Equilibrium Credit Spreads and the Macroeconomy

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ABSTRACT

Credit markets play an important role in the macroeconomy and credit market data is often used to predict future macroeconomic performance. In this paper we propose a tractable general equilibrium asset pricing model with heterogeneous firms that links movements in stock and bond markets to macroeconomic activity. The model suggests that movements in risk premia in corporate bond markets are an important determinant of aggregate fluctuations. We show that movements in credit and term spreads forecast recessions by predicting future movements in corporate investment. Endogenous movements in credit markets allow our model to match the observed conditional and unconditional movements in stock market returns and credit spreads with a reasonable amount of aggregate volatility.

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1 Introduction

Much economic research has documented the role of asset market movements in both predicting and amplifying macroeconomic fluctuations. In particular, credit spreads, measured as the difference between corporate and treasury yields, have been shown to carry important information both about future movements in real activity and equity markets. Credit spreads are both large, volatile, and strongly countercyclical. A long literature has documented that credit spreads forecast both output and investment growth as well as future stock returns and stock market volatility.[1,2]

In this paper we address these findings in the context of a one sector general equilibrium model with heterogeneous firms that make optimal investment and financing decisions under uncertainty. The model endogenously links movements in aggregate quantities such as investment and output to the prices of stocks and bonds. As a result movements in financial variables such as credit spreads and expected equity returns will forecast future economic activity. In our model these movements are largely driven by risk premia. Intuitively, investors incur losses on defaulted bonds in recessions, precisely when marginal utility is high giving rise to countercyclical credit spreads. In our equilibrium setting, endogenous default also increases the volatility of consumption during recessions, thereby rendering the market price of risk sharply countercyclical. As a consequence, expected returns on stocks are higher in recessions, which are naturally anticipated by movements in credit spreads.

Moreover because investors require higher compensation for default risk in bad times, firms find it especially costly to obtain debt financing in recessions. Accordingly firms

[1]Examples of the ability of credit spreads to forecast economic activity include studies by Bernanke, Gerler, and Gilchrist (1999), Stock and Watson (1991), Lettau and Ludvigson (2004), Gilchrist and Zakrajsek (2008), and Mueller (2008)

[2]Examples of the link between credit spreads and equity markets include Keim and Stambaugh (1986) and Schwert (1989).

will find it harder to obtain funding for investment expenditures, thereby depressing investment and output and amplifying macroeconomic conditions. Risk premia in corporate bond markets are thus propagated into the real economy and this accounts for the predictive power of credit spreads for output and investment. Thus credit risk premium emerges as the common link between credit markets, equity markets and macroeconomic aggregates.

Quantitatively, accounting for the premia in corporate bond prices allows the model to generate sizeable credit spreads. In addition, we also show how allowing for endogenous movements in credit markets also enhances the model's ability to match the observed conditional and unconditional movements in equity markets with a reasonable amount of aggregate volatility.

Existing theoretical work has focused either on the role of asset prices in signalling a decline in future cash flows or the fact that asset prices, and in particular credit spreads, often reflect exogenous changes in credit supply.[3] While all these mechanisms seem important empirically, the existing theoretical literature has mostly focused on a limited set of facts and has thus failed to provide a comprehensive explanation of the joint movements in aggregate quantities and asset prices. As a result the existing models can be viewed as offering at best a partial explanation for these co-movements and often have strongly counterfactual implications. In contrast, our model provides a unified explanation of the joint movements in macroeconomic quantities and asset prices. In particular, by adopting an equilibrium perspective it allows for a better understanding of the spillovers between asset markets and real variables, and in particular of the role of credit risk.

Our paper is closely related to a recent set of contributions in finance seeking to understand the level and dynamics of credit spreads in relation to macroeconomic shocks.

[3]Recent examples include Bernanke, Gertler, and Gilchrist [1999] and Philippon [2008].

A list of recent papers in this area includes Hackbarth, Miao, Morellec (2007), Chen, Collin Dufresne, Goldstein (2008), Chen (2008), Bhamra, Kuhn, and Strebualev (2008a). In contrast to this literature we adopt a general equilibrium perspective and study the mutual endogenous interactions between credit markets, equity markets and macroeconomic variables.

More broadly, our work adds to the literature on equilibrium asset pricing with heterogeneous firms (Gomes, Kogan, and Zhang (2003), Gourio (2006), Gala (2006)). From this point of view the novelty in our work is that we explicitly allow for deviations from the Modigliani-Miller theorems so that corporate financing decisions affect investment, and thus asset prices and output. Finally, our paper is related to recent work on corporate capital structure and financing decisions across the business cycle (Covas and Den Haan (2007), Bhamra, Kuhn and Strebulaev (2008b), Hennessy and Levy (2006)).

The paper is organized as follows. Section 2 describes our basic general equilibrium model and some of its properties, while section 3 discusses the details associated with solving it numerically. Our findings are covered in section 4 and the final section concludes.

2 The Model

In this section we describe a general equilibrium complete markets model with heterogeneous firms that are financed with both debt and equity. Debt is used because of its tax benefits. Although our economy is often stylized the model presented here preserves tractability and economic intuition. Nevertheless as we will see below this economy is also suitable for detailed quantitative analysis.

2.1 Production Sector

The production sector of the economy is made of a continuum of firms that differ in their productivity, size and leverage among other characteristics. In characterizing the problem of firms we take the stochastic discount factor for the economy, M, as given. We show below how this will be determined in general equilibrium by the optimal consumption and savings decisions of households.

2.1.1 Technology and Investment

All firms produce the same homogeneous final good that can be used for either consumption or investment. The production function denoting the instantaneous flow of output is described by the expression:

$$y(x, z, k) = xzk \tag{1}$$

where x and z denote the values of aggregate and firm specific productivity, respectively. These productivities are exogenous and evolve according to first order Markov transitions denoted $Q_x(x'|x)$ and $Q_z(z'|z)$, respectively. Here we have used the notation x' and z' to denote the future value of the variables x and z, respectively. The variable k denotes the firm's productive capacity. This capacity is installed when the firm begins to operate and remains fixed throughout the life of the firm.

2.1.2 Firm Entry and Financing

New firms can enter the market and start production if market conditions are sufficiently attractive. Entering firms draw the initial realization of the idiosyncratic shock z from the invariant distribution implied by the transition $Q_z(z'|z)$, denoted $G(z)$. This value is only observed *after* entry. We further assume that entering firms are not immediately productive.

Entering firms must invest to build their productive capacity, k. This investment can be finance with either debt or equity finance. Debt takes the form of a consol bond that pays a fixed coupon bk as long as the firm is in existence and does not default on its obligations. Note that writing the coupon as bk allows us to interpret b as a measure of book leverage.

2.1.3 Equity Value and Exit

Given production and leverage the firm's operating profits are given by the expression

$$\pi(x, z, b, k) = (xz - b)k \tag{2}$$

Denote by s the aggregate state of the economy, which includes the state of aggregate productivity, x. Taking the households pricing kernel, M, as given, the firm's equity value, $V(s, z, k, b)$ *after* entering the economy is determined through the Bellman equation

$$V(s, z, b, k) = \max\{0, (1 - \tau)(xz - b)k + E[M(s, s')V(s', z', b, k)]\} \tag{3}$$

where τ is the marginal tax on corporate profits, adjusted for taxes on distributions and personal interest income and we have used the notation s' to denote the future value of s. Note that our assumptions about the nature of cash flows implies that equity value is linear in k.

This Bellman equation implies that equity holders will default on their debt obligations when equity value falls to zero. This boundary condition yields a default cut-off value for the idiosyncratic shock, $\bar{z}_d(s, b)$, such that the firm will default whenever $z < \bar{z}_d(s, b, k)$. Formally, we define this default threshold with the condition

$$z_d(s, b, k) = \min\{z : V(s, z, b, k) = 0\} \tag{4}$$

2.1.4 Value of Debt and Credit Spreads

Bondholders receive the coupon b when the firm does not default and will receive a fraction $1-\phi$ of the book value of the firm in case of default. Formally then, we can define the market value of debt, $B(s,z,b,k)$, in recursive form as follows

$$B(s,z,b,k) = (bk + E[M(s,s')B(s',z',b,k)])\chi_{\{z>\bar{z}_d\}} + (1-\phi)k(1-\chi_{\{z>z_d\}}) \tag{5}$$

where χ is an indicator function that takes the value of 1 when $z > \bar{z}_d(s,b,k)$. A possible interpretation of this equation for the value of debt value is that at default the firm gets liquidated and its assets are transferred to bondholders at a discount, $1-\phi$.

Given our definition it is straightforward to define the yield $y(s,z,b,k)$ on corporate debt as

$$y(s,z,b,k) = b/B(s,z,b,k) \tag{6}$$

Moreover we can construct measures of the credit spread for this economy by comparing this quantity with the yield on a riskless bond of identical maturity. Formally:

$$cs(s,z,b,k) = y(s,z,b,k) - y(s) \tag{7}$$

2.1.5 Entry and Optimal Capital Structure

Given the expression for equity and debt value, the expected value of entry, for any level of leverage b, in aggregate state s is given by the expression:

$$\int V(s,z,b,k) + B(s,z,b,k)dG(z) \tag{8}$$

Upon entering the firm invests to build up the required productive capacity, k, at unit cost e. Thus, in our framework a minimum productive capacity is required to operate the firm, while the cost of installation varies across firms. These installation costs are randomly drawn from a continuous distribution $H(e)$. As we will see below this implies that only a subset of potential entrants actually begins production in a given period.

The firm finances these purchases of capital using an optimal mix of debt and equity. This optimal capital structure can be found by optimizing the expected firm value with respect to the instantaneous level of coupon payments, b. Formally then the optimal ex-ante value of the firm (i.e. debt plus equity) $A_0(s)$ is given by the expression:

$$A_0(s,k) = \max_{b \geq 0} \{ \int V(s,z,b,k) + B(s,z,b,k) dG(z) \} \tag{9}$$

It follows that firms will enter the economy if and only if the setup cost ek is less or equal the ex ante firm value $A_0(s,k)$. Formally, entry occurs whenever

$$e \leq \bar{e}(s) = A_0(s,k)/k \tag{10}$$

Note that because the value of the firm is homogenous of degree 1 in k the cutoff level of installation costs, e, does not depend on the scale of production.

2.2 Aggregation

Given the optimal behavior for individual firms described above we can construct aggregate quantities in this economy. First however let us define $\mu(b,z;x)$ as the cross-sectional distribution of firms over leverage and idiosyncratic shocks. Note that this distribution will in general vary over time according to the state of aggregate productivity x. However there is no cross-sectional variation in productive capacity since this is constant across firms.

We can now construct aggregate output as follows:

$$\mathbf{Y}(x,\mu) = \int_{z_{d(s,b)}}^{\infty} xzk \, d\mu \tag{11}$$

Similarly aggregate investment equals the sum of the setup costs for entering firms:

$$\mathbf{I}(x,\mu) = \int_{0}^{\bar{e}(s)} kdH(e) \, d\mu \tag{12}$$

Finally we can define aggregate market value of corporate equity and debt respectively with the expressions:

$$\mathbf{V}(x,\mu) = \int_{z_{d(s,b)}}^{\infty} V(s,z,k,b)\, d\mu \tag{13}$$

and

$$\mathbf{B}(x,\mu) = \int_{z_{d(s,b)}}^{\infty} B(s,z,k,b)\, d\mu \tag{14}$$

It follows from the definitions for the aggregate quantities that we can fully characterize the aggregate state of our economy s, with the pair (x,μ). Intuitively aggregate quantities and prices will depend both on the aggregate state of productivity and profits but also on the cross-sectional variation in firm productivities and balance sheet positions.

2.3 Households

The economy is populated by identical competitive households, who derive utility from the consumption flow of the single consumption good, C_t. The household maximizes the discounted value of future utility flows, defined through the Epstein-Zin (1991) and Weil (1990) recursive function:

$$U_t = \{(1-\beta)u(C_t)^{1-1/\sigma} + \beta\, \mathrm{E}_t[U_{t+1}^{1-\gamma}]^{1/\kappa}\}^{1/(1-1/\sigma)}. \tag{15}$$

The parameter $\beta \in (0,1)$ is the household's subjective discount factor and $\gamma > 0$ is the coefficient of relative risk aversion. The parameter $\sigma \geq 0$ denotes the elasticity of intertemporal substitution and $\kappa = (1-\gamma)/(1-1/\sigma)$.

We assume that there exists a complete set of financial markets, including an instantaneously riskless bond in zero net supply that earns a rate of interest r_t. There are no frictions and no constraints on short sales or borrowing. Accordingly there is a unique equilibrium pricing kernel, denoted $M_{t,t+s}$ which determines prices of all financial assets.

The representative household maximizes the expected utility of consumption (15), taking the prices of financial assets as given. In a complete financial market, the budget constraint is given by:

$$E_t\left[\int_0^\infty M_{t,t+s} C_{t+s}\, ds\right] \leq W_t. \tag{16}$$

where W_t denotes accumulated household wealth. This measure includes both the market value of equity and debt but also the present value of all tax rebates and bankruptcy costs.[4] Thus we assume that bankruptcy costs associated with the liquidation of corporations accrue to specialized firms (e.g. law practices) which are also owned by the representative agent.

Formally the equilibrium pricing kernel is then given by the expression

$$M_{t,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma} R_{W,t+1}^{1-1/\kappa}\right]^{\kappa}. \tag{17}$$

and, as is well known, the absence of arbitrage implies that gross asset returns satisfy:

$$\mathrm{E}_t[M_{t+1} R_{i,t+1}] = 1, \tag{18}$$

for all assets i.

2.4 Equilibrium

As we have seen given the stochastic discount factor firm's behavior determines aggregate investment and output as well as household wealth. On the other hand household behavior determines the equilibrium stochastic discount factor, given the level of wealth. A competitive equilibrium can then be constructed by imposing that:

$$C_t = C(x, \mu) = \mathbf{Y}(x, \mu) - \mathbf{I}(x, \mu) \tag{19}$$

so that the stochastic discount factor used by firms corresponds to that implied by optimal household behavior. Moreover, because tax revenues and bankruptcy costs are

[4] We are assuming that households invest directly in the aggregate stock and bond market portfolio.

redistributed to the representative household, Walras' law implies that equilibrium in the capital market also holds as long as equation (19) is satisfied.

3 Computation

This section describes our approach to solve the model in section 2. Although the model is fairly stylized the competitive equilibrium is a complex object and its computation requires the use of numerical methods. Below we explain our choices for the key parameters in the model and offer an overview of the numerical algorithm that we employ.

3.1 Parameter Choices

Table 1 reports the parameters that we use for our calibration. As usual we set the preference parameters β, γ and σ to generate the right levels for the risk free rate and the equity premium in the economy. Their values are also very similar to those used in most recent quantitative studies that rely on time non-separable preferences and persistent shocks to aggregate growth (e.g. Bansal and Yaron (2004)).

The stochastic behavior of the (log) aggregate and idiosyncratic productivity shocks is restricted to follow a first order autoregressive process with normal innovations. Specifically we assume that

$$\log(x_t) = \rho_x \log(x_{t-1}) + \sigma_x v_{xt},$$

$$\log(z_t) = \rho_z \log(z_{t-1}) + \sigma_z v_{zt},$$

where both v_{xt} and v_{zt} are independently and identically distributed shocks drawn from a standard normal distribution. Our assumptions combined with a linear technology imply that the aggregate economy experiences stochastic and persistent variation in its growth rate over time through fluctuations in aggregate productivity, x_t.

To pin down the volatility and persistence of the aggregate productivity process we require that our model matches the volatility and persistence of output growth in the data. This implies that $\rho_x = 0.9995$ and $\sigma_x = 0.025$. The parameters for idiosyncratic shocks determine the amount of cross-sectional variation in firm heterogeneity. Since we are especially concerned with the role of leverage and credit spreads in our economy we set these parameters to match the unconditional means of both of these variables. Thus we choose the value of $\rho_z = 0.95$ and $\sigma_z = 0.35$

Beyond these choices we only need to determine the value of the marginal corporate tax rate, τ and bankruptcy costs, α. In line with previous studies we assume that the marginal tax rate equals 0.2 a value that reflects both the effects of direct corporate income taxes and those of individual income taxes on distributions and interest. For an estimate of bankruptcy costs we turn to Werner (1977) who suggests that a value of about 0.25 reflects both the direct and indirect costs associated with this process.

3.2 Computation Details

Some computational complexity stems from the endogeneity of the pricing kernel, which embodies the equilibrium market clearing conditions. The solution algorithm exploits two devices to overcome this obstacle. First, following Khan and Thomas, the problem is rewritten in units of marginal utility. Second, simulation is used to back out the equilibrium marginal utility function.

Denote by $p = u'(C)$ the marginal utility of current consumption. Recall that $C = C(x)$. Then the Bellman equations are the following. The equity value of a firm becomes

$$\tilde{V}(x, z, b) = \max\{0, (1 - \tau)(xzk - b)p + E[\beta\tilde{V}(x', z', b)]\}$$

and the bond pricing equation

$$\tilde{B}(x, z, b) = (bp + E[\beta\tilde{B}(x', z', b)])\chi_{\{z>\bar{z}_d\}} + \theta xzk(1 - \chi_{\{z>\bar{z}_d\}})p$$

where $\tilde{V}(x,z,b) = u'(C(x))V(x,z,b)$ and $\tilde{B}(x,z,b) = u'(C(x))B(x,z,b)$ To make this tractable, we start by guessing a consumption function $C(x)$. That is, we approximate using a log linear specification. Given values for the coefficients of the log linear approximation, equity and bond problems can be solved by standard value function iteration on a discretized state space. To that end, we discretize x, z and b to lie on discrete grids. This yields values for the default cutoffs, $\bar{z}_d(x,b)$, and, upon maximizing over b, the investment threshold $\bar{e}(x)$.

Note that so far the coefficient vector has been arbitrary. It will now have to be determined in equilibrium by means of simulation. To do that, we have to check the market clearing in every period. The exact procedure is the following:

- Start with an initial guess for the coefficients in the log linear approximation of $C(x)$. With this guess solve the equity and bond Bellman equation on the discretized state space using value function iteration
- With equity and bond value functions at hand, maximize over the coupon b to determine firms' optimal capital structure.
- Preparing for the simulation draw shocks. Use the default and entry rules obtained in the value function iteration to determine entry and exit and compute the implied consumption from the aggregate budget constraint and then use a bisection routine to compute the market clearing C.
- After the simulation update the consumption function using OLS on the simulated data.
- Iterate on this procedure until the coeffcients converge and check that the R^2 is high enough.

irms.

3.3 Computation 2

Equilbrium Equations: new version

$$\begin{aligned} V(x,z,b) &= \max\left\{0,(1-\tau)\left(xz-\delta-b\right)+\int\int M(x,x')V(x',z',b)Q_xQ_z\right\} \\ & \quad \overline{z}(x,b) \\ B(x,z,b) &= \left(b+\int\int M(x,x')B(x',z',b)Q_xQ_z\right)\chi_{\{z>\overline{z}(x,b)\}}+(1-\theta)xz\chi_{\{z<\overline{z}(x,b)\}} \end{aligned}$$

Leverage

$$b(x)=\arg\max_b \int\left(V(x,z,b)+B(x,z,b)\right)dG(z)$$

Entry

$$\overline{e}(x)=\int\left(V(x,z,b(x))+B(x,z,b(x))\right)dG(z)$$

Aggregation

$$Y(x)=\int xzd\mu_z(z,b)$$

$$I(x,N)=N\int^{\overline{e}(x)}edH(e)-\int\delta d\mu_z(z,b)$$

Equilibrium

$$C(x,N)=Y(x)-I(x,N)$$

Note: we are redistributing the taxes and the default costs

3.3.1 Households

Dealing with Habit

$$\begin{aligned} U &= \max_{C_t}\sum_{t=0}^{\infty}\beta^t u(C_t,S_t) \\ C_t &= Y(x,S)-I(x,S) \end{aligned}$$

where $S_t=f(C_{t-1},S_{t-1})$

Pricing kernel is

$$M_{t,t+1}=\beta\frac{u'(C_{t+1},S_{t+1})}{u'(C_t,S_t)}$$

Equity Valuation becomes

$$V(x,z,b,S) = \max\left\{0, (1-\tau)(xz-\delta-b) + \int\int \beta \frac{u'(C',S')}{u'(C,S)} V(x',z',b,S') Q_x Q_z\right\}$$

Rewrite transformed problem as

$$V(x,z,b,S)u'(C,S) = \max\left\{0, (1-\tau)(xz-\delta-b)\, u'(C,S) + \int\int \beta u'(C',S') V(x',z',b,S') Q_x Q_z\right\}$$

and change variables to get normalized value function

$$\widetilde{V}(x,z,b,S) = \max\left\{0, (1-\tau)(xz-\delta-b)\, u'(C,S) + \int\int \beta \widetilde{V}(x',z',b,S') Q_x Q_z\right\}$$

Strategy is

- Constuct grids for x, z, b and S
- Guess a consumption function (or equivalently, a marginal utility function), $C = C(x,S)$. Using a log linear approximation to that function, this implies specifying an initial guess of the coefficients of the approximation.
- Iterate on $\widetilde{V}$ and $\widetilde{B}$. Use linear interpolation to find next period values of the function at the implied next period habit level, given by the law of motion for habit. This can be $S' = \kappa C$ for one-period habit, or $S' = \kappa C + (1-\kappa)S$ for slow moving habit a la Campbell-Cochrane.
- Compute value function on grid and re-normalize, $V(x,z,b) = \widetilde{V}(x,z,b,S)/u'(C,S)$
- Compute exit decision, $\overline{z}(x,b,S)$
- Compute bond prices $B(x,z,b,S)$
- Compute capital structure/coupon choice, $b(x,S)$
- Compute entry: by evaluating, $\overline{e}(x,S)$

- Aggregate to compute investment and output $I(x, S, N), Y(x, S, N)$
- Compute N to satisfy goods market equilibrium $C(x, N) = Y(x) - I(x, N)$
- Using simulation back out consumption levels implied by market clearing. Using equilibrium consumption and habit stocks update on guess (update coefficients in log linear approximation) and iterate to convergence.

Notes:

S_t completely summarizes the impact of history on the value function.

Does not impact equilibrium

Simulation allows us to update S over time Ensure that grid is fine enough

Dealing with Epstein-Zin

See Kaltenbrunner and Lochstoer for pricing kernel

$$M_{t,t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma-\theta} \left(\frac{A_{t+1} + C_{t+1}}{A_t} \right)^{\theta-1}$$

Equity Valuation becomes

$$V(x, z, b, A) = \max \left\{ 0, (1-\tau)(xz - \delta - b) + \int\int \beta^{\theta} \left(\frac{C'}{C} \right)^{1-\gamma-\theta} \left(\frac{A' + C'}{A} \right)^{\theta-1} V(x', z', b, A') Q_x Q_z \right\}$$

Define

$$S(C, A) = C^{1-\gamma-\theta} (A)^{\theta-1}$$

Rewrite transformed problem as

$$V(x, z, b, A) S = \max \left\{ 0, (1-\tau)(xz - \delta - b) S + \int\int \beta^{\theta} \left(\frac{A' + C'}{A'} \right)^{\theta-1} S' V(x', z', b, A') Q_x Q_z \right\}$$

and change variables to get normalized value function

$$\widetilde{V}(x, z, b, A) = \max \left\{ 0, (1-\tau)(xz - \delta - b) S(A, C) + \int\int \beta \left(\frac{A' + C'}{A'} \right)^{\theta-1} \widetilde{V}(x', z', b, A') Q_x Q_z \right\}$$

Strategy is

- Constuct grids for x, z, b and A

- Recall that $C = C(x, A)$. Guess consumption as a log linear function of x and A by specifying an initial set of coefficients. We also need to forecast A'. Use a log linear forecast as a function of x and A by specifying an initial set of coefficients.

- Using the approximated consumption function and wealth forecasts, iterate on the Bellman equations.

- Compute value function on grid and re-normalize, $V(x, z, b, A) = \widetilde{V}(x, z, b)/C^{1-\gamma-\theta}(A)^{\theta-1}$

- Compute exit decision, $\overline{z}(x, b, A)$

- Compute bond prices $B(x, z, b, A)$

- Compute capital structure/coupon choice, $b(x, A)$

- Compute entry: by evaluating, $\overline{e}(x, A)$

- Aggregate to compute investment and output $I(x, N, A), Y(x, N, A)$

- Compute N to satisfy goods market equilibrium $C(x, N, A) = Y(x, A) - I(x, N, A)$

- Simulate the economy using the decision rules. This allows to back out equilibrium consumption and wealth every period from the market clearing conditions. Update the coefficients of the approximated consumption function and the wealth forecast and iterate to convergence.

Note: we need to ensure that the value of A is consistent with the guess in the value functions. Alternative is to find equilibrium expression for wealth, A.

Dealing with Growth

$$
\begin{aligned}
Y(x) &= \int xz d\mu_z(z,b) \\
&= xE[z]\left(\overline{N}+N\right)
\end{aligned}
$$

$$
\begin{aligned}
I(x,N) &= \overline{N}\int^{\overline{e}(x)} edH(e) - \int \delta d\mu_z(z,b) \\
&= \overline{N}\int^{\overline{e}(x)} edH(e) - \delta\left(\overline{N}+N\right)
\end{aligned}
$$

4 Findings

We are now ready to describe our quantitative findings. We begin by summarizing the basic properties of the model as summarized by the unconditional means and volatilities in aggregate quantities and asset prices. We then examine the model's implications for the behavior of financial variables over the business cycle and compare those with the available empirical evidence. Finally we investigate the role of credit spreads in predicting future movements in both macro quantities and in equity markets.

To construct the statistics reported below we solve the model by numerical dynamic programming as detailed in 3. We then simulate the implied equilibrium policies at monthly frequency to construct 1000 independent panels of 57 years each. Macroeconomic data is aggregated at the quarterly frequency to match the available data. Unless otherwise noted we always report the relevant empirical moments for the sample period between 1951 and 2007.

4.1 Basic Properties

The first panel in Table 2 reports the volatility of the key macroeconomic variables as well as the share of investment in GDP. We can see that our parameter choices imply a very close match between the model and the data along these dimensions. Not only is the share of investment (and consumption) plausible but both variables also seem to exhibit as much variability as in the actual data.

The lower panel in Table 2 documents the implied properties of the model for the unconditional means and volatilities of the risk free rate and the equity premium. As we can see, our model does a very job in replicating these difficult objects. Both the level of the risk free rate and the equity premium are very close to those observed in the data, and this match does not require the very large movements in the risk free rate often associated with habit preferences.

4.2 Credit Market Statistics

Table 3 shows the basic properties of the key credit market statistics as well as its empirical counterparts. Unlike the previous table however which was focused on aggregate variables, the statistics reported in Table 3 are based on the average properties of the cross-sectional distribution of firms.

The Table shows that our parameter choices imply that the model matches almost exactly the cross-sectional average market leverage - usually constructed by dividing the ratio of book leverage by the value of market equity plus book leverage. Moreover the model also yields a realistic level of credit defaults in our model, at about 1.48% per year. More challenging, but equally successful, is the comparison with the average credit spread in the data. As in recent work by Bhamra et al (2008) and Chen (2008) we find that exploit the systematic variation in risk premium allows us to generate a sizable credit spread, even though default rates and costs are in line with the data. Although the intuition is exactly the same as in the earlier papers it is important to see that it survives in the more demanding setting of quantitative general equilibrium.

4.3 Investment and Finance over the Business Cycle

Table 4 documents the cyclical behavior of several investment and financing variables by reporting their cross-correlations with GDP. The table shows that all variables have the correct cyclical behavior in our model although the implied correlations are usually too high. Because our model has only one source of aggregate uncertainty the innovations in aggregate GDP growth are completely tied to those in aggregate productivity. As a result most of these relations can be understood by examining the effects of aggregate productivity of the various variables.

Intuitively the persistence in the aggregate shocks renders implies a strongly pro-cyclical behavior in aggregate investment as new firms enter the market and build up productive capacity in anticipation of higher future profits. As a result the market value of firms (and especially of equity) is also strongly pro-cyclical implying a countercyclical pattern in market leverage.

Also intuitive is the behavior of both default rates and credit spreads which are strongly countercyclical since default is becomes less attractive when profits are temporarily high. As a result of this improvement in credit market conditions which leads book leverage will rise. This is because new entrants will choose a typically higher level of debt thus raising average leverage in the economy.

4.4 Credit Spreads and Predictability in Equity Markets

We now turn to the role of credit markets, and in particular of credit spreads, in forecasting movements in equity markets. Table 7 shows the results of regressing the k period ahead return on the aggregate stock market, on the credit spread at time t. The tables shows that credit spreads in our model are able to predict future equity returns at horizons ranging between 1 quarter and 1 year ahead. For comparison we also report the recent empirical findings in Fama and French (2002).

As we can see from this table, although short horizon predictability is not statistically very significant it rises in importance with the horizon until it becomes fairly strong at a 1 year horizon. Thus, in the model as in the data, an increase in credit spreads anticipates a future rise in equity returns.

An explanation for this finding is suggested by the results in Table 8. Here we used the value of the credit spread at time t to forecast the volatility in the aggregate stock market over the next quarter. We also report the findings in Schwert (1989) who performs a similar exercise in the data. For completeness we also include the result of using market leverage in the forecasting regression used by Schwert (1989).

Our results show that, as in the data, a rise in credit spreads forecasts an increase in stock market volatility. This evidence seems to confirms that movement in credit spreads are associated with changes in risk premia. Intuitively this occurs because investors incur larger losses on defaulted bonds in recessions, precisely when marginal utility is high giving rise to countercyclical credit spreads. In our equilibrium setting then, endogenous default increases the volatility of consumption during recessions, thereby rendering the market price of risk sharply countercyclical. As a consequence, expected returns on stocks are higher in recessions, and this is naturally anticipated by movements in credit spreads.

4.5 Credit Spreads and Business Cycle Predictability

Finally tables 5 and 6 show the results of regressing the k period ahead growth in (log) output and investment, respectively, on the value weighted aggregate credit spread at time t.

These tables show that credit spreads in our model are able to forecast movements in both aggregate output and investment at horizons ranging between 1 quarter and 1 year. This finding is then consistent with much empirical evidence about the forecasting ability of credit spreads and document recently in Mueller (2008), Gilchrist et al (2008) and Lettau and Ludvigson (2004). In both the data and the model the forecasts are both statistically and economically meaningful. Moreover the estimated coefficients on the simulated panels are of very similar magnitudes to those found in recent empirical studies.

The intuition for these results follows from the fact that the cyclical nature of consumption implies that investors will require higher compensation for default risk in bad times. As a consequence firms find it especially costly to obtain debt financing during recessions. In our model, this makes it difficult for the young (new) firms to obtain funding for investment expenditures and depresses aggregate investment and output for a number of quarters thereafter. Risk premia in corporate bond markets are thus propagated into the real economy and this accounts for the predictive power of credit spreads for output and investment. Moreover these endogenous movements in risk premia play a key role in amplifying underlying macroeconomic conditions. Thus credit risk premium emerges as the common link between credit markets, equity markets and macroeconomic aggregates.

To summarize we find that, accounting for the premia in corporate bond prices allows the model to generate sizeable credit spreads. In addition, we also show how allowing for endogenous movements in credit markets also enhances the model's ability to match the observed conditional and unconditional movements in equity markets with a reasonable amount of aggregate volatility.

5 Conclusion

In this paper we propose a tractable general equilibrium asset pricing model with heterogeneous firms that links movements in stock and bond markets to macroeconomic activity. The model endogenously links movements in aggregate quantities such as investment and output to the prices of stocks and bonds. As a result movements in financial variables such as credit spreads and expected equity returns will forecast future economic activity. In our model these movements are largely driven by risk premia. In our equilibrium setting, endogenous default increases the volatility of consumption during recessions, thereby rendering the market price of risk sharply countercyclical. As a consequence, expected returns on stocks are higher in recessions, which are naturally anticipated by movements in credit spreads. Endogenous movements in credit markets allow our model to match the observed conditional and unconditional movements in stock market returns and credit spreads with a reasonable amount of aggregate volatility.

Table 1: **Calibration**

Parameter Values	
β	0.975
γ	10
σ	2
τ	0.2
ϕ	0.25
ρ_x	0.9995
σ_x	0.025
ρ_z	0.95
σ_z	0.35

This table reports parameter choices for our model. TO BE COMPLETED

Table 2: **Aggregate Moments**

Variable	Data	Model
Macro Moments		
$\sigma[\Delta_C]$	2.22	2.09
$\frac{\sigma[\Delta_C]}{\sigma[\Delta_Y]}$	0.51	0.44
$\frac{\sigma[\Delta_I]}{\sigma[\Delta_Y]}$	2.56	2.87
$\frac{I}{Y}$	0.19	0.21
Asset Pricing Moments		
$E[r^f]$	1.62	1.31
$\sigma[r^f]$	2.25	1.79
$E[r^e - r^f]$	6.18	5.92
$\sigma[r^e]$	16.54	15.26

This table reports unconditional sample moments generated from the simulated data of some key variables of the model. TO BE COMPLETED...

Table 3: **Credit Market Statistics**

Variable	Data	Model
Default rate	1.48	1.41
Credit Spread (10yr BAA-AAA)	101	97
Market Leverage	0.35	0.36

This table reports statistics related to credit markets and firms' capital structures. TO BE COMPLETED

Table 4: **Financing Over Business Cycle**

Correlation w/ GDP	Data	Model
Investment	0.37	0.72
Book leverage	0.13	0.87
Market leverage	-0.11	-0.69
Equity Issuance	0.32	0.31
Default rate	-0.33	-0.91
Credit Spread	-0.36	-0.77

This table reports business cycle properties of key financial variables in the model. TO BE COMPLETED

Table 5: **Forecasting Output Growth**

$\Delta Y_{t,t+k}$	Actual Data		
Horizon k	1 quarter	2 quarter	1 year
CS_t	-1.37	-1.47	-1.47
	(2.32)	(2.77)	(2.94)
$\Delta Y_{t,t+k}$	Simulated Data		
Horizon k	1 quarter	2 quarter	1 year
CS_t	-1.71	-2.14	-2.31
	(3.72)	(4.37)	(5.5)

This table reports regressions corresponding to Mueller (2009), table I. It regresses the k period ahead output log growth $\Delta Y_{t,t+k} = \log Y_{t+k} - \log Y_t$ on the value weighted aggregate credit spread at time t, CS_t. T-statistics are reported in parentheses below. These numbers are obtained by averaging the results from simulating the economy 1000 times over 57 years. The standard errors are corrected using Newey-West with 8 lags.

Table 6: **Forecasting Investment Growth**

$\Delta I_{t,t+k}$	Actual Data		
Horizon k	1 quarter	2 quarter	1 year
CS_t	-0.02	-0.03	-0.03
	(-3.59)	(-2.72)	(-1.64)
$\Delta I_{t,t+k}$	Simulated Data		
Horizon k	1 quarter	2 quarter	1 year
CS_t	-0.04	-0.04	-0.06
	(-2.28)	(-2.51)	(-2.16)

This table reports regressions corresponding to Lettau and Ludvigson (2002), table II. It regresses the k period ahead investment growth $\Delta I_{t,t+k} = I_{t+k} - I_t$ on the value weighted aggregate credit spread at time t, CS_t. T-statistics are reported in parentheses below. These numbers are obtained by averaging the results from simulating the economy 1000 times over 57 years. The standard errors are corrected using Newey-West with 8 lags.

Table 7: **Forecasting Stock Returns**

$R_{t,t+k}$	Actual Data		
Horizon k	1 month	1 quarter	1 year
CS_t	0.52	2.18	10.98
	(1.43)	(1.61)	(2.12)
$R_{t,t+k}$	Simulated Data		
Horizon k	1 month	1 quarter	1 year
CS_t	0.12	1.88	6.44
	(1.19)	(1.52)	(2.56)

This table reports regressions corresponding to Fama and French (2002), table III. It regresses the k period ahead value weighted aggregate stock market return $R_{t,t+k}$ on the value weighted aggregate credit spread at time t, CS_t. T-statistics are in parentheses. The numbers are obtained by averaging the results from simulating the economy 1000 times over 57 years. The standard errors are corrected using Newey-West with 8 lags.

Table 8: **Forecasting Return Volatility**

σ_{t+1}	Data	Model
CS_t	5.65	3.12
	(8.29)	(4.39)
σ_{t+1}	Model	Data
CS_t	0.05	0.03
	(0.02)	(0.01)

This table reports regressions corresponding to Schwert (1989), tables VII and VIII. It regresses the one month ahead conditional volatility σ_{t+1} of the value-weighted stock market return on i) the value weighted aggregate credit spread at time t, CS_t, and ii) on the value-weighted aggregate market leverage ratio in period t , $MLev_t$. Standard errors are in parentheses. The numbers are obtained by averaging the results from simulating the economy 1000 times over 57 years. The standard errors are corrected using Newey-West with 8 lags.