with return a_{t+1}, distributed according to the cumulative distribution function $F(a_{t+1})$ with $a_{t+1} \in [0, \infty)$, or to enjoy an outside option that gives utility $\bar{V}$. If she decides to undertake the risky project, she can borrow by issuing one-period discount bonds. Define p_t the price of the bonds issued at time t. The entrepreneur chooses how much to borrow and how much to consume, taking p_t as given. When she is young, her budget constraint is

$$c_t + k \leq p_t b_{t+1}, \tag{2}$$

where c_t represents consumption at time t and b_{t+1} represents the one-period discount bonds issued at time t. There is an upper bound $\bar{b}$ on how much entrepreneurs can borrow.

When she is old, she collects the project pay-off a_t and has the option to default on her debt b_{t+1}. If she defaults she does not repay the debt, but she suffers a cost of default in terms of output loss of $(1-\theta)\, a_t$, that is, she keeps only θa_t of the return on the project. If she does not default, she has to repay her debt and consume the rest. Her budget constraint when old is

$$c_{t+1} \leq a_{t+1} - \left(1 - \chi_{t+1}\right) b_{t+1} - \chi_{t+1} \left(1-\theta\right) a_{t+1}, \tag{3}$$

where $\chi_{t+1} : \mathbb{R}_+ \mapsto \{0, 1\}$ denotes the default decision that the agent is making at time $t+1$ after observing the realization of a_{t+1}, with $\chi_{t+1} = 1$ if there is default and $\chi_{t+1} = 0$, otherwise. Let us denote by q_t the ex-ante probability of default, that is, $q_t \equiv E_t[\chi_{t+1}]$.

The problem for an active entrepreneur born at t is to maximize her utility

$$u\left(c_t\right) + \beta \mathbb{E}\left[v\left(c_{t+1}\right) | p_t\right],$$

subject to (2) and (3), taking p_t as given. We assume that $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave and have continuous first and second derivative. Moreover, $-cu''(c)/u'(c) \geq 1$. The problem can be rewritten as

$$V(p_t) = \max_{b_{t+1} \leq \bar{b}, \chi_{t+1}} u(p_t b_{t+1} - k) + \beta E[v\{a_{t+1} - (1 - \chi_{t+1}) b_{t+1} - \chi_{t+1}(1-\theta) a_{t+1}\} | p_t] \quad (4)$$

Ex-ante, an entrepreneur will choose to undertake the risky project if and only if $V(p_t) \geq \bar{V}$. We denote the aggregate supply of bonds issued by entrepreneurs at a given price by $B_t(p)$.

3.2 Investors and fund managers

The financial market is populated by a mass Γ of risk-neutral investors. At any time t, each investor has one unit of capital and needs a fund manager to invest it. There are two types of risk-neutral fund managers: informed and uninformed. Informed managers know in advance the realization of the productivity of the risky project and, hence, can perfectly predict default. Uninformed managers, instead, expect the bond to default with probability q_t. There is a mass M^I of informed managers and a larger continuum of uninformed managers. Fund managers do not have any capital, and need to be employed by an investor to make any investment decision. Each investor can employ only one fund manager and a fund manager can work only for a single investor. All the managers have to pay a cost κ to become active unemployed and search for an investor. Moreover, investors do not know the type of the managers.

At time t there is a mass Γ_t^s of employed managers of type s, and all investors have a manager working for them, so that $\Gamma_t^I + \Gamma_t^U = \Gamma$. Then, employed managers choose to invest the unit of capital either in a risk-free asset with gross return R, or in the risky bond described in the previous section, with price p_t and supply $p_t b_{t+1}$. The return on the bond will be 0 if there is default, and $1/p_t$ if there is no default. Next, investors observe the return of their manager's investment and decide whether to fire him or not. Moreover, they receive a signal that reveals the type of an uninformed manager with probability $1-\omega$. Each manager has a probability $1-\delta$ to die, in which case a new manager of the same type is born.[6] Then, inactive managers decide whether to pay a cost κ to search for a job. Let N_t^s be the mass of managers of type s, for $s = I, U$, who decide to look for a job. At the same time, all investors who do not have a manager, either because they have fired him or because he is dead, randomly search for a new one. The probability for a manager to find a job, denoted by μ_t, is equal to the

[6] This ensures that the mass of informed M^I is constant over time.

ratio of the mass of investors searching for a manager to the mass of managers searching for an investor.

We look for a stationary equilibrium where the mass of informed and uninformed employed managers, Γ^I and Γ^U, and the matching probability μ are constant over time. Hence, from now on we can drop the time dependence for these objects. Moreover, for simplicity, we fix the contract between investors and fund managers: fund managers keep a share γ of the revenues and leave the rest to the investors. Both investors and managers fully consume their net revenues in each period.

At the beginning of time t, each investor has a manager j working for him that he believes is informed with probability η_t^j. Manager j has to choose how to invest his unit of capital. If he is informed he chooses a demand schedule $d_t^I(p_t, a_{t+1}) \in [0,1]$ and if he is uninformed he chooses a demand schedule (which will be a correspondence) $d_t^U(p_t) \in [0,1]$. There is an auctioneer, who collects the demand schedules of all the managers, selects the equilibrium price p_t, and assigns risky bonds to a fraction x_t^I and x_t^U of informed and uninformed managers. The bond allocation must be consistent with the demand schedules and with the bond market clearing. Moreover, θ_t^j represents an indicator variable, which is equal to 1 if manager j is allocated a unit of the risky bond, and to 0 otherwise. Notice that x_t^s represents the probability for agent of type s of receiving the risky bond.

At the end of time t, the investor gets his share of the realized returns, observes θ_t^j and whether there has been default or not. Moreover, if his manager is uninformed, he discovers his type with probability $1-\omega$. Then, he updates his belief η_{t+1}^j about the type of his manager using the Bayes' Rule. Using his posterior, he chooses his firing strategy ϕ_t^j, that is, whether to keep his manager for next period ($\phi_t^j = 0$), or to fire him and randomly hire a new one ($\phi_t^j = 1$).[7] Clearly, the investor's firing decision is affected by the probability that a new hire is informed. The key feature of our model is that manager j knows that his investment decision will affect the investors' firing decision by changing his posterior belief. This generates career concerns affecting the investment strategy that are at the core of our model.

To complete the description of the demand for the risky bond, at each time t there is a mass of noise traders who invest y_t dollars in the bond , where y_t is a random variable uniformly distributed on the support $[0, \bar{y}]$. The price of the risky bond has to clear the bond market,

[7] Notice that all the unemployed managers have the same probability of being informed, given that the ones who have a good history are never fired and the ones who have a bad history will never pay the cost κ to search for a new investor.

that is,

$$\Gamma^I x_t^I + \Gamma^U x_t^U + y_t = p_t b_{t+1}. \tag{5}$$

The right-hand side of the market clearing condition represents the value of the supply of bonds, which is determined by the optimal problem of the entrepreneurs described in the previous section, taking p_t as given. The left-hand side, instead, represents the demand of bonds, which comes from three different sources: 1) a proportion x_t^I of informed employed managers, 2) a proportion x_t^U of uninformed employed managers, and 3) the noise traders who demand a random value y_t of the bond. Let us define the total demand of informed managers and noise traders as $z_t \equiv \Gamma^I x_t^I + y_t$. Notice that the uninformed managers can potentially extract some information on the strategy of the informed managers from the equilibrium price.

4 Equilibrium

In this section, we first introduce the definition of a stationary equilibrium when $M^I > 0$. Then, we propose a type of stationary equilibrium we are interested in, an *interior equilibrium*, we verify that it is an equilibrium and we show under which conditions it exists. In the next section we are going to propose a limit version of this equilibrium, where $M^I \to 0$, which is going to be the focus of the rest of the paper.

Definition 1 *For a given $M^I > 0$, a stationary equilibrium is a demand function for informed managers $d^I(p_t, a_{t+1})$, a demand correspondence for uninformed managers $d^U(p_t)$, a firing strategy for investors $\phi(\eta_{t+1}^j)$ with beliefs $\eta_{t+1}^j = \zeta(\eta_t^j, \theta_t^j, z_t, a_{t+1})$, entrepreneurs' strategies $\{\chi(y_t, a_{t+1}), b(z_t)\}$, bond allocations for the informed and uninformed managers $\{x^I(z_t, a_{t+1}), x^U(z_t)\}$, a price $p(z_t)$, a constant mass of employed informed and uninformed managers, Γ^I and Γ^U, and a constant matching probability μ such that*

1. *investors maximize their expected utility, taking as given the price and the strategies of fund managers and entrepreneurs;*

2. *fund managers maximize their expected utility, taking as given the price and the strategies of investors and entrepreneurs;*

3. *entrepreneurs maximize their expected utility, taking as given the price;*

4. *the price and the bond allocations are consistent with the demand schedules of informed and uninformed managers and with market clearing;*

5. *there is free entry in the labor market for fund managers;*

6. *investors' beliefs are consistent with the Bayes' rule.*

4.1 Interior Equilibrium

When $M^I > 0$, employed uninformed managers face the risk of being fired and, hence, their investment decisions are affected by their expected future utility. We focus on equilibria where uninformed managers are typically the marginal traders, that is, they have to be indifferent between investing in the bond and in the risk-free asset whenever prices are not fully revealing. We call this type of equilibrium, an "interior" equilibrium.

Definition 2 *A stationary interior equilibrium is an equilibrium where prices are*

$$p(z_t) = \begin{cases} \underline{p} & \text{if } z_t \in [0, \Gamma^I) \\ p^* & \text{if } z_t \in [\Gamma^I, \bar{y}] \\ 1/R & \text{if } z_t \in (\bar{y}, \bar{y} + \Gamma^I] \end{cases}, \tag{6}$$

where $p^* \in (\underline{p}, 1/R)$*, and:*

(i) *the entrepreneurs' strategy is*

$$b(p_t) = \begin{cases} \bar{b} \text{ with prob. } \nu(z) \text{ and } 0 \text{ otherwise} & \text{if } p_t = \underline{p} \\ b^* & \text{if } p_t = p^* \\ \tilde{b} & \text{if } p_t = 1/R \end{cases}, \tag{7}$$

with $b^* > \tilde{b}$ *in* $[0, \bar{b}]$*, and* $\chi(p_t, a_{t+1}) = \mathbf{1}\{a_{t+1} < \hat{a}(p_t)\}$*, where* $\hat{a}(p_t) = b(p_t)/(1-\theta)$*.*

(ii) *the informed managers' demand function is* $d^I(p_t, a_{t+1}) = 1 - \chi(p_t, a_{t+1})$ *and their bond allocation is* $x^I(z_t, a_{t+1}) = d^I(p(z_t), a_{t+1})$*;*

(iii) *the uninformed managers' demand correspondence is* $d^U(p_t) = \{0, 1\}$ *if* $p_t \in (\underline{p}, 1/R]$ *and* $d^U(p_t) = 0$ *if* $p_t = \underline{p}$*, and their bond holdings are*

$$x^U(z_t) = \begin{cases} 0 & \text{if } z_t \in [0, \Gamma^I) \\ \frac{p^* b - z_t}{\Gamma^U} & \text{if } z_t \in [\Gamma^I, \bar{y}] \\ \frac{b/R - z_t}{\Gamma^U} & \text{if } z_t \in (\bar{y}, \bar{y} + \Gamma^I] \end{cases}; \tag{8}$$

(iv) *the investors' strategy is to fire manager j if the exogenous signal reveals that j is uninformed or $\theta_t^j \neq 1 - \chi(p_t, a_{t+1})$ and $p_t < 1/R$, and keep him otherwise.*

An interior equilibrium is characterized by three possible revelation regimes: $p_t = 1/R$ reveals that there is going to be default, $p_t = \underline{p}$ reveals that there is going to be no default, and, thanks to the uniform distribution of y, $p_t = p^*$ does not reveal any information. Notice that also the entrepreneurs do not have any superior information about a_{t+1} and, hence, learn information from the prices only when they are fully revealing. If there is no revelation, then the entrepreneurs choose to borrow b^* and to default iff their productivity is below a constant cut-off $\hat{a}$. If $p_t = 1/R$, then they choose to borrow $\tilde{b} < b^*$ and not to default, while if $p_t = \underline{p}$, they adopt a mixed strategy, that is, they will be indifferent between the outside option and becoming entrepreneurs, borrow the maximum possible level $\bar{b}$ and always default. The value of $\underline{p}$ is derived in the appendix exactly to ensure this indifference condition, that is, such that $V(\underline{p}) = \bar{V}$. On the financial side, informed managers know in advance if there is going to be default or not, and hence can time the market perfectly, by demanding the risky bond if and only if there is not going to be default. The uninformed managers, instead, do not have this superior information and will claim to be indifferent between obtaining the bond or not. Then, the auctioneer has to satisfy the demand of the informed managers and determine the fraction of uninformed who obtain the bond in order to guarantee that the bond market clears. Finally, investors fire their manager whenever they have an exogenous signal that he is uninformed and if $p_t < 1/R$ and their manager does not time the market correctly.

In Appendix A, we verify that a stationary interior equilibrium as defined above is indeed an equilibrium. We first show that there are three possible revelation regimes and then we show that the equilibrium strategies described in definition 2 are optimal, taking as given the equilibrium price and the equilibrium strategies of the other agents.

4.2 Existence

We make the following assumptions:

$$k \geq \bar{y} \geq 2M^I, \tag{A1}$$

$$M^I + \bar{y} < \underline{p}B(\underline{p}) \text{ and } \Gamma - M^I > \frac{1}{R}B\left(\frac{1}{R}\right), \tag{A2}$$

and

$$\omega < \frac{1}{1+\delta}. \tag{A3}$$

The first inequality of assumption A1 ensures that the supply of the risky bond is always big enough to cover the demand of the noise traders as long as the price is different from 0, while the second inequality ensures that prices are not fully revealing with positive probability. Assumption A2 ensures that there are always some uninformed managers investing in both the risky bond and the risk-less asset. Finally, assumption A3 ensures that the proportion of informed managers among those who are searching for a job is sufficiently small that if an uninformed manager does not make a mistake, he is not fired. This last assumption is not crucial, but it makes the analysis simpler.

Given Definition 2 and all the results derived in Appendix A, to characterize an interior equilibrium it remains to find the equilibrium values of p^* and q^*. These values can be found by solving the following fixed point problem: find a pair (p^*, q^*) such that $p^* = P(q^*)$ and $q^* = Q(p^*)$, where $P(\cdot)$ is the pricing function for a given probability of default and $Q(\cdot)$ is the endogenous probability of default for a given bond price. Both $P(\cdot)$ and $Q(\cdot)$ are derived in Appendix A.

In the next proposition, we find sufficient condition for the existence of a stationary equilibrium. Before stating the result, let us define

$$q_1 \equiv F\left(\frac{B(1/R)}{1-\theta}\right) \text{ and } q_2 \equiv F\left(\frac{\bar{b}}{1-\theta}\right).$$

Proposition 1 *For a given $M^I > 0$, such that assumptions A1-A3 hold and*

$$P(q_1) < 1/R \textit{ and } P(q_2) > \underline{p}, \tag{9}$$

an interior equilibrium exists.

5 Limit Equilibrium

In the description of our main results we will focus on a limit stationary equilibrium where $M^I \to 0$. This limit case is very tractable. We show that as $M^I \to 0$, the sequence of stationary equilibria converge to a limit where the equilibrium objects are constant over time.

In particular, we focus on a *limit interior equilibrium*, where the bond price never reveals any information, and is constant over time. Intuitively, this can be the case because as the fraction of informed manager is infinitesimal, the uninformed managers will have to demand all the bonds supplied and hence won't learn any information from the equilibrium price. Let us define a limit interior equilibrium.

Definition 3 *A limit interior equilibrium with* $M^I \to 0$*, is an equilibrium where* $p^* \in (\bar{y}/b, 1/R)$ *is determined by the indifference condition of the uninformed managers; the informed managers' demand function is* $d^I(\chi_{t+1}) = 1 - \chi_{t+1}$ *and their bond allocation is* $x^I(\chi_{t+1}) = d^I(\chi_{t+1})$*; the uninformed managers' demand correspondence is* $d^U = \{0, 1\}$ *and their bond allocation is* $x(y_t) = (p^* b - y_t)/\Gamma$*; the investors' strategy is to fire manager* j *if he receives a negative exogenous signal or* $\theta_t^j \neq 1 - \chi(a_{t+1})$*, and keep him otherwise.*

This definition implies that, in a limit interior equilibrium, career concerns affect the bond price by generating a reputational premium. The equilibrium behaves in a similar way to the general one, in the case in which the price does not reveal any information. Investors fire the managers who are not able to time the market, informed managers always time the market and are never fired, while only a fraction of the uninformed managers times the market. The market clearing condition for the risky bond determines the fraction $x(y_t)$ of uninformed managers who invest in the risky bond. Assumption A2 guarantees that $x(y_t) \in (0, 1)$ for any y_t, so that there are always some uninformed managers investing in the risky bond and some investing in the risk-free asset. Hence, it must be that the uninformed managers are indifferent between the two investment possibilities. This is the notion of interior solution we refer to when we call the equilibrium an interior equilibrium.

For a given default probability q, the equilibrium price p is determined by the indifference condition for the uninformed managers, that is,

$$(1 - q)(\gamma/p + \delta\omega W) = \gamma R + q\delta\omega W, \tag{10}$$

where W is the expected continuation utility of an uninformed manager who keeps the job. This condition is the analogous to condition (1) in the example in Section 2. The left-hand side of equation (10) represents the expected payoff of a manager who invests in the risky bond. With probability $1 - q$, there is no default, the manager gets a return γ/p. If there is no exogenous signal, he is not fired and gets expected continuation utility W. If instead there is default, the manager gets zero revenues, is fired, and gets 0 continuation utility, given that there is free entry. Similarly, the right-hand side of equation (10) represents the expected payoff of a manager who invests in the risk-free bond. He gets always a return γR, but, only if there is default and there is no exogenous signal, he is not fired and gets expected continuation utility W. Otherwise, the investor learns that he was not informed and fires him. The continuation utility W is given by

$$W = \frac{\gamma R}{1 - \delta \omega q}. \tag{11}$$

This expression is obtained by noticing that managers are always indifferent between investing in the risk-free asset and in the risky bond, and hence their expected utility can be calculated as the value of always investing in the risk-free asset.

As in Section 2, let the *reputational premium* Π be the difference between the expected repayment and the risk free rate R, that is,

$$\Pi \equiv \frac{1-q}{p} - R. \tag{12}$$

This premium characterizes the price distortion generated by the career concerns of the uninformed managers.

Consider a model with $M^I = 0$. In this case, all managers are uninformed, so investors will be indifferent between keeping the manager they started with and hiring a new one. Then, there exists an equilibrium where managers maximize their period by period profit and the bond price is determined by the standard no-arbitrage condition

$$(1-q)\,\frac{1}{p} = R. \tag{13}$$

We call this equilibrium, the *benchmark equilibrium.*

In the benchmark equilibrium, the standard arbitrage condition (13) immediately implies that Π is equal to zero. When instead there is a positive measure of informed managers, $M^I > 0$, the reputational premium can be negative or positive. Typically, it is positive when q is sufficiently large and negative when q is sufficiently small. Betting on large probability events is especially attractive for an uninformed manager with career concerns, because it increases the chance that he will not make an unsuccessful decision and will not be fired. The equilibrium price reflects this preference for large probability events. Fund managers are willing to get a lower expected return in exchange for a large probability of not being fired.

5.1 Equilibrium Characterization

Along the same lines of the general case, Definition 3 implies that the limit equilibrium can be characterized by a constant bond price and default rule, p^* and q^*. These values can be determined by solving the following fixed point problem: find a pair (p^*, q^*) such that

$p^* = P^L(q^*)$ and $q^* = Q(p^*)$. On the one hand, the pricing rule $P^L(\cdot)$ is obtained by combining conditions (10) and (11) and is given by

$$P^L(q) \equiv \frac{(1-q)(1-\delta\omega q)}{[1-\delta\omega(1-q)]R}.$$

On the other hand, the repayment rule $Q(\cdot)$ is derived in Appendix A from the entrepreneurs' optimal behavior and is given by $Q(p) \equiv F(B(p)/(1-\theta))$ with $B(p)$ implicitly defined by

$$pu'(pB(p)-k) - \beta \int_{\frac{B(p)}{1-\theta}}^{\infty} v'(a_{t+1} - B(p))\, dF(a_{t+1}) = 0.$$

Next lemma establishes some important properties of entrepreneurs' optimal behavior.[8]

Lemma 1 *In a limit equilibrium, as* p *increases, (i) the face value of debt* $B(p)$ *decreases, (ii) the probability of default* $Q(p)$ *decreases, (iii) the value of the bonds* $pB(p)$ *increases, and (iv) the ex-ante value of becoming active entrepreneurs* $V(p)$ *increases.*

As borrowing becomes cheaper, entrepreneurs need to borrow less and, hence, there is higher chance that they can repay their debt. This implies that the probability of default decreases as a function of the bond price, that is, q is downward sloping in the space (p, q) as shown in Figure 3. Moreover, the value of the borrowing pb increases, because entrepreneurs want to smooth consumption between the two periods of their life and, hence, they decrease b less than proportionally with respect to the initial increase of p. Finally, as intuition suggests, the value of being an entrepreneur is increasing with the price, given that the revenues from running the risky project are higher if the cost of borrowing is lower.

As a point of comparison, the benchmark equilibrium can also be characterized by a fixed point $(\hat{a}^B, p^B)$, where $q^B = Q(p^B)$, but the pricing rule is given by the standard no-arbitrage condition (13), that is, $p^B = P^B(q^B) \equiv (1-q^B)/R$.

Proposition 1 guarantees that a limit equilibrium exists. The equilibrium regime is determined jointly by the fundamentals of the risky project and the state of the financial market. Figure 3 represents graphically both the limit equilibrium for an economy with career concerns (E) and the benchmark equilibrium (B). The prices in the two equilibria, respectively, p^* and p^B, correspond to the intersections of the repayment rule $Q(p)$ and the corresponding pricing rule, that is, $P^L(q)$ and $P^B(q)$, graphed in the space (p, q).

[8] Note that the same Lemma applies to the general equilibrium defined in 2 when the economy is in a not revealing regime.

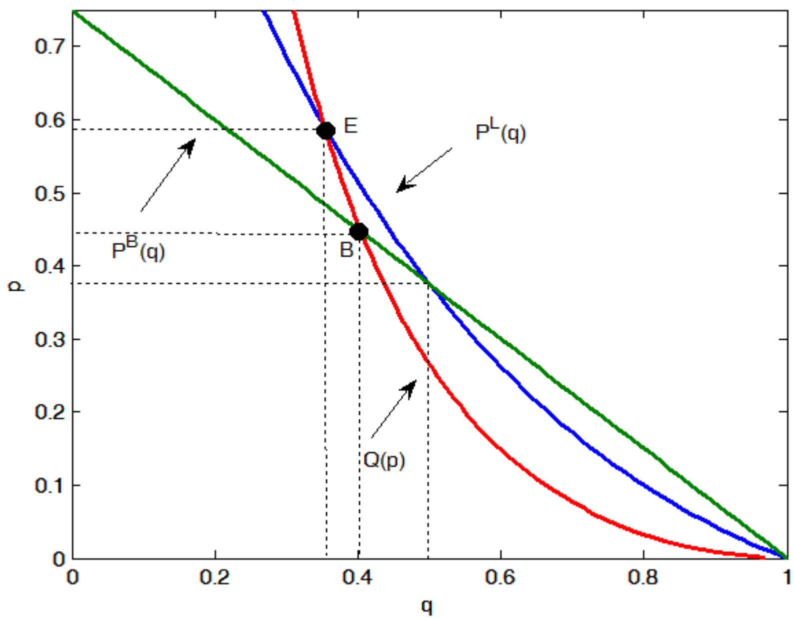

Figure 3: The red line represents the repayment rule and the blue curve and the green curve represent the pricing rule in the economy with career concerns and in the benchmark economy, respectively. Points E and B denote the equilibrium in the economy with career concerns and in the benchmark economy, respectively.

Notice that the reputational premium is zero iff $q^* = q^B = 1/2$ and $p^* = p^B = 1/2R$. Moreover, both $P^L(q)$ and $P^B(q)$ are decreasing in q. This proves the following proposition.

Proposition 2 *In equilibrium, one of the following regimes arises: (i) if* $q^* = 1/2$, *then* $\Pi^* = 0$, *(ii) if* $q^* < 1/2$, *then* $\Pi^* < 0$*; (iii) if* $q^* > 1/2$, *then* $\Pi^* > 0$.

In the baseline numerical exercise we assume that $u(c) = \log c$, $v(c) = c$ and $F(a) = 1 - \underline{a}^{\gamma} a^{-\gamma}$. We choose $\underline{a} = 1.5$, $\gamma = .6$, and $k = .45$. We work out this example in detail in Appendix C. In Figure 3, the parameters of the model are such that $q^* < 1/2$ and then $p^* > p^B$, that is, such that the reputational premium is positive. Changing the parameters, we can easily obtain the analogous figure where $p^* < p^B$ and a negative reputational premium. For example, this is the case if we decrease the lower bound for the productivity process $\underline{a}$ to 1.

5.2 Comparative statics

Next, we analyze some interesting properties of the limit equilibrium. In particular, we are interested in the reaction of the economy to shocks both to financial markets and to the fundamentals of the risky project. The first type of shocks affect the pricing rule and we refer to them as demand-side shocks; the second type affect the repayment rule and we label them supply-side shocks. Our main result is that there is an amplification effect that magnifies the reaction to both types of shocks of our equilibrium in comparison to the benchmark model. The mechanism behind this result is that both types of shocks can move the economy from one regime to the other, generating a natural amplification in the price and in the default probability.

Let us focus on the vector of parameters $\sigma = \{R, \alpha\}$, where R is the return on the risk-free asset and α represents a parameter affecting the distribution of the productivity shock a, such that if $\alpha'' > \alpha'$, then $F(a|\alpha'') < F(a|\alpha')$. A change in R represents a typical demand-side shock, that is, a change in the return of alternative investment opportunities. A change in α, instead, represents a first-order stochastic shift of the productivity distribution of the risky project, that is, a typical supply-side shock.

With a slight abuse of notation, let us denote by $(q^*(\sigma), p^*(\sigma))$ the default probability and price in the limit equilibrium when $M^I \to 0$ and the parameters are σ, and $(q^B(\sigma), p^B(\sigma))$ the default probability and price in the benchmark equilibrium when $M^I = 0$ and the parameters are σ. When there are multiple equilibria, let us focus on the equilibrium with the highest bond price.

Next proposition states our main amplification result.

Proposition 3 *Suppose there exists a pair (σ', σ'') such that $q^B(\sigma') < 1/2$, $q^B(\sigma'') > 1/2$, and $q^*(\sigma'') > 1/2$. Then, there is amplification, that is, $q^*(\sigma'') - q^*(\sigma') > q^B(\sigma'') - q^B(\sigma')$ and $p^*(\sigma'') - p^*(\sigma') > p^B(\sigma'') - p^B(\sigma')$.*

Proposition 3 shows that if there is a change in the parameters such that the equilibrium switches regime from a positive premium to a negative premium, then both prices and default probabilities respond more than in the benchmark model. Suppose, for example, we start from a regime where the reputational premium is negative. As the outside investment opportunities improve, that is, R increases, the bond price decreases making borrowing more expensive and default happening more often. If the shock is big enough, it can generates a shift in the sign of the premium and a switch of regime. Alternatively, the economy can move from a regime to another because of a change in the parameters on the supply-side of the model. For example, a big enough decrease in α can increase the default probability enough to make the reputational premium negative. The effect on both prices and quantities is amplified in comparison to the benchmark model.

Assume that $\lim_{\alpha \to -\infty} F(a_t|\alpha) = 1$. Next proposition shows that when there exists a unique interior equilibrium for a given set of parameters, it is possible to change R or α enough that the regime shifts.

Proposition 4 *Suppose that* $\lim_{\alpha \to -\infty} F(a_t|\alpha) = 1$ *and that there exists a unique interior equilibrium with* $\sigma' = \{R', \alpha'\}$ *such that* $Q(p^*|\alpha') < 1/2$, *where* $\alpha' \in [\underline{a}, \overline{a}]$ *for some* $\underline{a}$ *and* $\overline{a}$. *Then,*

1. *if* $Q(\underline{p}|\alpha') > 1/2$, *there is an* $\hat{R} > R'$ *such that for any* $R'' > \hat{R}$, $Q(p^*|\alpha') > 1/2$;
2. *there is an* $\hat{\alpha}$ *with* $\underline{\alpha} < \hat{\alpha} < \alpha'$ *such that for any* α'' *with* $\underline{\alpha} < \alpha'' < \hat{\alpha}$, $Q(p^*|\alpha'') > 1/2$.

In the example illustrated in figure 3 and worked out in Appendix C, there is a unique equilibrium.[9]

Propositions 3 and 4 show that as the financial environment or the fundamentals of the risky project change, the economy can switch from a regime with low bond spreads (high p) and high level of capital invested in the risky bond market (high pb) to a regime with high bond spreads (low p) and low level of capital invested (low pb). The first type of regimes are frequently described as regimes of *abundant liquidity* or with *traders reaching for yield*. To describe phenomena where the economy switch to the second type of regime, common terms are *flight-to-quality*, *flight-to-liquidity*, *disappeared liquidity*, or *drop in risk appetite*. In our model, phenomena of this type can arise even if fund managers are risk-neutral and their aggregate funds are constant. We argue that abrupt changes in prices can be caused by managers' career concerns. In good times, when the default probability of credit instruments is low, it is very attractive for uninformed fund managers to invest in these instruments because if they prefer less risky investment opportunities, they are likely to produce lower returns, lose reputation, and, hence, funds. If suddenly a negative shock hits either the demand or the supply side of

[9] In particular, it is possible to show that if $u(c) = \log c$, $v(c) = c$ and $F(a) = 1 - \underline{a}^{\gamma} a^{-\gamma}$, there exists a unique equilibrium if k is small enough.

the market, the probability of default increases, and investing in the risk-free asset increases the probability of losing their reputation. Hence, prices increase not only because of the higher probability of default, but also because of an additional premium coming from reputational concerns. This generates the amplification result we have discussed.

Our main result is that the impact of shocks can be amplified by the sign change in the reputational premium, leading to excess volatility of the bond price, default probability, and capital flows. This is consistent with the empirical evidence that shows that emerging market bond prices fluctuate more than what is accounted for by changes in probability of default. On the one hand, Broner, Lorenzoni and Schmukler (2007) argue that the premium over the expected repayment on emerging market bonds is especially high during crises times. On the other hand, Duffie et al. (2003) document that the implied short spread of Russian bonds was very low during the first 10 months of 1997. Moreover, their estimation shows that in one short interval in 1997, bond prices were so high that the implied default adjusted short spread was negative. Although this observation is model specific, it is still interesting to point out that this is inconsistent with most risk-aversion based explanation, but consistent with our model. Note also that the result that demand-side shocks can be important determinants of bond prices is broadly consistent with the empirical evidence that a large proportion of the variation in prices of both corporate bonds and emerging market bonds cannot be explained by the variation of fundamentals, and that a large part of this unexplained component is common across bonds (see Collin-Dufresne at al., 2000, Gruber et al., 2001, Westphalen, 2001).

6 Persistent Productivity Shock

In this section, we introduce persistency in the productivity process of the risky project. In particular, assume that a_{t+1} is distributed according to a first-order Markov process with cumulative density function $G(a_{t+1}|a_t)$. The environment is a natural generalization of the one with *i.i.d.* shock, where a_t represents an additional state variable. We look for Markovian equilibria.

6.1 Equilibrium characterization

The equilibrium we focus on is a natural generalization of the interior limit equilibrium described in definition 3.

Definition 4 *A Markovian interior equilibrium with $M^I \to 0$, is an equilibrium where $p^*(a_t) \in (0, 1/R)$ is determined by the indifference condition of the uninformed managers; the borrowers' strategy is $b^*(a_t) > 0$ and $\chi(y_t, a_{t+1}) = 0$ if either $a_{t+1} \geq \hat{a}^*(a_t)$, and $\chi(y_t, a_{t+1}) = 1$, otherwise; the informed managers' allocation is $x^I(y_t, a_{t+1}) = 1 - \chi(y_t, a_{t+1})$; the uninformed managers' allocation is $x^U(y_t, a_t) = (p^*(a_t) b^*(a_t) - y_t)/\Gamma$; the investors' strategy is to fire manager j if $\theta_t^j \neq 1 - \chi(y_t, a_{t+1})$ and keep him otherwise.*

When the process for a_t is not *i.i.d.*, the expected utility of the uninformed managers at time t depends on the realization of a_t, because they can use the past information to update the distribution of a_{t+1}, that is,

$$W(a_t) = \gamma R + \delta \int_0^{\hat{a}(a_t)} W(a_{t+1}) \, dF(a_{t+1}|a_t). \tag{14}$$

Moreover, their indifference condition becomes

$$(1 - q(a_t)) \frac{\gamma}{p(a_t)} + \delta \int_{\hat{a}(a_t)}^{\infty} W(a_{t+1}) \, dF(a_{t+1}|a_t) = \gamma R + \delta \int_0^{\hat{a}(a_t)} W(a_{t+1}) \, dF(a_{t+1}|a_t).$$

This condition implicitly defines the equilibrium price as a function of the state a_t, $P(a_t, q(a_t)) = p(a_t)$, and the default rule $\hat{a}(a_t)$ where $\hat{a}(a_t) = F^{-1}(q(a_t))$.

Also the borrowers update their expectation of the distribution of a_{t+1}, conditional on a_t. Their default rule is $Q(a_t, p(a_t)) = F(B(a_t, p(a_t)) / (1 - \theta) | a_t)$, where $B(a_t, p(a_t)) = b(a_t)$ is implicitly defined by

$$p(a_t) u'(p(a_t) b(a_t) - k) - \beta \int_{\frac{b(a_t)}{1-\theta}}^{\infty} v'(a_{t+1} - b(a_t)) \, dF(a_{t+1}|a_t) = 0.$$

Hence, a Markovian interior equilibrium is characterized by a fixed point such that $p^*(a_t) = P(a_t, q^*(a_t))$ and $q^*(a_t) = Q(a_t, p^*(a_t))$.

When $M^I = 0$ and the shock a_t is persistent, the benchmark equilibrium is defined as a fixed point such that $q^B(a_t) = Q\left(a_t, p^B(a_t)\right)$ and

$$p^B(a_t) = \frac{1 - q^B(a_t)}{R}.$$

6.2 Numerical example

Here we present some numerical exercises to illustrate the dynamic properties of our equilibrium when productivity shocks are persistent. In particular, we show how career concerns can magnify the reaction of the economy to shocks, hence, increasing the volatility of prices.

First, we show how the default probability, the bond price, the amount of capital borrowed by entrepreneurs, and the reputational premium vary with the realization of the productivity shock. Let us start with the equilibrium behavior in the benchmark economy. As a bad shock hits, the financial market will realize that, even for a given default rule, the probability of default will be higher and will require a lower bond price. As borrowing becomes more expensive, borrowers will then increase their default cut-off, magnifying the reduction in the bond price. A lower bond price also decreases the amount of capital entrepreneurs will borrow, so capital flows out from the market of risky bonds. Hence, for low realizations of productivity, the default cut-off will be higher and the bond price and the dollar value of outstanding bonds lower. However, the change in the bond prices is limited by the fact that the expected pay-off from holding the bonds will remain constant. Now, consider the economy with career concerns. Suppose the default probability is high enough that the reputational premium is positive. In this case, the financial market will require a bond price even lower than the benchmark economy because of the reputational premium. Given that productivity is persistent, a bad realization of the shock will further increase the probability of default, increasing the fear of the uninformed managers of being fired and pushing the bond price further down. This implies that the reputational premium itself is higher after bad shocks. Moreover, if the economy starts from a regime where the reputational premium is negative, a bad shock not only increases the premium, but can even make it switch sign. Thus, the effect of the productivity shock on the bond price, the probability of default and the capital flows is amplified by the career concerns of managers.

We report the numerical results for an example similar to the one illustrated on Figure 3 with $u(c) = \log c$ and $v(c) = c$. However, now $\log(a_t)$ follows an AR(1) process.[10] Figures (4) and (5) show how the reputational premium, the bond price, and the default probability vary in equilibrium with the different realizations of the productivity shocks.

Now, consider an economy that at time zero is hit by a shock. Figure 6 shows how the equilibrium prices react in expected terms to a bad and to a good shock, both with and without career concerns. The figure shows our amplification result: the economy with career concerns reacts more to the shocks than the benchmark economy. Moreover, notice that in the economy

[10] Figures (4), (5) and (6) use the following process for a_t: $\log(a_{t+1}) = (1-\rho)\mu + \rho \log(a_t) + \varepsilon_t$ with $\rho = .7$, $\mu = 2.8$ and $\varepsilon_t \sim N(0,2)$. Moreover, $\beta = .75$, $\delta = .5$, $\gamma = 1$, $k = .4$, and $\theta = .1$.

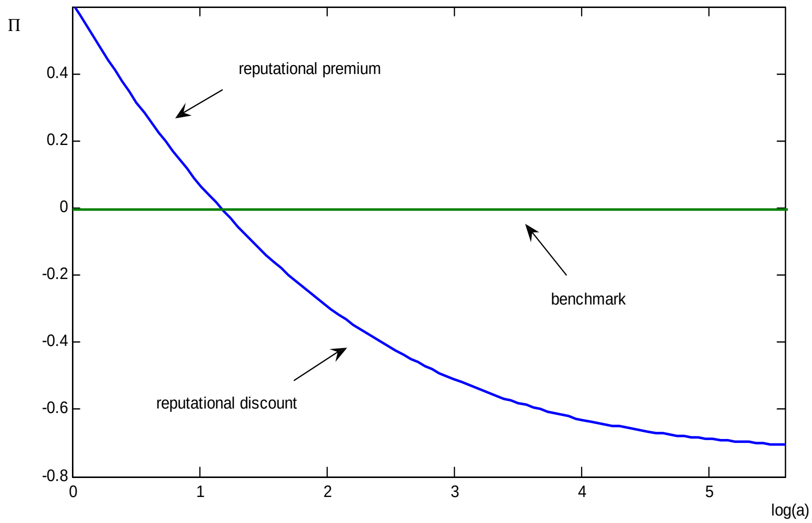

Figure 4: The figure shows the reputational premium as a function of the realization of log(a). The blue line is the premium with career concerns and the green line shows the premium in the benchmark case.

considered, the reputational premium would be negative in expected terms and a bad shock can actually make the economy shift regime.

7 Conclusion

In this paper, we have proposed a general equilibrium model of delegated portfolio management with endogenous default. Investors hire fund managers to invest their capital either in a defaultable bond or in a riskless one and only a small fraction of managers have precise information about the default risk. Looking at the past performance, investors update their beliefs on the information of their fund managers. This leads to career concerns that affect the managers' investment decisions, generating a "reputational premium". When the probability of default is sufficiently high, fund managers prefer to invest in safe bonds even at a lower expected return to reduce the probability of being fired. On the contrary, if the probability of default is low enough, investing in the risky bond has a reputational advantage. The reputational premium can switch sign in response to shocks, both to the financial market and to the fundamentals of the risky project that requires financing. This can generate an overreaction of the market leading to excess volatility of spreads and capital flows.

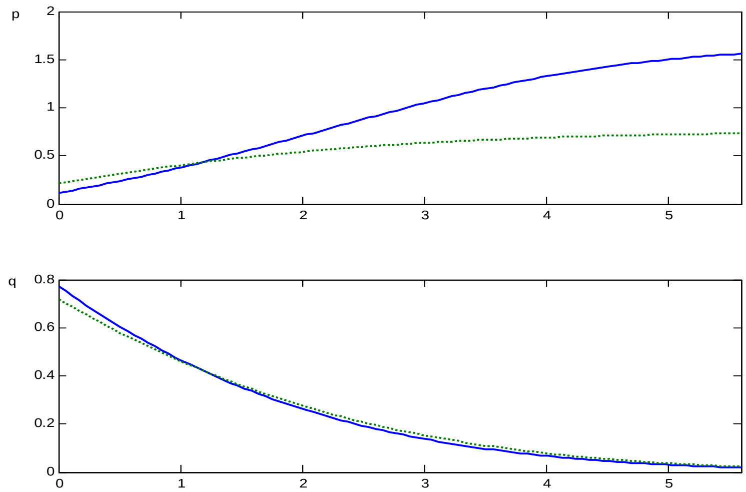

Figure 5: The upper panel shows the equilibrium price as a function of log(a), while the lower panel shows the equilibrium probability of default as a function of log(a). In both panels, the solid line represents the equilibrium with career concerns case and the dashed line represents the benchmark equilibrium.

For future research, it would be interesting to introduce alternative risky assets in the portfolio choice of the managers. In this case, our mechanism would generate contagion. Imagine that there are two risky bonds and a risk-less asset. The reputational cost of investing in the risk-less asset depends on the default probability of both the risky bonds. If none of them defaults, the manager who invests in the risk-less bond will loose his reputation. Thus, if the probability of default of any of the risky bonds decreases, the risk-less asset will be less attractive, and the prices of both bonds will have to increase in order to make uninformed managers indifferent between different investment opportunities.

Finally, an interesting application of our model is to emerging market bonds. A large literature on business cycle characteristics of emerging markets [11] highlights that emerging market bond spreads are very volatile. Also, capital flows are more volatile in small emerging market economies than in developed economies of comparable size. In particular, the magnitude of volatility of interest rates is hard to reconcile with models where bond prices are determined by the standard no-arbitrage condition. Our model provides an appealing framework to think

[11] See Neumeyer and Perri (2005), Uribe and Yue (2006), Arellano (2006), Aguiar and Gopinath (2006), Longstaff et al (2007)

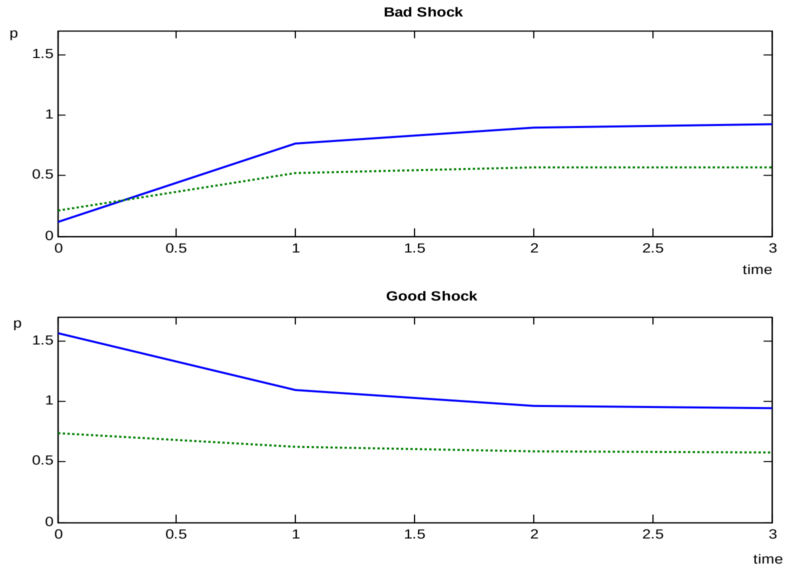

Figure 6: The two panels show the reaction of the equilibrium prices to a bad and a good shock, respectively. The blue line represents the price in the benchmark economy, and the green line the price in the economy with career concerns. At time zero productivity drops to the lowest possible realization in the first case and rises to the highest possible one in the second case.

about this excess volatility. It would be interesting to calibrate our model to quantify how much of the volatility of specific emerging markets bonds can be explained with our mechanism.

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Appendix A

Revelation regimes

First of all, let us show that in an interior equilibrium, each period there are three possible regimes: the bond price is fully revealing and the entrepreneurs default for sure, the price is fully revealing and the entrepreneurs do not default, or the price does not reveal any information and the probability of default is equal to $q^* \equiv F(\hat{a}^*)$. Taking as given the strategies of all the players and the bond allocation of the informed managers, the market clearing condition can be rewritten as

$$\Gamma^I \left(1 - \chi_{t+1}\right) + \Gamma^U x_t^U + y_t = p_t b_{t+1},$$

Given that y_t is uniformly distributed on $[0, \bar{y}]$, $z_t = \Gamma^I \left(1 - \chi_{t+1}\right) + y_t$ must be in $\left[0, \bar{y} + \Gamma^I\right]$. It follows that if $p_t = \underline{p}$, then $z_t \in [0, \Gamma^I)$ and the uninformed managers will know that $\chi_{t+1} = 1$. If, instead, $p_t = 1/R$, then $z_t \in (\bar{y}, \bar{y} + \Gamma^I]$ and they will know that $\chi_{t+1} = 0$. In both these cases, the price will be fully revealing. When instead $p_t = p^*$, then $z_t \in \left[\Gamma^I, \bar{y}\right]$ and the uninformed managers will update their beliefs about the probability of default as follows:

$$\Pr\left(\chi_{t+1} = 1 | p_t = p^*\right) = \frac{\Pr\left(\chi_{t+1} = 1, z_t \in [\Gamma^I, \bar{y}]\right)}{\Pr\left(\chi_{t+1} = 1, z_t \in [\Gamma^I, \bar{y}]\right) + \Pr\left(\chi_{t+1} = 0, z_t \in [\Gamma^I, \bar{y}]\right)}.$$

Notice that z_t can be in $\left[\Gamma^I, \bar{y}\right]$ in two cases: when $\chi_{t+1} = 1$ and $y_t \in \left[\Gamma^I, \bar{y}\right]$ and when $\chi_{t+1} = 0$ and $y_t \in \left[0, \bar{y} - \Gamma^I\right]$. Given that y_t is uniformly distributed, the first case arises with probability $q^* \left(\bar{y} - \Gamma^I\right)/\bar{y}$ and the second with probability $(1 - q^*)\left(\bar{y} - \Gamma^I\right)/\bar{y}$. It follows that $\Pr\left[\chi_{t+1} = 1 | p = p^*\right] = q^*$. This shows that, thanks to the uniform distribution of y_t, the price p^* does not reveal any information. Let us define $\pi^* \equiv \Pr\left\{z_t \notin \left[\Gamma^I, \bar{y}\right]\right\}$, so that in equilibrium the price is fully revealing with probability π^* and fully not revealing with probability $1 - \pi^*$.

Borrowing decision and repayment rule

Next, we show that the equilibrium strategy of the entrepreneurs is optimal, taking as given the equilibrium prices.

Lemma 2 *Taking as given the equilibrium price (6) in Definition 2, the entrepreneurs optimally choose (7), with* $\tilde{b} < b^*$ *in* $[0, \bar{b}]$, *and* $\chi(p_t, a_{t+1}) = \mathbf{1}\{a_{t+1} < \hat{a}(p_t)\}$, *where* $\hat{a}(p_t) = b(p_t)/(1 - \theta)$.

Active entrepreneurs choose their default rule and how much to borrow and to consume in order to solve problem (4), taking p_t as given. Let us first consider the default decision of an old entrepreneur. For a given realization of the shock a_{t+1}, she will default if and only if $a_{t+1} - b_{t+1} < \theta a_{t+1}$. Then $\chi(p_t, a_{t+1}) = 1$ if $a_{t+1} \leq \hat{a}(p_t)$ and $\chi(p_t, a_{t+1}) = 0$, otherwise, with

$$\hat{a}(p_t) = \frac{b(p_t)}{1-\theta}. \tag{15}$$

Notice that the threshold $\hat{a}(p_t)$ is increasing in $b(p_t)$, that is, as intuition suggests, the more the entrepreneurs borrow, the higher is the probability of default.

Given that y_t is uniformly distributed between 0 and 1, the signal of the uninformed $z_t = \Gamma^I(1-\chi_{t+1}) + y_t$ must be in $[0, \bar{y} + \Gamma^I]$. It follows that if $z_t \in (\bar{y}, \bar{y} + \Gamma^I]$, the uninformed managers will know that $\chi_{t+1} = 0$ and if $z_t \in [0, \Gamma^I)$, they will know that $\chi_{t+1} = 1$. There are going to be two possible cases with fully revealing prices: $z_t \in (\bar{y}, \bar{y} + \Gamma^I]$ and $p_t = 1/R$, or $z_t \in [0, \Gamma^I)$ and $p_t = \underline{p}$.

In the first case it must be that $y_t \geq 1/\Gamma^I$ and $\chi_{t+1} = 0$. Define $\hat{a}^e$ the default rule expected by the informed managers. If the price is equal to $1/R$, it must be that $a_{t+1} \geq \hat{a}^e$. In order for this to be an equilibrium it must be that when $p_t = 1/R$, entrepreneurs choose $\hat{a}^1 = \hat{a}^e$. When $p_t = 1/R$, the entrepreneurs problem can be written as

$$\begin{aligned} b^1 &= \arg\max_b u\left(\frac{b}{R} - k\right) + \frac{\beta}{1-F(\hat{a}(b^e))} \int_{\hat{a}^e}^{\min\left\{\frac{b}{1-\theta}, \hat{a}^e\right\}} v(\theta a)\, dF(a) \\ &\quad + \frac{\beta}{1-F(\hat{a}^e)} \int_{\min\left\{\frac{b}{1-\theta}, \hat{a}^e\right\}}^{\infty} v(a-b)\, dF(a), \end{aligned}$$

given that they know that the informed managers are investing in the risky bond and hence, that they believe that $a_{t+1} \geq \hat{a}^e$. Hence to have an equilibrium, b^1 must be such that $b^1 = (1-\theta)\hat{a}^e$ and such that $g(b^1) = 0$, where

$$g(b) \equiv \frac{1}{R} u'\left(\frac{b}{R} - k\right) - \frac{\beta}{1 - F\left(\frac{b}{1-\theta}\right)} \int_{\frac{b}{1-\theta}}^{\infty} v'(a-b)\, dF(a).$$

Moreover, to be sure that this is an equilibrium, we have to check that if prices where not revealing, entrepreneurs would default for any $a_{t+1} < \hat{a}^e$, that is, that, when $p_t \in (0, 1/R)$, they choose b^* such that $b^* \leq b^1$ and $\hat{a}^* \leq \tilde{a}$, where $\hat{a}^* \equiv \hat{a}(b^*)$ and $\tilde{a} \equiv \hat{a}(b^1)$. We know that, for any $p_t \in (0, 1/R)$, b^* solves $\tilde{g}(b^*) = 0$, where

$$\tilde{g}(b) \equiv p_t u'(p_t b - k) - \beta \int_{\frac{b}{1-\theta}}^{\infty} v'(a-b)\, dF(a).$$

Hence, it is enough to show that $\tilde{g}\left(b^1\right) > 0$. This comes straight from the assumption that $-cu''(c)/u'(c) > 1$.

In the second case, when $z_t \in [0, \Gamma^I)$, the price will be $p_t = \underline{p}$. In this case, it must be that $y < \Gamma^I$ and $\chi_{t+1} = 1$. Hence, if the price is equal to $\underline{p}$, it must be that $a_{t+1} < \hat{a}^e$. In order for this to be an equilibrium it must be that when $p_t = \underline{p}$, entrepreneurs choose $b^2 = (1-\theta)\,\hat{a}^e$. When $p_t = \underline{p}$, the entrepreneurs problem can be written as

$$\begin{aligned} b^2 &= \arg\max_b u(\underline{p}b_t - k) + \frac{\beta}{F(\hat{a}^e)} \int_0^{\min\left\{\frac{b}{1-\theta}, \hat{a}^e\right\}} v(\theta a)\, dF(a) \\ &\quad + \frac{\beta}{F(\hat{a}(b^e))} \int_{\min\left\{\frac{b}{1-\theta}, \hat{a}^e\right\}}^{\hat{a}^e} v(a-b)\, dF(a), \end{aligned}$$

given that they know that the informed managers are investing in the risk-free asset and hence, that they believe that $a_{t+1} < \hat{a}^e$. It is easy to see that when $p_t = \underline{p}$, entrepreneurs will choose $b^2 = \bar{b}$ and, hence, will choose always to default.

In order for this to be an equilibrium, it must be that $\underline{p}$ is such that

$$\bar{V} = u(\underline{p}\bar{b} - k) + \frac{\beta}{F\left(\frac{\bar{b}}{1-\theta}\right)} \int_0^{\frac{\bar{b}}{1-\theta}} v(\theta a)\, dF(a),$$

so that when $p_t = \underline{p}$, the entrepreneurs are indifferent between undertaking the risky project, borrow $\bar{b}$ and always default, or take their outside option $\bar{V}$. This, guarantees that they can choose a mixed strategy to ensure that the bond market clears. When $p_t = \underline{p}$ the demand for the risky bond is given by y_t, so that to have market clearing it must be that the entrepreneurs choose to borrow with probability $\nu(y_t) \equiv y_t/(\underline{p}\bar{b})$, and take their outside option and borrow 0 with probability $1 - \nu(y_t)$.

Finally, when $z_t \in [\Gamma^I, \bar{y}]$, the price p^* does not reveal any information, and the entrepreneurs have no information on the realization of a_{t+1}. After substituting the default decision $\chi(p_t, a_{t+1})$, problem (4) for a given $p \in (\underline{p}, 1/R)$ becomes

$$\max_b u(pb - k) + \beta \int_0^{\frac{b}{1-\theta}} v(\theta a_{t+1})\, dF(a_{t+1}) + \beta \int_{\frac{b}{1-\theta}}^{\infty} v(a_{t+1} - b)\, dF(a_{t+1}). \tag{16}$$

Let us define, $B(p)$ the optimal borrowing policy for a given non fully revealing price $p \in (\underline{p}, 1/R)$ such that

$$pu'(pB(p) - k) - \beta \int_{\frac{B(p)}{1-\theta}}^{\infty} v'(a_{t+1} - B(p))\, dF(a_{t+1}) = 0. \tag{17}$$

Using equation (15), the default probability is $F(B(p)/(1-\theta))$. Define the function $Q(p) \equiv F(B(p)/(1-\theta))$. We will refer to $Q(p)$ as the borrowers' optimal *repayment rule*. Then, when prices are not fully revealing, the equilibrium borrowing level is $b^* = B(p^*)$, the equilibrium default rule is $\hat{a}^* = b^*/(1-\theta)$, and the equilibrium default probability is $q^* = F(\hat{a}^*)$.

In order to have a well-behaved problem we need to make the following assumption.

Assumption 1 *For any $p \in (\underline{p}, 1/R)$, assume that the objective function*

$$\Phi(p,b) \equiv u(pb-k) + \beta \int_0^{\frac{b}{1-\theta}} v(\theta a)\, dF(a) + \beta \int_{\frac{b}{1-\theta}}^{\infty} v(a-b)\, dF(a)$$

is quasi-concave in b for $b \in [0, \bar{b}]$ and there exists an optimum $V(p) = \max_{b\in[0,\bar{b}]} \Phi(p,b)$.

Under Assumption 1, the problem has a unique solution. Intuitively, we have to rule out the possibility that the marginal cost of default is not large enough compared to the advantage of additional borrowing. In that case, the entrepreneur would always like to borrow more and default more often, so that problem (16) could not have a finite solution. This trivially implies the following proposition.

Proposition 5 *Under assumption 1, for given $p \in (\underline{p}, 1/R)$, problem (16) has a unique solution with $pb > k$.*

Assumption 1 is satisfied for many different parametric assumptions. As we show in Appendix C, it is satisfied by the example illustrated on Figure 3.

Investment decision and pricing rule

Next, let us turn to the managers, who maximize their expected utility, taking as given the equilibrium regime, the equilibrium strategies of entrepreneurs and investors, and the equilibrium price.

First of all, in an interior equilibrium, thanks to free-entry, the uninformed managers make zero profits ex-ante, that is,

$$\mu W - \kappa = 0, \tag{18}$$

where W represents the expected utility of an employed uninformed manager. On the contrary, the informed managers make positive profits, given that they never make a mistake and are never fired. Hence, all of them will choose to pay the cost κ at each point in time, so that $N_t^I = M^I - \delta\Gamma_t^I$.

Moreover, in a interior equilibrium, Γ^I, Γ^U and μ are constant over time. In particular, Γ^I must be such that the mass of employed informed managers stay constant after δ of them die and μ of the unemployed ones are hired, that is,

$$\Gamma^I = \delta\Gamma^I + \mu\left(M^I - \delta\Gamma^I\right). \tag{19}$$

Also, Γ^U must be such that the mass of employed uninformed managers stay constant after δ of them die, $1 - \omega\xi_t$ of them are fired and μ of the unemployed uninformed are hired, that is,

$$\Gamma^U = \delta\Gamma^U - (1 - \omega\xi_t)\,\delta\Gamma^U + \mu N_t^U, \tag{20}$$

where ξ_t denotes the proportion of uninformed managers who do not make mistakes, that is,

$$\xi_t = \begin{cases} \left(1 - x_t^U\right)\chi_{t+1} + x_t^U\left(1 - \chi_{t+1}\right) & \text{if } z_t \in \left[\Gamma^I, \bar{y}\right] \\ 1 & \text{if } z_t \notin \left[\Gamma^I, \bar{y}\right] \end{cases}. \tag{21}$$

Condition (20) shows that in order to obtain a constant equilibrium value for Γ^U, the mass of uninformed managers who are unemployed N_t^U must change over time together with ξ_t. However, these two variables won't affect the behavior of the equilibrium, which will be stationary.

Next Lemma shows that the investment decisions of the hired managers are consistent with an interior equilibrium.

Lemma 3 *Taking as given the equilibrium price (6) and the equilibrium strategies (i) and (iv) in Definition 2, the informed managers choose* $d^I(p_t, a_{t+1}) = 1 - \chi(p_t, a_{t+1})$ *and their bond allocation is* $x^I(z_t, a_{t+1}) = d^I(p(z_t), a_{t+1})$.

Investors fire managers investing in the bond when there is default and the ones investing in the risk-free asset when there is no default. Hence, informed managers will always choose to invest in the risk-free asset if there is going to be default, and in the bond if there is not going to be default, that is, $d^I(p_t, a_{t+1}) = 1 - \chi(p_t, a_{t+1})$. They do not have an incentive to deviate, given that the bond price is always smaller or equal to $1/R$. To satisfy the demand, the auctioneer has to allocate $x^I(z_t, a_{t+1}) = d^I(p(z_t), a_{t+1})$.

Lemma 4 *Suppose assumption A2 holds. Then, taking as given the equilibrium price (6), the bond allocation (ii), the equilibrium strategies (i) and (iv) in Definition 2, the uninformed managers choose* $d^U(p_t) = [0, 1]$ *if* $p_t \in (\underline{p}, 1/R]$ *and* $d^I(p_t) = 0$ *if* $p_t = \underline{p}$, *and, to ensure market clearing, the bond allocation to uninformed managers is (8).*

The uninformed managers submit a demand correspondence to an auctioneer, as follows: $d^U(p_t) = \{0, 1\}$ if $p_t \in (\underline{p}, 1/R]$ and $d^I(p_t) = 0$ if $p_t \leq \underline{p}$. Then the auctioneer will chose the equilibrium bond allocation $x(z_t)$ to ensure that the bond market clears.

If $p_t = \underline{p}$, there is full revelation and the uninformed managers know that there is going to be default. Hence, they will always demand 0 bonds in order to avoid to be fired. If $p_t = 1/R$, there is still full revelation, but now the uninformed managers know that there is not going to be default and that they are not going to be fired. Hence, they are indifferent between investing in the risky bond and in the riskless asset, which both give a return of $1/R$ for sure.

If prices do not reveal any information, that is, $p_t \in (\underline{p}, 1/R)$, then from the market clearing condition it must be that

$$x^U(z_t) = \frac{b^* p^* - z_t}{\Gamma - \Gamma^I}. \tag{22}$$

Assumption A2 guarantees that $x^U(z_t) \in (0, 1)$ when $z_t \in [\Gamma^I, \bar{y}]$, so that when prices are not fully revealing, there are always some uninformed managers investing in the risky bond and some investing in the risk-free asset. Hence, it must be that the uninformed managers are indifferent between the two investment possibilities. This is the notion of interior solution we refer to when we call the equilibrium an interior equilibrium.

For a given default probability q, the equilibrium price p conditional on a not fully revealing equilibrium is determined by the indifference condition for the uninformed managers, that is,

$$(1 - q)(\gamma/p + \delta\omega W) = \gamma R + q\delta\omega W, \tag{23}$$

where W is the expected continuation utility of an uninformed manager who keeps the job and is given by

$$W = \gamma R + \delta\omega [\pi(q) + q(1 - \pi(q))] W, \tag{24}$$

where

$$\pi(q) \equiv [q + (1 - F(\tilde{a}))] \frac{\Gamma^I}{\bar{y}}, \tag{25}$$

and Γ^I is defined by the combination of (18), (19), and (24). When the price is fully revealing, with probability $\pi(q)$, uninformed managers always gain γR, given that if they learn that there is default they invest in the risk-free asset, while if they learn that there is no default, the bond price will be equal to $1/R$. Moreover, they are never fired and get the continuation utility W, as long as there is no exogenous signal. When instead the price does not reveal any information, with probability $1 - \pi(q)$, uninformed managers are indifferent in each point in time between investing in the risk-free asset or in the risky bond, and hence their expected utility can be calculated as the value of always investing in the risk-free asset. Again, they will always gain γR, but, in this case, they will get the continuation utility W only if there is default and no exogenous signal.

By combining equations (23) and (24) we immediately obtain the not revealing price as a function of the default probability q, that is, we obtain the following *pricing rule*:

$$P(q) \equiv \frac{(1-q)(1-\delta[\pi(q)+q(1-\pi(q))])}{[1-\delta(1-q)(1+\pi(q))]R}, \tag{26}$$

where $\pi(q)$ is defined in equation (25). Hence, we have proved the following lemma.

Lemma 5 *Given the equilibrium strategies (i)-(iv) in Definition 2, the equilibrium price satisfies (6), where* $p^* = P(q^*)$*.*

Firing rule

Finally, taking all the other equilibrium objects as given, investors make their firing decision in order to maximize their expected utility.

Lemma 6 *Suppose assumption A3 holds. Then, given the equilibrium price (6) and the equilibrium strategies (i), (ii), and (iii) in Definition 2, the investors choose to fire manager j if* $\theta_t^j \neq 1 - \chi(y_t, a_{t+1})$ *and* $p_t \in [\underline{p}, 1/R)$ *and keep him otherwise.*

First of all, notice that the restriction $p_t \leq 1/R$ ensures that the investors will always prefer to hire informed managers. Given that, investors need to assess the probability that their manager is informed to make their firing decision. At the end of time t, each investor observes the investment realization of his manager θ_t^j and the default realization χ_{t+1}. Then if $p_t < 1/R$ and either $\chi_{t+1} = 0$ and $\theta_t^j = 1$, or $\chi_{t+1} = 1$ and $\theta_t^j = 0$, he realizes that his manager is not informed, that is, $\eta_{t+1}^j = 0$. In this case, the investor fires him, given that there is always a positive probability that a new manager is informed and the probability of finding a new manager is always equal to 1. For the same reason, the investor also fires an uninformed manager who reveal his type with probability $1 - \omega$. On the other hand, if the manager does not make a mistake, that is, if $\theta_t^j = \chi_{t+1}$ and/or $p_t = 1/R$, so that he does not reveal to be uninformed, then he is not fired if and only if his updated belief η_{t+1}^j is higher than the probability that a new hire is informed, ε_{t+1}, which satisfies

$$\varepsilon_{t+1} = \frac{M^I - \delta\Gamma^I}{M^I - \delta\Gamma^I + N_t^U} > 0.$$

When manager j realizes $\theta_t^j = 1 - \chi_{t+1}$ and/or $p_t = 1/R$, the investor's belief is updated as follows:

$$\eta_{t+1}^j = \frac{\eta_t^j}{\eta_t^j + \omega\xi_t(1-\eta_t^j)},$$

where ξ_t represents the proportion of uninformed managers who make the same investment decision of the informed managers given by (21). Next, we show that assumption A3 is sufficient to make sure that in equilibrium $\eta_{t+1}^j \geq \varepsilon_{t+1}$ for any ξ_t and $\eta_t^j > 0$.

First, consider an investor who has just hired manager j and hence, by definition, has prior belief $\eta_t^j = \varepsilon_t$. In this case, if $\theta_t^j = 1 - \chi_{t+1}$, then

$$\eta_{t+1}^j = \frac{\varepsilon_t}{\varepsilon_t + \omega\xi_t\,(1-\varepsilon_t)}.$$

Next, we want to show that $\eta_{t+1}^j \geq \varepsilon_{t+1}$. This condition can be rewritten as

$$\frac{1-\varepsilon_{t+1}}{\varepsilon_{t+1}} \geq \left(\frac{1-\varepsilon_t}{\varepsilon_t}\right)\omega\xi_t, \tag{27}$$

Using the expression for ε_t, we have that

$$\frac{1-\varepsilon_{t+1}}{\varepsilon_{t+1}} = \frac{N_t^U}{M^I - \delta\Gamma^I},$$

and hence, condition (27) can be rewritten as

$$\frac{N_t^U}{N_{t-1}^U} \geq \omega\xi_t,$$

where

$$N_t^U = (1 - \omega\xi_t\delta)\,\frac{\Gamma^U}{\mu}$$

Hence, it must be that

$$1 - \delta\omega\xi_t > \left(1 - \delta\omega\xi_{t-1}\right)\omega\xi_t,$$

which is always satisfied, given assumption A3.

Let us now consider managers that were working for an investor for longer than 1 period. First, notice that the investors' beliefs about any manager who is still working but was hired at time $t-\tau$ with $\tau \in [0,t]$, must be higher than the initial belief $\varepsilon_{t-\tau}$, given that if he was not fired he never made any mistake, that is, $\eta_t^j \geq \varepsilon_{t-\tau}$. Hence, the belief about a manager who was hired at time $t-\tau$ and did not make a mistake at time t is

$$\eta_{t+1}^j = \frac{\eta_t^j}{\eta_t^j + \omega\xi_t(1-\eta_t^j)} \geq \frac{\varepsilon_{t-\tau}}{\varepsilon_{t-\tau} + \omega\xi_{t-\tau}(1-\varepsilon_{t-\tau})}.$$

Hence, a sufficient condition for this guy not being fired is

$$\frac{1-\varepsilon_{t+1}}{\varepsilon_{t+1}} \geq \left(\frac{1-\varepsilon_{t-\tau}}{\varepsilon_{t-\tau}}\right)\omega\xi_{t-\tau},$$

which, by the same argument, is satisfied when assumption A3 holds. For the same reason, when $p_t = 1/R$, no manager is fired, given that there is no information in their action, completing the proof.

Appendix B

Proof of Lemma 1

We prove Lemma 1 for the general equilibrium when prices do not reveal any information. Let us start to prove condition (i). First, notice that

$$B(p) \equiv \arg\max_b \Phi(p, b)$$

Assumption 1 implies that there exists a unique $B(p)$ and

$$\begin{aligned} \Phi_b(p, B(p)) &= 0, \\ \Phi_{bb}(p, B(p)) &< 0. \end{aligned} \tag{28}$$

Then, by total differentiating (28), we obtain

$$\Phi_{bp}(p, B(p)) + \Phi_{bb}(p, B(p))\, B'(p) = 0,$$

and then

$$B'(p) = -\frac{\Phi_{bp}(p, B(p))}{\Phi_{bb}(p, B(p))}. \tag{29}$$

Given that $\Phi_{bb}(p, B(p)) < 0$, to show that $B'(p) < 0$, it is enough to show that $\Phi_{bp}(p, B(p)) < 0$. We can calculate that

$$\Phi_{bp}(p, B(p)) = u'(p_t b_{t+1} - k) + p_t b_{t+1} u''(p_t b_{t+1} - k), \tag{30}$$

and hence $\Phi_{bp}(p, B(p)) < 0$ given that by assumption $k > 0$ and

$$-\frac{cu''(c)}{u'(c)} \geq 1.$$

From this, it is immediate that

$$\frac{dF(\hat{a})}{dp} = \frac{dF(\hat{a})}{\hat{a}}\left(\frac{1}{1-\theta}\right)B'(p) \leq 0.$$

proving condition (ii).

Finally, we need to prove condition (iii). Notice that

$$\frac{d(pb)}{dp} = b + pB'(p),$$

where $B'(p)$ satisfies equation (29), with $\Phi_{bp}(p, B(p))$ given in equation (30) and

$$\Phi_{bb}(p, B(p)) = p^2 u''(pb - k) + \beta v'\left(\frac{\theta}{1-\theta} b\right) + \beta \int_{\frac{b}{1-\theta}}^{\infty} v''(a - b)\, dF(a) < 0, \tag{31}$$

given assumption 1. Then, after some algebra, we obtain

$$\frac{d(pb)}{dp} = \frac{b}{\Phi_{bb}(p, B(p))} \left\{ \beta \left[v'\left(\frac{\theta}{1-\theta} b\right) + \int_{\frac{b}{1-\theta}}^{\infty} v''(a-b)\, dF(a) \right] - \frac{p}{b} u'(p_t b_{t+1} - k) \right\}.$$

Hence $d(pb)/dp \geq 0$ iff the term in parenthesis is negative. We can show that this is the case combining condition (31) and the fact that $-cu''(c)/u'(c) \geq 1$, hence completing the proof.

Proof of Proposition 1

We need to show that there exists a pair (p^*, q^*) that solves the fixed point defined by $q^* = Q(p^*)$ and $p^* = P(q^*)$. Recall that

$$\begin{aligned} Q(p) &= F\left(\frac{B(p)}{1-\theta}\right), \\ P(q) &= \frac{(1-q)(1-\delta[\Pi(q) + q(1-\Pi(q))])}{[1-\delta(1-q)(1+\Pi(q))]R}, \end{aligned}$$

where $B(p)$ is implicitly define by

$$pu'(pB(p) - k) - \beta \int_{\frac{B(p)}{1-\theta}}^{\infty} v'(a - B(p))\, dF(a) = 0. \tag{32}$$

and $\Pi(q)$ is implicitly defined by

$$\Pi(q) = \psi(\Pi(q); q) \equiv \frac{[q + (1-\tilde{q})]\gamma R M^I}{[\kappa(1-\delta)(1-\delta[\Pi(q)(1-q)+q]) + \delta\gamma R]\bar{y}}. \tag{33}$$

First, Proposition 5 shows that there exists a unique $B(p)$ that solves equation (32) on the interval $[\underline{p}, 1/R]$ and Lemma 1 shows that $B'(p) < 0$. Then, $Q(p)$ is decreasing on the interval $[\underline{p}, 1/R]$, given that it is immediate that $Q'(p) \propto B'(p)$.

Next, we show that there is a unique $\Pi(q)$ that solves equation (33). First, we show that $\psi(\Pi(q); q)$ has derivative with respect to $\Pi(q)$ smaller than 1, that is,

$$\frac{\pi\delta(1-\delta)(1-q)}{(1-\delta(1-\mu))(1-\delta q - \delta\pi(1-q))} < 1,$$

given that $\pi < 1$, $(1-\delta) < (1-\delta(1-\mu))$, and $1-\delta(1-\pi)+\delta\pi q > 0$. Then, we only need to show that $\psi(0;q) > 0$ and $\psi(1;q) < 1$. With some algebra, it is easy to show that

$$\psi(0;q) = \frac{[q+(1-\tilde{q})]\,\gamma R M^I}{[\kappa(1-\delta)(1-\delta q)+\delta\gamma R]\,\bar{y}} > 0,$$

and

$$\psi(1;q) = \frac{[q+(1-\tilde{q})]\,\gamma R M^I}{\left[\kappa(1-\delta)^2+\delta\gamma R\right]\bar{y}}.$$

A sufficient condition for $\psi(1;q) > 1$ is $2M^I/\bar{y} < 1$, which is ensured by assumption A1.

This implies that equation (33) has a unique interior solution for $\Pi(q)$. Then there exists a fixed point (q^*, p^*) with $Q(1/R) < q^* < Q(\underline{p})$, given that by assumption $P(q_1) < 1/R$ and $P(q_2) > \underline{p}$.

Proof of Proposition 3

With a slight abuse of notation, define

$$\begin{aligned} g^L(q;\sigma) &\equiv q - Q\left(P^L(q;\sigma);\sigma\right), \\ g^B(q;\sigma) &\equiv q - Q\left(P^B(q;\sigma);\sigma\right). \end{aligned}$$

Then, a limit equilibrium is characterized by $q^*(\sigma)$ such that $g^L(q^*;\sigma) = 0$ and the benchmark equilibrium is characterized by $q^B(\sigma)$ such that $g^L(q;\sigma) = 0$.

First, we show that if $q^B(\sigma') < 1/2$, then $q^*(\sigma') < q^B(\sigma') < 1/2$. If $q^B(\sigma') < 1/2$, then $P^L\left(q^B(\sigma');\sigma'\right) > P^B\left(q^B(\sigma');\sigma'\right)$. Moreover, Lemma 1 implies that $Q(p;\sigma)$ is decreasing in p, and we obtain

$$g^L\left(q^B(\sigma');\sigma'\right) = q^B(\sigma') - Q\left(P^L\left(q^B(\sigma');\sigma'\right);\sigma'\right) > q^B(\sigma') - Q\left(P^B\left(q^B(\sigma');\sigma'\right);\sigma'\right) = 0.$$

Moreover, we assumed that $Q(1/R;\sigma) > \underline{q}(\sigma)$ where $\underline{q}(\sigma)$ is such that $P^L(\underline{q}(\sigma);\sigma) = 1/R$. Hence,

$$g^L(\underline{q}(\sigma');\sigma') = \underline{q}(\sigma') - Q(P^L(\underline{q}(\sigma');\sigma');\sigma') < 0.$$

It follows that $q^*(\sigma')$ is such that $\underline{q}(\sigma') < q^*(\sigma') < q^B(\sigma') < 1/2$, as we wanted to show.

Second, we show that if $q^*(\sigma'') > 1/2$, then $q^B(\sigma'') > 1/2$. If $q^*(\sigma'') > 1/2$, then $P^L(q^*(\sigma'');\sigma') < P^B(q^*(\sigma'');\sigma'')$. It follows that

$$g^B\left(q^*(\sigma'');\sigma''\right) = q^*(\sigma'') - Q\left(P^B\left(q^*(\sigma'');\sigma''\right);\sigma''\right) > q^*(\sigma'') - Q\left(P^L\left(q^*(\sigma'');\sigma''\right);\sigma''\right) = 0,$$

and $q^B(\sigma'') < q^*(\sigma'')$ as we wanted to show.

These first two steps immediately imply that $q^*(\sigma') < q^B(\sigma') < 1/2 < q^*(\sigma'') < q^B(\sigma'')$, and hence $q^*(\sigma'') - q^*(\sigma') > q^B(\sigma'') - q^B(\sigma')$. This also implies that $p^*(\sigma') > p^B(\sigma') > 1/(2R) > p^B(\sigma'') > p^*(\sigma'')$, implying that $p^*(\sigma'') - p^*(\sigma') > p^B(\sigma'') - p^B(\sigma')$. This completes the proof.

Proof of Proposition 4

Claim 1. Let us fix α'. We know that $P^L(1;\sigma) = 0$ for any σ. Moreover we can calculate

$$\frac{dP^L(q;\sigma)}{dq} = -\frac{1}{R}\frac{\delta\omega(1-q)(1-\delta\omega(1-q)) + (1-\delta q)}{(1-\delta\omega(1-q))^2} < 0,$$

and $\lim_{R\to 0} dP^L(q;\sigma)/dq = -\infty$. Hence, if $Q(\underline{p}) > 1/2$ and we start with R' such that $Q(p^*(R',\alpha')|\alpha') < 1/2$. A graphical argument available upon request implies that there exists an $\hat{R}$, such that if $R'' > \hat{R}$ then $Q(p^*(R'',\alpha')|\alpha') > 1/2$.

Claim 2. First, we prove that for any fixed price p there is a α_1 such that for any $\alpha < \alpha_1$, $\partial Q(p)/\partial\alpha < 0$.

From $q(b) = F(b/(1-\theta))$, with a slight abuse of notation let us define $b_\alpha(q) = (1-\theta)\left(F_\alpha^{-1}(q)\right)$ be the bond holding which implies a probability of default of q when the parameter is α. Then, for a fixed p, we can write the first order condition of entrepreneurs as

$$\tilde{\Phi}(\alpha, b_\alpha(q)) = pu'(pb_\alpha(q) - k) - \beta\int_{\frac{b_\alpha(q)}{1-\theta}}^{\infty} v'(a_{t+1} - b_\alpha(q))\, dF_\alpha(a_{t+1}).$$

We show that there is a α_1 such that

$$\frac{dq}{d\alpha} = -\frac{\frac{\partial\tilde{\Phi}(\alpha,b_\alpha(q))}{\partial b}\frac{\partial b_\alpha(q)}{\partial q} + \frac{\partial\tilde{\Phi}(\alpha,b_\alpha(q))}{\partial\alpha}}{\frac{\partial\tilde{\Phi}(\alpha,b_\alpha(q))}{\partial b}\frac{\partial b_\alpha(q)}{\partial q}}|_{\alpha<\alpha_1} < 0. \tag{34}$$

From the properties of a cumulative density function and our assumptions on α, we know that $\partial b_\alpha(q)/\partial q > 0$ and $\partial b_\alpha(q)/\partial\alpha > 0$. Furthermore, from the result of $\Phi_{bb}(p,b) < 0$, we know that $\tilde{\Phi}_b(\alpha, b_\alpha(q)) < 0$. Note that

$$\frac{\partial\tilde{\Phi}(\alpha, b_\alpha(q))}{\partial\alpha} = -\beta\frac{\partial\int_{\frac{b_\alpha(q)}{1-\theta}}^{\infty} v'(a_{t+1} - b_\alpha(q))\, dF_\alpha(a_{t+1})}{\partial\alpha}.$$

As $\lim_{\alpha\to-\infty} F_\alpha(a_{t+1}) = 1$, we can choose an α_1 that $\partial((F_\alpha(a_{t+1}) - F_{\alpha+\varepsilon}(a_{t+1}))/\partial a_{t+1} < 0$ for any fixed $\alpha < \alpha_1$ and $a_{t+1} > b_\alpha(q)/(1-\theta)$. To show 34, it is sufficient to show that for any $\alpha < \alpha_1$

$$\frac{\partial \tilde{\Phi}\left(\alpha, b_\alpha\left(q\right)\right)}{\partial \alpha}|_{\alpha<\alpha_1} < 0.$$

This is a consequence of the following chain of inequalities for any $\alpha < \alpha_1$

$$\begin{aligned}
&\int_{\frac{b_a(q)}{1-\theta}}^{\infty} v'\left(a_{t+1} - b_\alpha\left(q\right)\right) d\left(F_\alpha\left(a_{t+1}\right) - F_{\alpha+\varepsilon}\left(a_{t+1}\right)\right) = \\
&= -v'\left(\frac{b_a\left(q\right)}{1-\theta} - b_\alpha\left(q\right)\right)\left(F_\alpha\left(\frac{b_a\left(q\right)}{1-\theta}\right) - F_{\alpha+\varepsilon}\left(\frac{b_a\left(q\right)}{1-\theta}\right)\right) \\
&- \int_{\frac{b_a(q)}{1-\theta}}^{\infty} v''\left(a_{t+1} - b_\alpha\left(q\right)\right)\left(F_\alpha\left(a_{t+1}\right) - F_{\alpha+\varepsilon}\left(a_{t+1}\right)\right) da_{t+1} \\
&< -v'\left(\frac{b_a\left(q\right)}{1-\theta} - b_\alpha\left(q\right)\right)\left(F_\alpha\left(\frac{b_a\left(q\right)}{1-\theta}\right) - F_{\alpha+\varepsilon}\left(\frac{b_a\left(q\right)}{1-\theta}\right)\right) \\
&- \left(F_\alpha\left(\frac{b_a\left(q\right)}{1-\theta}\right) - F_{\alpha+\varepsilon}\left(\frac{b_a\left(q\right)}{1-\theta}\right)\right)\int_{\frac{b_a(q)}{1-\theta}}^{\infty} v''\left(a_{t+1} - b_\alpha\left(q\right)\right) da_{t+1} = \\
&= \left(F_\alpha\left(\frac{b_a\left(q\right)}{1-\theta}\right) - F_{\alpha+\varepsilon}\left(\frac{b_a\left(q\right)}{1-\theta}\right)\right)\left[-v'\left(\frac{b_a\left(q\right)}{1-\theta} - b_\alpha\left(q\right)\right)\right. \\
&\left. - \lim_{a_{t+1}\to\infty} v'\left(a_{t+1} - b_\alpha\left(q\right)\right) + v'\left(\frac{b_a\left(q\right)}{1-\theta} - b_\alpha\left(q\right)\right)\right] \leq 0
\end{aligned}$$

where the first equation is by partial integration and the inequality is the consequence of $\partial\left(\left(F_\alpha\left(a_{t+1}\right) - F_{\alpha+\varepsilon}\left(a_{t+1}\right)\right)/\partial a_{t+1} < 0\right.$ and $v''\left(\cdot\right) \leq 0$.

Moreover, if $\lim_{\alpha\to-\infty} Q\left(p\right)$ exists, then $\lim_{\alpha\to-\infty} Q\left(p\right) = 1$. This is a simple consequence of the first order condition and $\lim_{\alpha\to-\infty} F_\alpha\left(a_t\right) = 1$.

To summarize, so far we have shown that $\partial Q_\alpha\left(p\right)/\partial\alpha < 0$ for any $\alpha < \alpha_1$ and $\lim_{\alpha\to\infty} Q\left(p\right) = 1$ whenever it exists. So if $\underline{\alpha}$ is sufficiently low, then $Q\left(p^*\left(R', \underline{\alpha}\right)|\underline{\alpha}\right) < \frac{1}{2}$. Then there is an $\alpha_2 = \min\alpha$ such that $Q\left(p^*\left(R', \alpha\right)|\alpha\right) = 1/2$. The second claim of the proposition follows immediately if we set $\hat{\alpha} \equiv \min\left(\alpha_1, \alpha_2\right)$.

Appendix C

Example

Assume $u\left(c\right) = \log\left(c\right)$, $v\left(c\right) = c$ and $1 - F\left(a\right) = \underline{a}^\gamma a^{-\gamma}$, with $\underline{a} > 1/\beta\left(1-\theta\right)$ and $\gamma < 1$. In this case we can write

$$\Phi_b\left(p, b\right) = b^{-\gamma}\left[\frac{pb^\gamma}{pb - k} - \beta\underline{a}^\gamma\left(1-\theta\right)^\gamma\right].$$

For a given p, there is a unique solution to $\Phi_b\left(p, b\right) = 0$ which solves

$$\frac{pb^\gamma}{pb - k} = \beta\underline{a}^\gamma\left(1-\theta\right)^\gamma.$$

Then, it is easy to verify that the left-hand side of this condition is decreasing in b whenever $\gamma < 1$, and it converges to ∞ for $b \to k/p$, and to 0 for $b \to \infty$. Given that the right-hand side is a positive constant, there must be a unique optimum b that is implicitly define as follows:

$$p = \frac{\beta \underline{a}^{\gamma} (1-\theta)^{\gamma} k}{\beta \underline{a}^{\gamma} (1-\theta)^{\gamma} b - b^{\gamma}}.$$

The function $\Phi(p, b)$ is quasi-concave because if $b_1 < b_2$ and $\Phi_b(p, b_1) > 0$, then $\Phi_b(p, b_2) > 0$, given that $b^{-\gamma} > 0$ for any $b > k/p$. For the same reason if $b_1 < b_2$ and $\Phi_b(p, b_2) < 0$, then $\Phi_b(p, b_1) < 0$, completing the proof.

Moreover, we can show that there always exists a limit equilibrium for k small enough. The fixed point problem for (q^*, p^*) can be rewritten as

$$\begin{aligned} p^* &= \frac{k\beta (1-q^*)}{(1-\theta) \beta \underline{a} (1-q^*)^{-\frac{1-\gamma}{\gamma}} - 1}, \\ p^* &= \frac{(1-q^*)(1-\delta q^*)}{[1-\delta(1-q^*)] R}. \end{aligned}$$

Let us define

$$\begin{aligned} h_1(q) &\equiv \frac{k\beta}{(1-\theta) \beta \underline{a} (1-q)^{-\frac{1-\gamma}{\gamma}} - 1} \\ h_2(q) &\equiv \frac{1-\delta q}{[1-\delta+\delta q] R}, \end{aligned}$$

then it is immediate that there exists a fixed point iff there exists a q^* such that $h_1(q^*) = h_2(q^*)$. This is easy to show given that both $h_1(q)$ and $h_2(q)$ are decreasing in q. Moreover, $h_1(0) = k\beta / [(1-\theta) \beta \underline{a} - 1] > 0$, $h_2(0) = 1/[(1-\delta) R] > 0$, $h_1(1) = 0$ and $h_2(1) = (1-\delta)/R > 0$. Hence, in order for a fixed point to exists, it is enough that $h_2(0) < h_1(0)$, that is,

$$k < \frac{(1-\theta) \beta \underline{a} - 1}{(1-\delta) \beta R}.$$

Moreover, if $k = 0$, then there is a unique solution, given by

$$\begin{aligned} q^* &= 1 - \left(\frac{1}{(1-\theta) \beta \underline{a}} \right)^{-\frac{\gamma}{1-\gamma}}, \\ p^* &= \frac{(1-q^*)(1-\omega\delta q^*)}{[1-\omega\delta(1-q^*)] R}. \end{aligned}$$

By continuity if k is small enough the equilibrium is also unique.