Crises and Liquidity in Over-the-Counter Markets*

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Abstract

In this paper we study the functioning of over-the-counter markets in times of crisis. Our model focuses on the key trading frictions of actual over-the-counter markets: investors search for dealers, and trades are bilateral, with quantities and prices determined by bargaining. We trigger a financial crisis by hitting investors' asset demands with a negative shock that lasts until some random recovery time. We show that dealers may provide liquidity to outside investors during the crisis by acting as counterparties in trades, and by accumulating asset inventories. However, in markets with severe trading frictions, even well capitalized dealers may fail to provide liquidity when it would be socially efficient for them to do so. In such cases, a capital injection by the government would hoarded by dealers, unless it is combined with market reforms that facilitate trades and erode dealers' bargaining power.

Keywords: liquidity, asset inventories, execution delays, search, bargaining

J.E.L. Classification: C78, D83, E44, G1

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1 Introduction

A crucial aspect of the recent financial turmoil has been the severe liquidity dry-up in over-the-counter (OTC) markets, and its role in the unfolding of the crisis. Many of the financial instruments at the core of the crisis— mortgage-backed securities, collateralized debt obligations, credit default swaps— are indeed traded over-the-counter through bilateral trades, outside of organized exchanges. Liquidity in those markets is provided on a voluntary basis by brokers-dealers, such as large investment banks. While liquidity provision seems inconspicuous in normal times, it has proved vulnerable recently, as revealed by the rapid drying-up of liquidity in markets for credit derivatives, commercial paper and municipal bonds alike.[1] The lack of resilience of OTC markets has been attributed to the inadequate regulation of these markets, as well as to the absence of "designated" marketmakers or dealers.[2] The lax regulations, it is argued, may have led dealers to commit too little marketmaking capital. In addition, even if dealers are well capitalized in normal times, they may have their credit lines severed during a crisis, and effectively withdraw from market marking. In extreme cases, they can even go bankrupt, as shown by the collapse of Lehman Brothers, one of the most active participants in OTC derivatives markets.

Analyzing the policies that aim at restoring liquidity in OTC market requires a framework describing explicitly the frictions and trading mechanisms in such markets. The objective of this paper is to propose such a model of liquidity provision in OTC markets and to study its implications for the functioning of markets and their allocative efficiency in times of crisis. A key insight of our analysis is to show that in some very illiquid and opaque markets, even well capitalized dealers may fail to provide liquidity when it would be socially efficient to do so. Injecting more capital would be ineffective because dealers would hoard the added "funding liquidity" instead of purchasing outside investors' assets on their own account. We show that, in order for capital injections to be effective, they should be combined with market reforms that facilitate trades (e.g., by promoting information disclosure, standardization of assets, electronic trading platforms). For instance, the difficulty to trade in OTC markets may be the symptom of some underlying adverse selection problem induced by unobserved counterparty risk.[3] Reducing this friction could mean, for instance, setting a Central Counterparty Clearing

[1] Brunnermeier (2009) provides a detailed description of the financial turmoil in 2007-2008 and its consequences for funding and market liquidity. See also "Subprime: Tentacles of a crisis" by Randall Rodd in Finance and Development (December 2007).

[2] Financial experts have also pointed out to the lack of transparency and defects in the infrastructure in support of OTC markets that typically do not occur in well-organized exchanges. See the report titled "Financial reform: A framework for financial stability" published on January 15, 2009, by the Group of Thirty, a panel of financial experts including Paul Volcker, Timothy Geithner, and Lawrence Summers.

[3] Hopenhayn and Werner (1996); Lester, Postelwaite, and Wright (2009a,b); Rocheteau (2009) show how information problems in bilateral meetings may reduce the probability of a trade, effectively worsening the search friction.

House (CCP), a new financially sound intermediary, providing safe collateral, or letting a public agency purchase assets, with a profit maximizing objective. Similarly, our model recommends combining capital injections with market reforms that erodes dealers' market power which could be achieved, for instance, by promoting greater price transparency.

The market setting we consider is based on the search-theoretic model of OTC markets by Duffie, Gârleanu, and Pedersen (2005) (DGP hereafter) and its extensions by Weill (2007) and Lagos and Rocheteau (2007, 2009). As in these models, we assume that outside investors cannot trade continuously in a centralized Walrasian market. Instead, they receive infrequent and random trading opportunities with intermediaries called "dealers," who are able to trade continuously with each others. While this search friction provides a natural description of bilateral trades in OTC markets, it also applies more broadly: it captures a wide range of frictions that make it more difficult to trade financial assets during crises – such as disruptions in communication systems, adverse selection problems with distressed intermediaries or outright dealers' failures. To create a crisis, we hit our search market with an aggregate negative shock that reduces investors' willingness to hold the asset. The crisis persists until some random time at which point investors receive the opposite shock and the economy recovers. Then, we determine the conditions under which well-capitalized dealers provide liquidity to outside investors, accumulating assets in their inventory during the crisis and unloading these assets when the economy recover.

Since our dealers are profit maximizing, their incentives to provide liquidity are driven by anticipated capital gains. Therefore, dealers are more likely to accumulate inventories if the initial price drop is large and followed by a quick rebound: i.e. if the crisis is severe enough and anticipated to be short-lived. We find that the amount of liquidity provided by well-capitalized dealers varies non-monotonically with the magnitude of two trading frictions: the search friction and the market power of dealers. Precisely, consider a spectrum of OTC markets ranging from those with very small frictions, for instance markets for Treasury securities or wholesale foreign exchange, to those with large trading frictions, such as some markets for subprime mortgage-backed securities. Then, under natural conditions, we find that dealers provide no liquidity in markets at both end of the spectrum, and some liquidity in markets lying in the the middle of the spectrum. When frictions are small, investors choose to take more extreme positions because they know that they can re-balance their asset holdings very quickly. Some investors supply so much liquidity to other investors, that dealers don't find it profitable to step in. At the other end of the spectrum when trading frictions are very large, investors become reluctant to hold extreme asset positions, because they anticipate these will be very difficult to unwind. In an equilibrium, all investors end up with a similar "average" asset position, do not re-balance much, and therefore do not demand much liquidity from dealers. We show that, because of this endogenous negative shift in liquidity demand, in equilibrium dealers do not provide any liquidity. As mentioned above, the main policy implication of this finding is that injecting capital may prove ineffective in markets with very large friction: dealers would hoard it and would not provide liquidity by accumulating asset inventories.

Our work belong to the recent literature that studies trading frictions in asset markets.[4] DGP focused on steady states and their analysis is silent about liquidity provision by dealers. Weill (2007) studies the timing of liquidity provision by dealers along the transitional path in a dynamic version of DGP. Weill and the literature spurred by DGP, however, keep the framework tractable by imposing a restriction on asset holdings, namely, that investors can only hold either 0 or 1 unit of the asset. Lagos and Rocheteau (2007, 2009) study a version of DGP where investors can hold unrestricted asset positions, creating a non-trivial extensive margin in investors' demand for liquidity. In this paper, we go beyond previous studies by studying the out-of-steady-state dynamics induced by aggregate shocks while allowing both dealers and investors to hold unrestricted asset positions. This generates new implications for the demand and the supply of liquidity that are relevant for the study of crisis. For instance we find that, in contrast to Weill (2007), even well-capitalized dealers may sometimes *not* find it in their interest to provide liquidity during the crisis when it would be socially optimal to do so, implying that capital injection may be ineffective. Weill (2007) also assumed that, after the initial shock, the recovery path was deterministic. While this is a useful simplifying assumption, it is clearly at odd with the considerable uncertainty faced by market participant during crises. We go beyond this analysis by making the recovery a random event. This makes the model more realistic and generates the new implication that our rational dealers find it optimal to buy asset while the market price continues to decline, and re-sell them while the market price continues to go up.

2 The environment

Time is continuous, runs forever, and is indexed by $t \geq 0$. There is one asset and one perishable good, which we use as a numéraire. The asset is durable, perfectly divisible and in fixed supply, $A > 0$. The numéraire is produced and consumed by all agents. The economy is made up of two types of infinitely-lived agents who discount the future at the same rate $r > 0$: a unit measure of investors and a unit measure of dealers.

The instantaneous utility function of an investor is $\zeta(t)u_i(a)+c$, where $a \geq 0$ represents the

[4] Examples include, Gârleanu (2009); Vayanos and Weill (2008); Longstaff (2009). Conceptually, our analysis is also related to the inventory models of Stoll (1978) and Ho and Stoll (1983) (see Chapter 2 in O'Hara, 1995, for a review of this earlier market-microstructure literature).

investor's asset holdings, c is the net consumption of the numéraire good ($c < 0$ if the investor produces more than he consumes), $i \in \{1, ..., I\}$ indexes an idiosyncratic preference shock, and $\zeta(t)$ represents an aggregate preference shock. The utility function $u_i(a)$ is strictly increasing, concave, continuously differentiable and satisfies the Inada condition that $u_i'(0) = \infty$. We also assume that it is either bounded below or above. Investors receive idiosyncratic preference shocks that occur at Poisson arrival times with intensity $\delta > 0$. When the preference shock hits, the investor draws preference type i with probability π_i. These preference shocks capture the notion that investors value the services provided by the asset differently over time, and will generate a need for them to periodically change their asset holdings.[5] At time zero the distribution of investors across the preference types $\{1, ..., I\}$ is at its steady state $\{\pi_i\}_{i=1}^I$.

We trigger a financial crisis with an aggregate preference shock. Namely, we assume that $\zeta(t) = \theta < 1$ for all $t \in [0, T_\rho)$ and $\zeta(t) = 1$ for all $t \geq T_\rho$, where T_ρ is an exponentially distributed random variable with mean $1/\rho$, independent from everything else.[6] A small θ indicates that the crisis is severe, and a small ρ that it is expected to be long-lived. Although assuming a preference shock is admittedly a reduced form model of a crisis, it is in the spirit of the aggregate endowment shocks that are commonly used in the literature (see, e.g. Grossman and Miller, 1988) and it admits several reasonable interpretation: a shock the riskiness (or "toxicity") of the asset, or a "flight to liquidity" (Longstaff, 2004).

To capture the intuitive notion that dealers are not the final holders of the asset, we assume that their instantaneous utility is c, i.e., that they derive no utility from holding the asset. We assume that dealers can continuously buy and sell the asset in an inter-dealer market, at price $p(t)$. Investors, on the other hand, can only trade periodically and through a dealer. Specifically, we assume that investors contact a randomly chosen dealer at Poisson arrival times with intensity $\alpha > 0$. Once the investor and the dealer have made contact, they negotiate the quantity of assets that the dealer will acquire (or sell) in the inter-dealer market on behalf of the investor and the intermediation fee that the investor will pay the dealer for

[5] Our specification associates a certain utility to the investor as a function of his asset holdings. This is a feature that we have borrowed from DGP. The utility the investor gets from holding a given asset position could be simply the value from enjoying the asset itself, as would be the case for real assets such as houses or physical capital. As yet another possibility, one could adopt the preferred interpretation of DGP, namely that $u_i(a)$ is in fact a reduced-form utility function that stands in for the various reasons why investors may want to hold different quantities of the asset, such as differences in liquidity needs, financing or financial-distress costs, correlation of asset returns with endowments (hedging needs), or relative tax disadvantages. By now, several papers that build on the work of DGP have formalized the "hedging needs" interpretation. Examples include Duffie, Gârleanu, and Pedersen (2007), Gârleanu (2009) and Vayanos and Weill (2008).

[6] Although we follow the spirit of Grossman and Miller (1988), we depart from their model in two ways. First, in our model, the length of the crisis is stochastic, so dealers' uncertainty about the recovery will shape their incentive to provide liquidity. Second, in Grossman and Miller, dealers provide liquidity in order to share risk with outside investors while, in the present model, dealers have no such direct utility motive for holding assets. They derive some value indirectly because they are continuously present in the market, so they can "time the market" better than outside investors. This leads them to hold inventory and, in the aggregate, speeds up the allocation of assets to their final holders.

his services. After completing the transaction, the dealer and the investor part ways. The trading arrangement is illustrated in Figure 1.

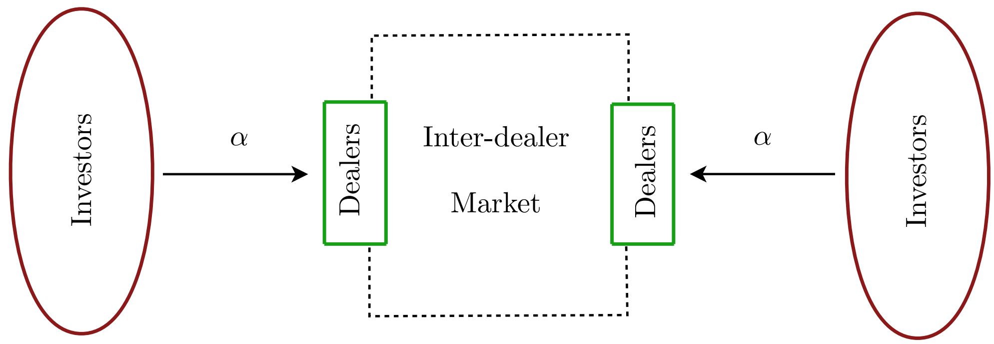

Figure 1: Trading arrangement

3 Equilibrium

We characterize an equilibrium in two steps: we first solve for the equilibrium after the recovery time, for every possible T_ρ. Then, we solve for the equilibrium during the crisis, before T_ρ realizes.

3.1 The path to recovery

In this section we describe the path of the economy following T_ρ. The aggregate preference component is $\zeta(t) = 1$ for all $t \geq T_\rho$. We take as given the realization of T_ρ, and the dealers' inventories at the time where the economy recovers, $A^d(T_\rho)$.

3.1.1 The terms of trades in bilateral meetings

Consider a meeting at time $t \geq T_\rho$ between a dealer who is holding inventory a_d and an investor of type i who is holding inventory a. Let a' denote the investor's post-trade asset holding and ϕ be the intermediation fee.[7] The pair (a', ϕ) is taken to be the Nash solution of a bargaining problem where the dealer has bargaining power $\eta \in [0, 1]$. Let $V_i(a, t)$ denote the value (maximum attainable expected discounted utility) of an investor with preference type i who is holding a quantity of asset a at time $t \geq T_\rho$. The investor's gain from trade is

$$V_i(a', t) - V_i(a, t) - p(t)(a' - a) - \phi. \tag{1}$$

[7] In our formulation we assume that the investor pays the dealer a fee. However, the bargaining problem can be readily reinterpreted as one in which the dealer pays the investor a bid price which is lower than the market price if the investor wants to sell, and charges an ask price which is higher than the market price if the investor wants to buy. See Lagos and Rocheteau (2009) for details.

Analogously, let $W(a_d, t)$ denote the value of a dealer who is holding inventory a_d at time $t \geq T_\rho$. Then, the utility of the dealer is $W(a_d, t) + \phi$ if an agreement (a', ϕ) is reached and $W(a_d, t)$ in case of disagreement, so the dealer's gain from trade is equal to the fee, ϕ.[8] The outcome of the bargaining is given by

$$[a_i(t), \phi_i(a,t)] = \arg\max_{(a',\phi)} [V_i(a',t) - V_i(a,t) - p(t)(a'-a) - \phi]^{1-\eta}\phi^{\eta}.$$

Hence, the investor's new asset holding solves

$$a_i(t) = \arg\max_{a'} \left[V_i(a',t) - p(t)a'\right], \tag{2}$$

and the intermediation fee is

$$\phi_i(a,t) = \eta \left\{V_i[a_i(t), t] - V_i(a,t) - p(t)[a_i(t) - a]\right\}. \tag{3}$$

According to (2), the investor's post-trade asset holding is the one he would have chosen if he were trading in the asset market himself, rather than through a dealer. According to (3), the intermediation fee is set so as to give the dealer a share η of the gains associated with readjusting the investor's asset holdings.[9]

3.1.2 The dealers' problem

The value function of a representative dealer who is holding asset position a at time t solves

$$W(a,t) = \max_{q(s)} \left\{ - \int_t^{\infty} e^{-r(s-t)} p(s) q(s) ds \right\} + \Phi(t), \tag{4}$$

subject to the law of motion $\dot{a}_d(s) = q(s)$, the short-selling constraint $a_d(s) \geq 0$, and the initial condition $a_d(t) = a$. Here, $a_d(s)$ represents the stock of assets that the dealer is holding and $q(s)$ is the quantity that he trades for his own account at time s. The dealer gets utility $-p(s)q(s)$ from changing this inventory. The function $\Phi(t)$ is the expected present discounted value of future intermediation fees from time t onward which, by (3), is independent of the dealer's asset holdings. This formulation makes it clear that dealers trade assets in two ways: continuously, in the competitive market, or at random times, in bilateral negotiations with investors. Since dealers have linear preferences and they can trade

[8] The outcome of the bilateral trade does not affect the dealer's continuation payoff, $W(a_d, t)$, because he has continuous access to the asset market and his trades are executed instantaneously. The dealer may fill an investor's order partially or in full by trading out of, or for his own inventory of the asset. A dealer following an optimal trading plan must be indifferent between using his inventories or not to execute a trade because he has continuous access to the asset market and all the transactions he is with involved in are instantaneous.

[9] Our choice of notation for the bargaining solution in (2) and (3) emphasizes the fact that the terms of trade depend on the investor's preference type but are independent of the dealer's inventories. In addition, the investor's post-trade asset holding is independent of his pre-trade holding, while the intermediation fee is not.

instantaneously and continuously in the competitive asset market, their optimal choice of asset holdings is independent from what happens in bilateral negotiations with investors. The following lemma describes the solution dealer's inventory accumulation problem which is in the first term on the right-hand side of (4).

Lemma 1. *Suppose that the price path $p(s)$ is differentiable and satisfies the no-bubble condition $\lim_{s\to\infty} e^{-rs}p(s) = 0$. Then, a bounded inventory path, $a_d(s)$ with initial condition $a_d(t) = a$ solves the dealer's problem, (4), if and only if for all $s > t$:*

$$\dot{p}(s) - rp(s) \leq 0 \text{ with equality if } a_d(s) > 0. \tag{5}$$

Several comments are in order. First the assumption of differentiability and no-bubble is only made to simplify the exposition: in Lagos, Rocheteau, and Weill (2007) we show that these two conditions must, in fact, hold in any equilibrium. Second, the Lemma restricts attention to bounded inventory paths because this property must also hold in equilibrium. Indeed, a group of agents can hold an unbounded positive position only if some other group hold the opposite negative one, which is ruled out by the short-selling constraint. Then, the "only if" part of the Lemma provides restrictions on the equilibrium price path, given any bounded solution $a_d(t)$ of the dealers' problem. The "if" part of the Lemma is a standard sufficient condition for "speculator" optimality: a dealer holds positive inventory if the flow cost of buying the asset, $rp(s)$, is equal the the capital gain, $\dot{p}(s)$, and no inventory otherwise.

3.1.3 The investor's problem

We now proceed with an analysis of the investor's problem. The value function corresponding to an investor with preference type i who is holding a assets at time t, $V_i(a,t)$, satisfies

$$\begin{aligned} V_i(a,t) &= \mathbb{E}_i\Big[\int_t^T e^{-r(s-t)}u_{k(s)}(a)ds + \\ &\quad e^{-r(T-t)}\{V_{k(T)}[a_{k(T)}(T),T] - p(T)[a_{k(T)}(T) - a] - \phi_{k(T)}(a,T)\}\Big], \end{aligned} \tag{6}$$

where T denotes the next time the investor meets a dealer, and $k(s) \in \{1,...,I\}$ denotes the investor's preference type at time s. The expectations operator, $\mathbb{E}_i$, is taken with respect to the random variables T and $k(s)$, and is indexed by i to indicate that the expectation is conditional on $k(t) = i$. Over the interval of time $[t,T]$ the investor holds a assets and enjoys the discounted sum of the utility flows associated with this holding a (the first term on the right-hand side of (6)). The length of this time interval, $T - t$, is an exponentially distributed random variable with mean $1/\alpha$. The flow utility is indexed by the preference type of the investor, $k(s)$, which follows a compound Poisson process. At time T the investor contacts a random dealer and readjusts his holdings from a to $a_{k(T)}(T)$. In this event the dealer purchases a quantity $a_{k(T)}(T) - a$ of the asset in the market (or sells if this quantity is negative) at price $p(T)$ on behalf of the investor, and the investor pays the dealer an intermediation fee, $\phi_{k(T)}(a, T)$. Both the fee and the asset price are expressed in terms of the numéraire good.

Substituting the terms of trade (2) and (3) into (6), it is apparent that from the investor's standpoint, the stochastic trading process and the bargaining solution are payoff-equivalent to an alternative trading mechanism in which the investor has all the bargaining power in bilateral negotiations with dealers, but he only gets to meet dealers according to a Poisson process with arrival rate $\kappa = \alpha(1-\eta)$. Consequently, we can rewrite (6) as

$$V_i(a,t) = \mathbb{E}_i\left[\int_t^{\tilde{T}} u_{k(s)}(a)\, e^{-r(s-t)}\, ds + e^{-r(\tilde{T}-t)}\{p(\tilde{T})a + \max_{a'}[V_{k(\tilde{T})}(a',\tilde{T}) - p(\tilde{T})a']\}\right], \quad (7)$$

where the expectations operator, $\mathbb{E}_i$, is now taken with respect to the random variables $\tilde{T}$ and $k(s)$, where $\tilde{T} - t$ is exponentially distributed with mean $1/\kappa$.

After subtracting $p(t)a$ from (7) and ignoring all the terms that do not depend on the asset holding a, we find that the problem of an investor with preference shock i, who gains access to the market at time t, consists of choosing $a \geq 0$ in order to maximize

$$\mathbb{E}_i\left[\int_t^{\tilde{T}} u_{k(s)}(a) e^{-r(s-t)}\, ds - \left(p(t) - e^{-r(\tilde{T}-t)} p(\tilde{T})\right) a\right]. \quad (8)$$

Intuitively, the investor chooses his asset holdings in order to maximize the present value of his utility flow net of the cost of purchasing the asset at time t and re-selling a the next time $\tilde{T}$ when he can readjust his holdings. The following lemma offers a simpler, equivalent formulation of the investor's problem.

Lemma 2. *Let*

$$U_i(a) = \frac{(r+\kappa)\, u_i(a) + \delta \sum_{j=1}^{I} \pi_j u_j(a)}{r+\kappa+\delta} \quad (9)$$

$$\xi(t) = (r+\kappa)\left(p(t) - \int_0^\infty \kappa e^{-(r+\kappa)s} p(t+s) ds\right), \quad (10)$$

and assume that $p(t)e^{-rt}$ is decreasing and goes to zero as t goes to infinity. Then a bounded process $a(t)$ solves the investor's problem if and only if, when the investor contacts the market with current type i,

$$a(t) = a_i(t) \text{ where } U_i'\left[a_i(t)\right] = \xi(t). \quad (11)$$

The assumption that $p(t)e^{-rt}$ is decreasing is without loss of generality, because it will be true in an equilibrium: otherwise if there were two times $t_1 < t_2$ such that $p(t_1) < e^{-r(t_2-t_1)}p(t_2)$, then a dealer could make unbounded profit by purchasing at t_1 and re-selling at t_2. The assumption that $p(t)e^{-rt} \to 0$ is the no-bubble condition we already used in Lemma 1.

Intuitively, $U_i(a)$ is the flow expected utility the investor enjoys from holding a assets until his next opportunity to rebalance his holdings, and $\xi(t)$ is an investor's effective cost of holding the asset during the inter-contact time period: the purchasing price minus the expected discounted resale value of the asset, all expressed in flow term. Differentiating (10), we can reexpressed the relationship between $\xi(t)$ and $p(t)$ as

$$\dot{p}(t) - rp(t) = \frac{\dot{\xi}(t)}{r+\kappa} - \xi(t). \tag{12}$$

From (12) the dealer's first-order condition, (5), can be rewritten as

$$\frac{\dot{\xi}(t)}{r+\kappa} - \xi(t) \leq 0 \text{ with an equality if } a_d(t) > 0. \tag{13}$$

Equations (11) and (13) illustrate the main differences between dealers and investors in our setup. Relative to investors, dealers enjoy no direct utility from holding the asset but they get an extra return captured by $\dot{\xi}(t)/(r+\kappa)$. This reflects a dealer's ability to make capital gains by exploiting his continuous access to the asset market.

3.1.4 The equilibrium recovery path

Given the solutions to the investors' and dealers' problem, we are now ready to study the determination of the asset price. Since each investor faces the same probability to access the market irrespective of his asset holdings, and since these probabilities are independent across investors, we appeal to the law of large numbers to assert that the flow supply of assets by investors is $\alpha[A - A_d(t)]$, where $A_d(t)$ is the aggregate stock of assets held by dealers. The measure of investors with preference shock i who are trading in the market at time t is $\alpha\pi_i$, where π_i is the ergodic measure of investors with preference type i. Therefore, the investors' aggregate demand for the asset is $\alpha\sum_{i=1}^{I}\pi_i a_i(t)$, and the net supply of assets by investors is $\alpha[A - A_d(t) - \sum_{i=1}^{I}\pi_i a_i(t)]$. The net demand from dealers is $\dot{A}_d(t)$, the change in their inventories. Therefore, market clearing requires

$$\dot{A}_d(t) = \alpha\left\{A - A_d(t) - \textstyle\sum_{i=1}^{I}\pi_i U_i'^{-1}[\xi(t)]\right\}, \tag{14}$$

after substituting in investors' first-order condition (11). This market-clearing condition determines the inventory path given some $\xi(t)$. Aggregating (13) across all dealers, we find the condition:

$$\dot{\xi}(t) - (r+\kappa)\xi(t) \leq 0 \text{ with an equality if } A_d(t) > 0 \tag{15}$$

An equilibrium following the recovery time T_ρ is a solution $[A_d(t), \xi(t); t \geq T_\rho]$ to the system of differential equations (14) and (15), with given initial condition $A_d(T_\rho)$. While we do not include $p(t)$ in the definition of an equilibrium, it can be recovered from the following properties of any equilibrium price path. Indeed, with the no-bubble condition $\lim_{t\to} e^{-rt}p(t)$, equation (12) implies

$$p(t) = \int_t^\infty e^{-r(s-t)} \left[\xi(s) - \frac{\dot{\xi}(s)}{r+\kappa}\right] ds. \tag{16}$$

In a steady-state, $\xi(t) = \bar{\xi}$ and $\dot{\xi}(t) = 0$ so that (15) hold with a strict inequality and $A_d(t)$ must be equal to zero. Plugging $A_d(t) = 0$ into equation (14), we find that the steady-state $\bar{\xi}$ is the unique solution of

$$\sum_{i=1}^{I} \pi_i U_i'^{-1}(\bar{\xi}) = A. \tag{17}$$

In addition, with equation (12) we find that the steady-state price solves $r\bar{p} = \bar{\xi}$. Turning to the equilibrium path towards the steady state, we show:

Proposition 1 (The equilibrium path to recovery)**.** *There is a unique equilibrium path* $\{\xi(t), A_d(t) : t \geq T_\rho\}$ *and it is such that:*

(a) For all $t \in (T_\rho, T]$*,*

$$\xi(t) = \bar{\xi} e^{-(r+\kappa)(T-t)} \tag{18}$$

$$A_d(t) = e^{-\alpha(t-T_\rho)} A_d(T_\rho) + \alpha \int_{T_\rho}^t e^{-\alpha(t-s)} \left[A - \sum_{i=1}^{I} \pi_i U_i'^{-1}[\xi(s)]\right] ds, \tag{19}$$

where $T < \infty$ *is the unique solution to* $A_d(T) = 0$*.*

(b) For all $t \geq T$*,* $\{\xi(t), A_d(t)\} = (\bar{\xi}, 0)$*.*

According to (18), the investor's effective cost of holding the asset, $\xi(t)$, increases at rate $r+\kappa$ while dealers hold inventories, meanwhile according to (19), the stock of assets held by dealers decreases monotonically until it is fully depleted at time T. The condition $A_d(T) = 0$ provides a relationship between the effective cost of holding the asset at the recovery time, $\xi(T_\rho) = \bar{\xi} e^{-(r+\kappa)(T-T_\rho)}$, and dealers' initial inventories, $A_d(T_\rho)$. We represent this relationship by the function $\psi(A)$ such that $\xi(T_\rho) = \psi[A_d(T_\rho)]$. Notice that $\psi'(A) < 0$, so $\xi(T_\rho)$ is decreasing in $A_d(T_\rho)$, and $\psi(0) = \bar{\xi}$. Intuitively, the larger the stock of inventories that dealers are holding at the time of the recovery, the lower the effective cost of holding the asset at the recovery time, and the longer it will take to deplete dealers' inventories once the recovery has occurred.

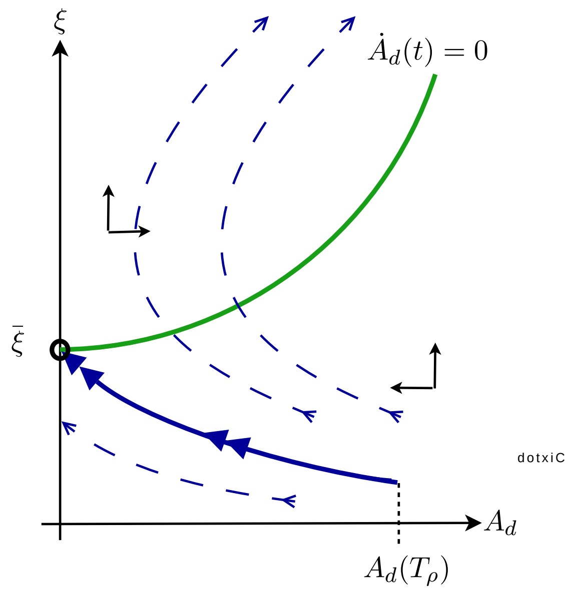

Figure 2: Phase diagram for the equilibrium recovery path

Figure 2 shows the phase diagram of the dynamic system $\{A_d(t), \xi(t)\}$ following the recovery. From (14) we see that the A_d-isocline is upward-sloping and intersects the vertical axis at the steady-state point. The equilibrium trajectory of the economy is indicated in the figure by arrows along the saddle-path, namely, $\xi(t) = \psi\,[A_d(t)]$. The initial condition $A_d(T_\rho)$ determines the starting point on the saddle path. The trajectories marked with dotted lines that do not follow the saddle path are solutions to the differential equations (14) and (15) but they either fail to satisfy the no-bubble condition or the requirement that the equilibrium path $\xi(t)$ be continuous.

3.2 The crisis

In this section, we analyze the economy over the initial crisis period, $t < T_\rho$. The value functions and the asset price following the recovery time, $t \geq T_\rho$, are characterized in Section 3.1. We parameterize these functions by the time of the recovery, T_ρ and denote them by $V_i(T_\rho, t)$, $W(T_\rho, t)$, and $p(T_\rho, t)$ respectively. For all value functions and prices during the crisis, $t < T_\rho$, we add the superscript "C".

3.2.1 Dealers and Investors' problem

At any time t before the recovery has occurred, the dealer solves

$$\max_{q(s)} \mathbb{E}\left[\int_t^{T_\rho} -e^{-r(s-t)}p^C(s)q(s)ds + e^{-r(T_\rho - t)}W\left[a_d\left(T_\rho\right), T_\rho, T_\rho\right]\right], \tag{20}$$

subject to $\dot{a}_d(s) = q(s)$, $a_d(s) \geq 0$ for all $s \geq t$ and the initial condition $a_d(t) = a$. The optimality conditions become:

Lemma 3. *Suppose that the price path $p^C(s)$ during the crisis is differentiable and satisfies the no-bubble condition. Then, a bounded inventory path, $a_d^C(s)$ with initial condition $a_d^C(t) = a$ solves the dealer's problem if and only if for all $s > t$:*

$$\dot{p}^C(s) + \rho\left(p(s,s) - p^C(s)\right) - rp^C(s) \leq 0 \text{ with equality if } a_d^C(s) > 0 \tag{21}$$

From Lemma 3 we see that the flow of profit of dealers during the crisis has three components: the opportunity cost of holding the asset, $rp^C(s)$, the capital gain while the economy remains in the crisis state, $\dot{p}^C(s)$, and the expected capital gain $\rho(p(s,s) - p^C(s))$ if the economy recovers with Poisson intensity ρ.

Following the same steps as in the previous section, one shows easily that an investor who gains access to the market at time t with preference type i, chooses $a \geq 0$ in order to maximize

$$\mathbb{E}_i\left[\int_t^{\tilde{T}} \left(\theta + (1-\theta)\mathbb{I}_{\{s \geq T_\rho\}}\right) u_{k(s)}(a)e^{-r(s-t)}\, ds - \left(p(t) - e^{-r(\tilde{T}-t)}p(\tilde{T})\right) a\right]. \tag{22}$$

where

$$p(\tilde{T}) = \mathbb{I}_{\{\tilde{T} < T_\rho\}}p^C(\tilde{T}) + \mathbb{I}_{\{\tilde{T} \geq T_\rho\}}p(T_\rho, \tilde{T}).$$

There are only two differences relative to program (8) in the previous section: first the period utility for the asset is scaled down by θ whenever $s \leq T_\rho$, and second an investor expects that the economy may have recovered by the time $\tilde{T}$ when he is able to re-sale the asset. As before, the investor's problem can be reformulated simply as:

Lemma 4. *Let $u_i^C(a) \equiv \frac{r+\kappa}{r+\kappa+\rho}\theta u_i(a) + \frac{\rho}{r+\kappa+\rho}U_i(a)$ and*

$$U_i^C(a) = \frac{(r+\kappa+\rho)\, u_i^C(t) + \delta\sum_{j=1}^I \pi_j u_j^C(a)}{r+\kappa+\rho+\delta} \tag{23}$$

$$\xi^C(t) = (r+\kappa)\left[p^C(t) - \int_t^\infty \kappa e^{-(r+\kappa)(\tau_\kappa - t)}\left(e^{-\rho(\tau_\kappa - t)}p^C(\tau_\kappa) + \int_0^{\tau_\kappa} \rho e^{-\rho(\tau_\rho - t)}p(\tau_\rho, \tau_\kappa)\, d\tau_\rho\right) d\tau_\kappa\right] \tag{24}$$

Assume that $\mathbb{E}_t\left[p(s)e^{-r(s-t)}\right]$ *is decreasing in* s *and that* $p(s)$ *satisfies the no-bubble condition. Then a bounded process* $a^C(t)$ *solves the investor's problem if and only if, when the investor contacts the market with current type* i*,*

$$a^C(t) = a_i^C(t) \text{ where } U_i^{C\prime}\left[a_i^C(t)\right] = \xi^C(t). \tag{25}$$

This is the natural counterpart of Lemma 2. As before $\mathbb{E}_t\left[p(s)e^{-r(s-t)}\right]$ has to be decreasing in an equilibrium, otherwise the dealer's problem would not have a bounded solution. Note that the formula for $U_i^C(a)$ is similar to the one for $U_i(a)$, except that the period utility $u_i(a)$ is replaced by

$$\frac{r+\kappa}{r+\kappa+\rho}\theta u_i(a) + \frac{\rho}{r+\kappa+\rho}U_i(a).$$

Intuitively, the period utility that is relevant for portfolio choice has to be adjusted for the preference shock θ while keeping in mind that, before the next contact time, the recovery may arrive with Poisson intensity ρ, in which case the flow continuation utility becomes $U_i(a)$. The formula for $\xi^C(t)$, which is also similar, takes into account the expected capital gain that will be realized the next time the investor gains access to the market, which may be before or after the economy recovers. As before the last two terms on the right-hand side of (24) represent the expected resale price of the asset.

3.2.2 The equilibrium path

After differentiating condition (24) we find that:

$$-rp^C(t) + \dot{p}^C(t) + \rho\left[p(t,t) - p^C(t)\right] = -\xi^C(t) + \frac{\dot{\xi}^C(t) + \rho\left[\xi(t,t) - \xi^C(t)\right]}{r+\kappa}, \tag{26}$$

where

$$\xi\left(T_\rho, t\right) = \psi\left[A_d(T_\rho)\right]e^{(r+\kappa)(t-T_\rho)}. \tag{27}$$

Plugging (26) back into the dealer's first-order condition (20) and aggregating, we obtain:

$$\left\{\dot{\xi}^C(t) + \rho\psi[A_d^C(t)] - (r+\kappa+\rho)\,\xi^C(t)\right\}A_d^C(t) \leq 0 \text{ with an equality if } A_d^C(t) > 0. \tag{28}$$

Now the market clearing condition is

$$\dot{A}_d^C(t) = \alpha\left\{A - A_d^C(t) - \sum_{i=1}^{I}\pi_i U_i^{C\prime-1}[\xi^C(t)]\right\}, \tag{29}$$

which is the same as before except for the fact that $U_i'(a)$ is replaced by $U_i^{C\prime}(a)$. We can now define an equilibrium during the crisis to be a $\left\{\xi^C(t), A_d^C(t)\right\}$, satisfying (28) and (29).

One easily shows that the system (28) and (29) has a unique steady state $(\bar{\xi}^C, \bar{A}_d^C)$ characterized by

$$\bar{\xi}^C \geq \frac{\rho}{r+\kappa+\rho}\psi(\bar{A}_d^C) \text{ with an equality if } \bar{A}_d^C > 0 \quad (30)$$

$$A = \bar{A}_d^C + \sum_{i=1}^{I} \pi_i U_i^{C\prime -1}(\bar{\xi}^C) \quad (31)$$

Analyzing the system (28) and (29) of ODEs yields:

Proposition 2 (The equilibrium path during the crisis)**.** *Assume that* $\bar{A}_d^C > 0$ *and suppose* $A_d^C(0) = 0$. *Then, the equilibrium path is unique, starts with* $\xi^C(0) > \bar{\xi}^C$, *and converges monotonically to the steady state,* $\{\bar{\xi}^C, \bar{A}_d^C\}$.

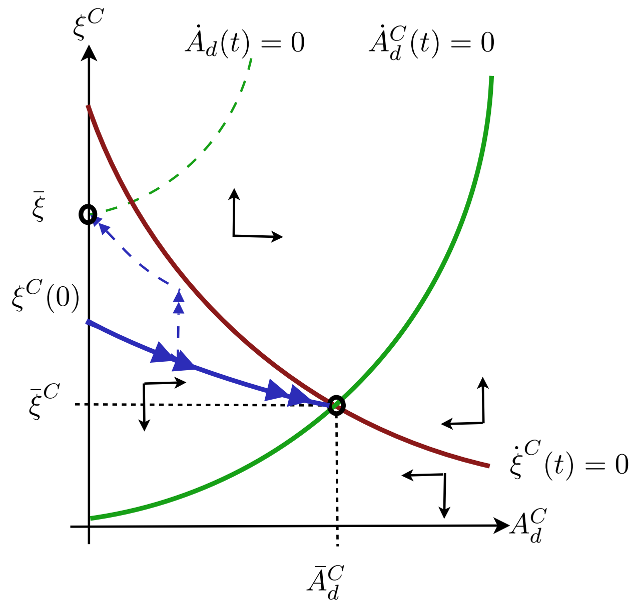

Figure 3: Phase diagram

These properties can be intuitively derived using the phase diagram of Figure 3. The isocline $\dot{A}_d^C = 0$ during the crisis is represented by the upward slopping plain curve. Note that it is located to the right of the recovery isocline $\dot{A}_d = 0$, represented by the upward slopping dashed curve. This is because, for any given ξ, investors demand less assets so dealers need to hold more of the asset in order to clear the market. The isocline $\dot{\xi}^C = 0$ is represented by the downward-sloping plain curve. As it is standard, the sign of the derivatives $\dot{A}_d^C(t)$ and $\dot{\xi}^C(t)$ in various regions of the plane are indicated by horizontal and vertical arrows. The Proposition shows that, given the initial condition $A_d^C(0) = 0$, there is a unique saddle-path during the crisis, represented in the Figure by the plain curve with double arrows, leading to the steady state, $\left(\bar{A}_d^C, \bar{\xi}^C\right)$.

The equilibrium unfolds as follows. The economy starts at $A_d^C(0) = 0$, and at the time of the crisis, $\xi^C(t)$ jumps down to the saddle-path leading to $(\bar{A}_d^C, \bar{\xi}^C)$. The economy then evolves along the crisis saddle-path until the random recovery shock occurs. In the meantime, along this path, dealers' inventories increase and $\xi^C(t)$ decreases. At the random recovery time, the system jumps to the recovery saddle-path leading to $(0, \bar{\xi})$. This is the saddle path of Proposition 1 and it is indicated in the figure by the dashed curve with double arrows. At the time the recovery shock occurs, the cost $\xi(t)$ of holding the asset jumps up, and dealers begin selling their inventories gradually until they are completely depleted. We summarize these findings in:

Proposition 3 (Crisis and recovery dynamics)**.** *Suppose $\bar{A}_d^C > 0$. At the time of the crisis, $t = 0$, the price $p(t)$ jumps down. Then, as the crisis unfold, $t \in (0, T_\rho)$, dealers' inventories increase towards $\bar{A}_d^C$ while the price continues to decrease. At the time of the recovery, $t = T_\rho$, the price jumps up. During the recovery, $t \in [T_\rho, \infty)$, dealers' inventories decrease towards zero, and the price continues to increase towards $\bar{\xi}$.*

While analysis of the previous paragraph showed that the effective cost $\xi^C(t)$ is decreasing during the crisis, the present Proposition shows that this is also true for the price, $p^C(t)$. Thus, our profit maximizing and atomistic dealers find it optimal to buy in a down market (see Ross and Sofianos, 1998, for evidence of such behavior). One may wonder why they don't prefer to wait and buy at a lower price: this is because waiting may cause them to "miss" the capital gain of the recovery jump. In fact, there is a simple equilibrium intuition why the price has to fall during the crisis: dealers anticipate that, as they accumulate inventories, it will take them longer and longer to unwind their asset positions. Thus, they have to be compensated by a larger and larger capital gain, implying that the price has to fall by more and more. Therefore, we obtain a divergence effect similar to that of Kondor (2009), but without having to make the assumption that dealers' capital is limited.

Finally, in terms of the normative properties of the environment, the socially efficient allocation is the one that maximizes the sum of the utilities of investors and dealers. One can show that a necessary and sufficient condition for the equilibrium allocation to be socially efficient is that dealers have no bargaining power, $\eta = 0$. The intermediation fee, ϕ, paid by investors to dealers is a transfer that does not affect social welfare directly. However, it does affect investors' asset positions. Remember that if $\eta > 0$, from the view point of investors, it is as if they live in an economy with direct access to the market at the arrival rate $\kappa = \alpha(1 - \eta) < \alpha$. Because investors perceive that they have fewer opportunities to readjust their asset holdings, they choose asset positions that are closer to their average preferences and, as a consequence, the distribution of asset holdings becomes more concentrated. This in turn reduces investors' demand for liquidity when the crash occurs, and hence it affects dealers' decision to accumulate assets. In Lagos, Rocheteau, and Weill (2007) we establish formally the following proposition.

Proposition 4. *The equilibrium is constrained efficient if and only if* $\eta = 0$.

Given that $\eta = 0$, it should also be emphasized that social welfare is (weakly) monotone increasing in α. This is so because any allocation that can be implemented in a low-α economy can also be implemented in a high-α economy.

4 When do dealers provide liquidity?

In our economy dealers face no liquidity constraints and have permanent access to a credit line allowing them to borrow at the interest rate r. Would such unconstrained dealers have incentives to provide liquidity in a distressed market by accumulating asset inventories?

4.1 General conditions

Perhaps the easiest way to answer this question is to derive the conditions under which dealers *do not* provide liquidity. We know from Lemma 3 that dealers do not find it optimal to provide liquidity during the crisis if and only if:

$$\frac{\rho\left[p(t,t) - p^C(t)\right] + \dot{p}^C(t)}{p^C(t)} < r. \tag{32}$$

From (32) the expected return obtained by purchasing the asset at time t and re-selling it at time $t + dt$ must be less than the rate at which dealers can borrow funds. If (32) holds, then dealers' aggregate inventory position is equal to zero during the crisis, $A_d^C(t) = \dot{A}_d^C(t) = 0$. Together with (29), this implies that $\xi^C(t)$ is constant and equal to $\bar{\xi}_0^C$ solution to

$$\sum_{i=1}^{I} \pi_i U_i^{C\prime -1}\left(\bar{\xi}_0^C\right) = A.$$

At the recovery time the economy jumps to its long-run steady state, $\xi(t,t) = \bar{\xi}$ and

$$p(t,t) = \bar{p} = \bar{\xi}/r, \tag{33}$$

for all $t \leq T_\rho$. Thus, using (26), we obtain that the asset price during the crisis, $p^C(t)$, is constant and equal to:

$$p^C(t) = \bar{p}_0^C = \frac{\kappa\rho\bar{p} + (r+\kappa+\rho)\,\bar{\xi}_0^C}{(r+\kappa)\,(r+\rho)}, \tag{34}$$

where $\kappa = \alpha(1-\eta)$. Taken together, (32), (33), and (34) give the following proposition.

Proposition 5. *Dealers hold inventories during the crisis if and only if*

$$\frac{\rho\left(\bar{p}-\bar{p}_0^C\right)}{\bar{p}_0^C} > r \Leftrightarrow \sum_{i=1}^{I} \pi_i U_i^{C\prime -1}\left(\frac{\rho\bar{\xi}}{r+\kappa+\rho}\right) < A. \tag{35}$$

From Proposition 5, one derives simple conditions under which dealers *do not* accumulate inventories.

Corollary 1. *If α is sufficiently large (the market is liquid), or ρ sufficiently close to zero (the crisis is anticipated to be permanent), then dealers do not hold inventories during the crisis.*

It is instructive to consider the limiting case as α goes to infinity (which implies κ goes to infinity) and the economy approaches the frictionless Walrasian benchmark. In this case, dealers no longer have the advantage of trading continuously vis-à-vis investors, and their ability to realize capital gains that would cover their borrowing costs vanishes. Put differently, as frictions vanish, the market provides dealers no incentive to buy assets early in the crisis, and they do not intervene regardless of the severity of the crisis. Formally, $U_i^C(a) \to \theta u_i(a)$ and, because of the Inada condition, the left side of condition (35) approaches infinity.

Next, consider the case where the crisis becomes permanent, $\rho \to 0$. The left side of (35) approaches infinity implying that dealers have no incentives to buy assets. That is, if the crisis is anticipated to last a long time, it is as if the economy switches to a new steady state, and dealers find that the prospective capital gains from asset inventories are smaller than the opportunity cost of holding the asset.

4.2 A parametric example

We now show that there are parameterizations for which condition (35) is satisfied and dealers buy assets at the beginning of the crisis, hold them for a while and sell them off as the investors' selling pressures subside. In these cases, dealers choose positive asset positions (foregoing interest on their stock of the numéraire good) even though they get no utility from holding these assets. The reason why dealers may be willing to carry assets is that they have continuous access to the market while investors do not: This trading advantage allows dealers to "time the market" continuously in order to capture capital gains that investors cannot realize.

For the reminder of the paper we will assume that investors have an iso-elastic utility function $u(a) = a^{1-\sigma}/(1-\sigma)$ with $\sigma > 0$, and that the idiosyncratic preference shock is multiplicative, i.e. $u_i(a) = \varepsilon_i u(a)$ with $\varepsilon_i \in \{\varepsilon_1, ..., \varepsilon_I\}$. From Lemma 2 an investor's flow expected utility during the recovery is $U_i(a) = \bar{\varepsilon}_i u(a)$, where

$$\bar{\varepsilon}_i = \frac{(r+\kappa)\varepsilon_i + \delta \sum_{j=1}^{I} \pi_j \varepsilon_j}{r+\kappa+\delta}.$$

Similarly, from Lemma 4 an investor's flow expected utility during the crisis is $U_i^C(a) = \bar{\varepsilon}_i^C u(a)$, where

$$\bar{\varepsilon}_i^C = \frac{(r+\kappa+\rho)\varepsilon_i^C + \delta \sum_{j=1}^{I} \pi_j \varepsilon_j^C}{r+\kappa+\rho+\delta},$$

and $\varepsilon_i^C = \frac{(r+\kappa)\theta\varepsilon_i + \rho\bar{\varepsilon}_i}{r+\kappa+\rho}$. With these functional forms, Proposition 5 becomes:

Corollary 2. *Let $u_i(a) = \varepsilon_i a^{1-\sigma}/(1-\sigma)$. Dealers hold inventories during the crisis if and only if*

$$\frac{\sum_{i=1}^{I} \pi_i \left(\bar{\varepsilon}_i^C\right)^{1/\sigma}}{\sum_{i=1}^{I} \pi_i \left(\bar{\varepsilon}_i\right)^{1/\sigma}} < \left(\frac{\rho}{r+\kappa+\rho}\right)^{\frac{1}{\sigma}}. \tag{36}$$

The green shaded regions in Figure 4 represent parameter values for which condition (36) is satisfied. In each panel, we let the two parameters in the axes vary and keep the rest fixed at some benchmark values. All panels have α—our index of the degree of the trading frictions—on the horizontal axis.

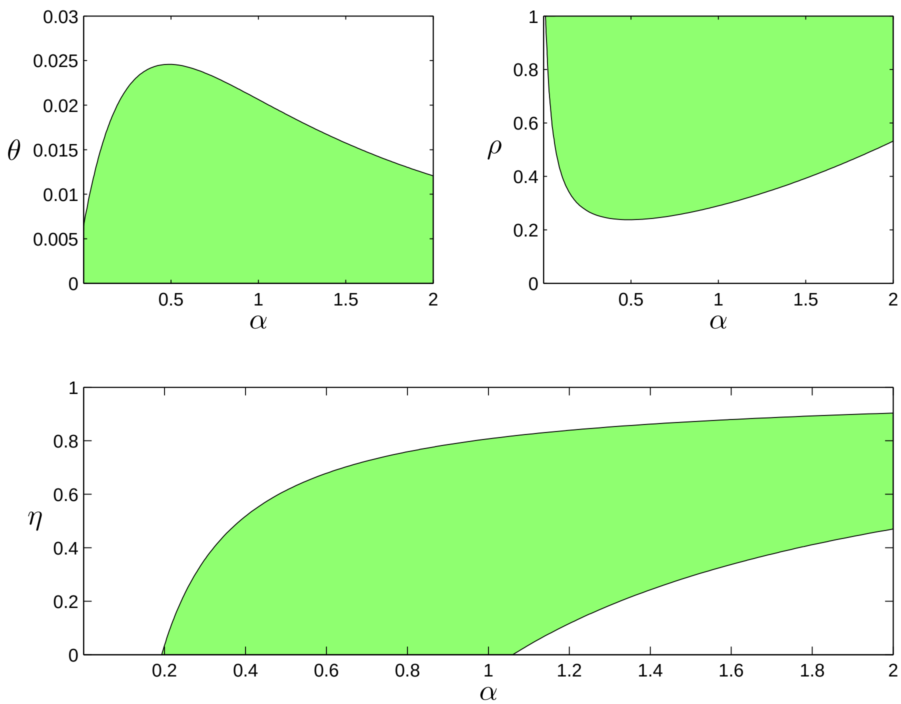

Figure 4: Parameterizations for which dealers provide liquidity. The benchmark parametrization is: $\sigma = 0.5$, $r = 0.05$, $\pi_1 = \pi_2 = 0.5$, $\alpha = 0.5$, $\delta = 1$, $\rho = 0.3$, $\theta = 0.02$, $\eta = 0$, and $A = 1$.

The first and second panels of Figure 4 relate liquidity provision to the characteristic of the crisis, θ and ρ. They show that dealers are more likely to accumulate asset inventories when the market crash is severe (θ low) and expected to be short-lived (ρ large). Intuitively, a very severe but short-lived crisis generates large enough expected capital gains to compensate dealers for their borrowing costs and to give them incentives to step in.

The distinctive features of OTC markets are twofold: (i) there is no organized exchanges, so that locating counterparties is time-consuming; (ii) there is no commitment on publicly posted prices, i.e., prices are determined through bargaining. In our theoretical market setup, these OTC features are captured by two parameters, α and η. All panels in Figure 4 show that, for a given size of the aggregate shock, dealers provide liquidity if trading frictions are neither too severe nor too small. For large α, investors face short delays to rebalance their asset holdings, $1/\alpha$ on average, which increases their willingness to take more extreme positions. In particular, investors with higher-than-average utility (i.e., a high ε_i) become more willing to hold larger-than-average positions and absorb more of the selling pressure. In some cases, when α is large enough, they end up supplying so much liquidity to other investors (those with a low ε_i) that dealers don't find it profitable to step in. Conversely, if α is very small, then $\bar{\varepsilon}_i$ and $\bar{\varepsilon}_i^C$ become close to the average preference shock over time, so all investors choose very similar asset holdings regardless of their preference type. In this case, the economy becomes similar to an economy without idiosyncratic preference shocks in which there is no need for the asset to be traded, and hence no need for dealers to reallocate assets across investors. So if finding counterparties becomes harder, either because of informational problems that make investors more reluctant to trade, or because of the failure of some existing dealers, then the remaining dealers are less likely to provide liquidity.

In order to interpret the third panel of Figure 4, it is useful to remember that the condition in Proposition 5 depends on $\kappa = \alpha(1-\eta)$, the effective amount of frictions in the economy, but not on α and η individually. So an increase in dealers' bargaining power produces the same effects in terms of liquidity provision as a reduction in α. We also discussed the observation that an increase in α has nonmonotonic effects on dealers' incentives to provide liquidity. In the context of the third panel of Figure 4 this implies that the effect of a change in the bargaining power of dealers on the provision of liquidity depends on the extent of the trading frictions. Suppose first that the trading frictions are severe, $\alpha \in (0.2, 1)$ in our example. If η is close to 1, then investors incur large transaction costs when they re-balance their asset holdings. Since the size of the intermediation fees increases with the quantity of assets that investors buy or sell, investors can incur less transaction costs by re-balancing smaller amounts of asset, choosing asset positions that reflect their average preferences and not their current preference shock. As a consequence, the need for investors to readjust their asset holdings over time is reduced and there is less scope for dealers to hold asset inventories. A reduction of dealers' bargaining power induces investors to take more disperse asset positions, which gives dealers incentives to provide liquidity and accumulate asset inventories. Suppose next that the trading frictions are not too severe, $\alpha > 1.1$ in our example. Dealers will provide liquidity if their bargaining power is neither too large nor too small. If dealers have a low bargaining power, then investors are better able to compete for the capital gains as the economy recovers, which reduces dealers' incentives to buy assets. An increase in dealer's bargaining power would help dealers secure larger capital gains, which would give them higher incentives to buy assets. If bargaining power becomes too large, however, investors incur larger intermediation fees whenever they trade, so they find it optimal to re-balance less their holdings. This reduces the amount of liquidity demanded by outside investors and the equilibrium amount of liquidity supplied by dealers.

5 Policy implications

The liquidity dry-up during the recent financial crisis has been widely discussed. Brunnermeier and Pedersen (2009) have distinguished two notions of liquidity that are important for an optimal design of a policy response to the crisis.[10] There is "market liquidity" that represents the ease with which an asset can be traded. Among other things, market liquidity depends on whether or not information is distributed evenly among market participants, the extent to which the market is intermediated (the presence of brokers, dealers, and market-makers), and on the trading venue (e.g., organized exchanges, electronic trading platform, OTC markets). Informational asymmetries about assets' fundamental values or about the credit worthiness of trade counterparties, the failure of large dealers, and the fact that the relevant assets for the crisis (mortgage-backed securities, collateralized debt obligations, credit default swaps) were traded over-the-counter have contributed to the decline in market liquidity in the current crisis. The second notion of liquidity is "funding liquidity" that represents the extent to which financial institutions can borrow to meet their current obligations. Funding liquidity dried up during the current crisis as intermediaries have had a harder time borrowing the capital needed to buy and sell assets on their own account, either because they were under-capitalized or because the markets where they could get financed ceased functioning adequately.

When turning to our model, market liquidity is represented by the parameters α, the ease to trade assets, and η, the bargaining power of dealers. As it is known from Hopenhayn and Werner (1996); Lester, Postelwaite, and Wright (2009a,b); Rocheteau (2009), among others, a heightening of asymmetric information problems make trade less likely in bilateral meeting,

[10] See the IMF Global Financial Stability Report (Chapter 3, 2008) for a discussion of the determinants of "market" and "funding" liquidity.

effectively increasing the search frictions. As for the large η, it translates into the large transaction costs observed in opaque (see, e.g., Edwards, Harris, and Piwowar, 2007) and distressed markets (see of US Securities and Exchange Commission, 1988, pages 23–24). The funding liquidity is represented by the ability of dealers to buy assets for their own account. DGP and Lagos and Rocheteau (2009) assume that dealers cannot hold asset inventories, which can be interpreted as an extreme form of a funding liquidity problem. Weill (2007) considers various degrees of funding liquidity.

The two notions of market and funding liquidity are not independent. Funding illiquidity by reducing dealers' absorptive capacity can increase market illiquidity. Proposition 5 illustrates this point by providing a condition under which dealers accumulate asset inventories in times of crisis. If this condition is satisfied but dealers have not enough capital to fund their desire to buy assets, then funding liquidity affects adversely market liquidity. In the following, we explore further the link between market and funding liquidity. We first look at the predictions of the model in terms of the demand for funding liquidity. We determine the maximum quantity of asset inventories that dealers are willing to hold, which gives us a proxy for the capital that dealers will need to absorb the selling pressures in the market. Next, we will discuss the conditions under which capital injection is ineffective in the sense that this capital is not used by dealers to build up asset inventories in order to improve the liquidity of the market. We then ask whether market reforms that improve market liquidity can make capital injections more effective. Finally, we will discuss how to diagnose inefficient liquidity provision.

5.1 The demand for funding liquidity

In order to assess the extent to which funding liquidity can be problematic, one needs to know how much capital the dealers in OTC market would be willing to commit to build up asset inventories in case of a crisis. The next proposition provides a closed-form solution for the maximum amount of asset inventories that dealers are willing to hold.

Proposition 6. *The maximum quantity of assets that dealers are willing to accumulate is*

$$\lim_{t\to\infty} A_d^C(t) = \bar{A}_d^C = A\left\{1 - \Omega\left[\frac{\alpha}{\gamma} + \left(\frac{\gamma-\alpha}{\gamma}\right)\Omega\right]^{\frac{\gamma}{\alpha-\gamma}}\right\} \quad (37)$$

where

$$\Omega \equiv \left(\frac{r+\kappa+\rho}{\rho}\right)^{1/\sigma} \frac{\sum_{i=1}^{I}\pi_i\left(\bar{\varepsilon}_i^C\right)^{1/\sigma}}{\sum_{i=1}^{I}\pi_i\left(\bar{\varepsilon}_i\right)^{1/\sigma}} \text{ and } \gamma \equiv \frac{r+\kappa}{\sigma}. \quad (38)$$

In Figure 5 we plot $\bar{A}_d^C$ for our baseline parameter values. The first panel confirms the nonmonotonic relationship between dealers' asset inventories and the degree of frictions that prevail in the market. The dealers' willingness to provide liquidity is maximum when the market is neither too liquid nor too illiquid. As discussed earlier, when α is low the demand for liquidity by investors is low; when α is high, investors supply liquidity to each other. The second panel shows that the prospects for capital gains and hence dealers' willingness to provide liquidity increase with the severity of the crisis. According to the third panel, the relationship between $\bar{A}_d^C$ and the expected duration of the crisis $(1/\rho)$ is nonmonotonic. As indicated in Corollary 1, if the crash is very persistent (ρ small), then dealers are not willing to accumulate large positions since the expected discounted capital gain from holding asset inventories is small. If the crash is anticipated to be short-lived (ρ large), dealers will not accumulate large inventories because the crash has a small effect on investors' asset demand. The fourth panel shows that dealers' inventories decrease as dealers' bargaining power increases. When dealers have a greater ability to extract rents, then the demand for liquidity falls. As indicated in Figure 4, this result holds for intermediate values of α. If α is sufficiently large, then an increase in dealers' bargaining power can have a nonmonotonic effect on $\bar{A}_d^C$.

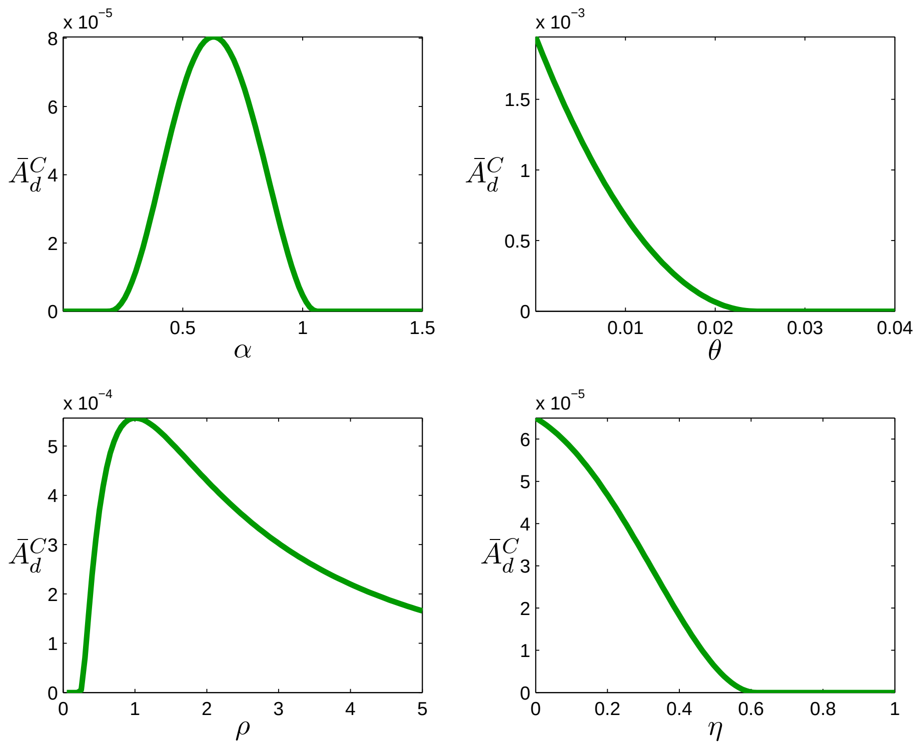

Figure 5: The steady-state level of inventories during the crisis, $\bar{A}_d^C$.

When dealers are undercapitalized and when their sources of financing dry up, they have a limited absorptive capacity. In this case, a shortage of funding liquidity is more likely if

$\bar{A}_d^C$ is large. Our results suggest that policy-makers should inject capital when the market crash is severe and when the crisis is expected to be persistent but not permanent. Moreover, the policy-maker should target financial intermediaries trading in markets that are neither too illiquid nor too liquid (e.g., OTC markets for Treasury securities or wholesale foreign exchange).

5.2 Ineffective capital injection

In the face of a shortage of funding liquidity, the Federal Reserve has put into place new facilities to ease intermediaries' liquidity constraints. In March 2008 it introduced the Term Securities Lending Facility (TSLF), an auction facility that allows primary dealers (investment banks, brokers-dealers) to borrow Treasury securities for longer periods against less liquid collateral.[11] The Federal Reserve also introduced the Primary Dealer Credit Facility (PDCF), an overnight loan facility that provides funding to primary dealers.

Our model suggests that the provision of funding liquidity might prove ineffective even when markets are dysfunctional. Intermediaries might not take advantage of the provision of funding liquidity, or they might end up hoarding it. For instance, most of the auctions from the TSLF were undersuscribed: primary dealers were bidding less than the quantity offered by the facility (e.g., Cecchetti, 2008). Our model can provide instances where, when market liquidity declines, intermediaries hoard funding liquidity and fail to make welfare improving purchases of outside investors' assets. To make this argument, the decline in market liquidity can be captured by either a low α or a large η. Consider the combination of low α and large η indicated by the red "dot" in figure 6. The social optimum, which corresponds to the equilibrium allocation holding α the same but setting η to 0, prescribes that dealers hold inventories during the crisis. In equilibrium, however dealers provide no liquidity even though, in the baseline model, they have no funding liquidity problem. It follows that injecting more funding liquidity will prove totally ineffective: dealers will hoard the liquidity but they will not use it to purchase assets from outside investors. Capital injections are ineffective because the root cause of the problem is not funding liquidity but market liquidity. In the face of a highly illiquid market, a low α high η combination, outside investors respond by taking less extreme asset positions: they adjust their asset demand so as to re-balance less and consume less market liquidity. As a consequence, in an equilibrium, even well-capitalized dealers do not find it profitable to step in.

[11] For a more detailed description of the TSLF, see Fleming, Hrung, and Keane (2009), "The Term Securities Lending Facility: Origin, Design, and Effects," Federal Reserve Bank of New York Current Issues in Economics and Finance, 15, 1-11. See also Cecchetti (2008), "Crisis and responses: The Federal Reserve and the the financial crisis of 2007-2008," NBER Working Paper 14134, and the speech of William Dudley on "The Federal Reserve's liquidity facilities" held on April 18, 2009.

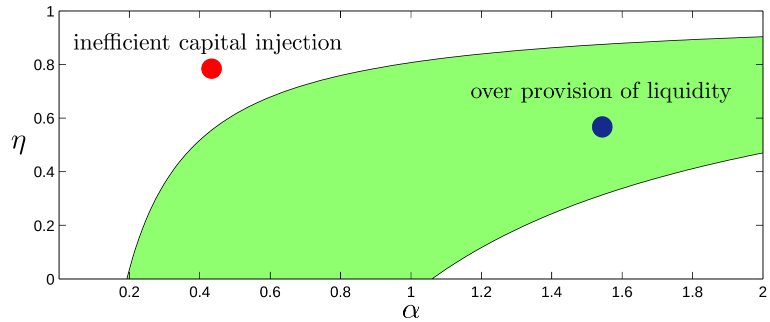

Figure 6: Parameterizations of the trading frictions, (α, η), for which dealers provide liquidity.

In order for an infusion of funding liquidity to be effective, it has to be accompanied by a market reform that facilitates trades between investors and dealers and that erodes dealers' market power. In practice, the trading frictions can be reduced in several ways. The regulator can promote standardization of the assets traded in OTC markets, as in OTC markets for Treasury securities and wholesale foreign exchange, disclosure of information regarding the characteristics of these assets, and the development of electronic trading platforms that facilitate and speed up trades. The policymaker can also maintain market liquidity by offering safer and more recognizable collateral, which reduces counterparties risk, and by preventing the failure of large dealers-brokers. This is what the Federal Reserve did on March 14, 2008, by lending directly to Bear Stearns.

Our policy implications are broadly consistent with the framework for financial stability of the Group of Thirty which recommends "improvements to the infrastructure supporting the OTC derivatives markets" and an enhanced "disclosure and dissemination regime for asset-backed and other structured fixed income financial products". They are also consistent with the recommendations of the chairman of the Federal Reserve to strengthen the financial infrastructure.

> "Since September 2005, the Federal Reserve Bank of New York has been leading a major joint initiative by the public and private sectors to improve arrangements for clearing and settling credit default swaps (CDS) and other over-the-counter (OTC) derivatives. As a result, the accuracy and timeliness of trade information has improved significantly. However, the infrastructure for managing these derivatives is still not as efficient or transparent as that for more mature instruments. (...) To help alleviate counterparty credit concerns, regulators are also encouraging the development of well-regulated and prudently managed central clearing counterparties for OTC trades."
>
> Speech of Ben Bernanke at the Council on Foreign Relations, March 10, 2009

5.3 Diagnosing inefficient liquidity provision

Our model suggests that commonly used indicators of welfare-improving liquidity provision could be misleading. For instance, in our model, dealers always incur capital losses on the assets they purchase as the crisis unfold. The fact that dealers are buying in a down market is often considered as an indication that they are acting for the greater social good rather than for their own private interest. While this behavior by dealers is consistent with individual optimality, the provision by dealers is not necessarily socially optimal. Indeed, as indicated by the blue "dot" in Figure 6, it can be the case that dealers provide liquidity when $\eta > 0$, but do not provide liquidity when $\eta = 0$. Since the equilibrium under $\eta = 0$ corresponds to the socially efficient allocation, it follows that, for this combination of parameter values, liquidity provision in equilibrium is inefficient.

Another finding of our model is that, at the time of the crisis, the price jumps down. The discontinuity occurs even in the constraint-efficient case, $\eta = 0$. This suggest that it could be misleading to assess the effectiveness of dealers as providers of liquidity on their ability or willingness to maintain "price continuity" (as in the New York Stock Exchange), or to "support" the level of asset prices.

We also established that the market performs efficiently when dealers have no market power, $\eta = 0$. In our model, as η decreases trading fees decrease and trading volume increase (see Lagos and Rocheteau, 2009). Thus, directly observable indicators of good liquidity provision are the size of the trading fee or bid-ask spread charged by dealers and trade volume. That is, as long as dealers are well capitalized, one should expect markets where the trading fees are smaller and trade volume is larger to provide better liquidity during crises. This has implication for the design of dealers' compensation structure in a regulated market: one should grant dealers little market power when they trade with their outside customers. This view agrees, for instance, with the Euronext rule that Designated Market Markers should commit to a minimum spread (see Menkveld and Wang, 2009).

A Proofs

A.1 Proof of Lemma 1

First, we substitute the constraint $\dot{a}_d(s) = q(s)$ into the dealers objective and integrate by part:

$$\int_t^T e^{-r(s-t)} p(s)\dot{a}_d(s)\,ds = a_d(t)p(t) + \int_t^T e^{-r(s-t)}\left[\dot{p}(s) - rp(s)\right] a_d(s)\,ds - a_d(T)p(T)e^{-r(T-t)}.$$

Keeping in mind that $a_d(t) = a$, letting $T \to \infty$ and using the no-bubble condition, we find that the value of the inventory path $a_d(s)$ is:

$$p(t)a(t) + \int_t^\infty e^{-r(s-t)}\left[\dot{p}(s) - rp(s)\right] a_d(s)\,ds.$$

Clearly, the condition of the Lemma is sufficient. For necessity, note that if there was some time $s > 0$ such that $\dot{p}(s) - rp(s) > 0$, then a dealer could improve her utility by accumulating more inventory around s, and the dealer's problem would not have any bounded solution.

A.2 Proof of Lemma 2 and Lemma 4

A.2.1 Preliminary Results

We let the random flow utility of an investor at time t be $u(a,t)$, where we use the time argument "t " as a short-hand for the investor's current idiosyncratic and aggregate preference shock. To simplify notations, we measure time from the point of a given contact with a dealer. We let $0 = T_0 < T_1 < T_2 < \ldots$ be the sequence of the investor's contact times with dealers, N_t be the number of contact times during $[0,t]$, and θ_t be the last contact time before t. Then, for any asset plan, a, we calculate the inter-temporal utility over $[0,t]$:

$$V_0^t(a) \equiv \int_0^t u[a(s),s]e^{-rs}\,ds - \sum_{n=1}^{N_t} p(T_n)e^{-rT_n}\left[a(T_n) - a(T_{n-1})\right],$$

along a realization of the contact time and type processes. This utility can be decomposed as

$$V_0^t = U_0^t - B_0^t + p(T_1)e^{-rT_1}a(0) - p(\theta_t)a(\theta_t)e^{-r\theta_t},$$

where

$$\begin{aligned} U_0^t(a) &= \int_0^t u\left[a(s),s\right]e^{-rs}\,ds, \\ B_0^t(a) &= \sum_{n=1}^{N_t-1} a(T_n)\left[p(T_n)e^{-rT_n} - p(T_{n+1})e^{-rT_{n+1}}\right]. \end{aligned}$$

We consider portfolio plans a that are bounded, and such that the intertemporal utility $\mathbb{E}\left[V_0^\infty(a)\right]$ is well defined. We first establish:

Lemma 5. *As $t \to \infty$, $\mathbb{E}\left[U_0^t(a)\right]$ and $\mathbb{E}\left[B_0^t(a)\right]$ converge to finite limits, and $\mathbb{E}\left[p(\theta_t)e^{-r\theta_t}a(\theta_t)\right]$ converges to zero.*

Because of the no-bubble condition $\lim_{t\to\infty} p(t)e^{-rt} = 0$ and the fact that $a(t)$ is bounded, we have that $\lim_{t\to\infty} p(t)a(t)e^{-rt} = 0$. Since θ_t goes to infinity almost surely, it follows that $\lim_{t\to} \mathbb{E}\left[p(\theta_t)e^{-r\theta_t}a(\theta_t)\right] = 0$ as well.

Let's turn to with $\mathbb{E}\left[U_0^t(a)\right]$. When the investor's utility is bounded below, then the result follows from the assumption that the portfolio plan, a, is bounded. When the investor's utility is unbounded below and bounded above, we can assume without loss of generality that it is negative. Then $\mathbb{E}[U_0^t]$ is decreasing and thus converges either to some finite or some infinite limit. The limit, in turn, must be finite because

$$\mathbb{E}[U_0^t] = \mathbb{E}[V_0^t] + \mathbb{E}[B_0^t] - \mathbb{E}[p(T_1)e^{-rT_1}a(0)] \geq \mathbb{E}[V_0^t] - \mathbb{E}[p(T_1)e^{-rT_1}a(0)],$$

where the inequality follows because $p(t)e^{-rt}$ is decreasing and B_0^t is therefore positive. Because $\mathbb{E}[V_0^\infty]$ is well defined, the right-hand side of the inequality is bounded below, implying that $\mathbb{E}[U_0^t]$ has a finite limit. It then immediately follows that

$$\mathbb{E}\left[B_0^t\right] = -\mathbb{E}\left[V_0^t\right] + \mathbb{E}\left[U_0^t\right] + \mathbb{E}[p(T_1)e^{-rT_1}a(0)] - p(\theta_t)e^{-r\theta_t}a(\theta_t).$$

also converges to some finite limit.

Lemma 6. *An investor's intertemporal utility is*

$$\mathbb{E}\left[V_0^\infty\right] = (r+\kappa)^{-1}\mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n}\left\{U\left[a(T_n), T_n\right] - \xi(T_n)a(T_n)\right\}\right]. \tag{39}$$

where

$$\begin{aligned} U\left[a(T_n), T_n\right] &= (r+\kappa)\mathbb{E}\left[\int_{T_n}^{T_{n+1}} u\left[a(s), s\right] e^{-r(s-T_n)}\, ds \,\middle|\, T_n\right] \\ \xi(T_n) &= p(T_n) - \mathbb{E}\left[p(T_{n+1})e^{-r(T_{n+1}-T_n)} \,|T_n\right]. \end{aligned}$$

To show that result, write

$$\begin{aligned} \mathbb{E}\left[B_0^\infty\right] &= \mathbb{E}\left[\sum_{n=1}^{\infty} a(T_n)\mathbb{E}\left[p(T_n)e^{-rT_n} - p(T_{n+1})e^{-rT_{n+1}} \,\middle|\, T_n\right]\right] \\ &= (r+\kappa)^{-1}\mathbb{E}\left[\sum_{n=1}^{\infty} a(T_n)\xi(T_n)e^{-rT_n}\right], \end{aligned}$$

by definition of $\xi(T_n)$. In addition note that, when u is bounded below, we can without loss of generality assume that it is positive, and we have

$$u[a(s), s]e^{-rs}\mathbb{I}_{\{s\leq\theta_t\}} \leq u[a(s), s]e^{-rs}\mathbb{I}_{\{s\leq t\}} \leq u[a(s), s],$$

and $u[a(s), s]\mathbb{I}_{\{s\leq\theta_t\}} \nearrow u[a(s), s]$ as t goes to infinity. The same reasoning go through with opposite inequalities when u is negative. Therefore, an application of the dominated convergence theorem implies that

$$\begin{aligned} \mathbb{E}\left[U_0^\infty\right] &= \lim_{t\to\infty} \mathbb{E}\left[\int_0^{\theta_t} u(a(s), s)e^{-rs}\, ds\right] = \lim_{t\to\infty} \mathbb{E}\left[\sum_{n=1}^{N_t-1} \int_{T_n}^{T_{n+1}} u(a(s), s)e^{-rs}\, ds\right] \\ &= (r+\kappa)^{-1}\mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n} U(a(T_n), T_n)\right], \end{aligned}$$

where the last equality follows by taking expectations of each term in the sum with respect to T_n.

A.2.2 Necessary and sufficient condition

For the "only if" part of the two Lemma, it is clear from (39) that an optimal portfolio strategy should maximize each term $U\left[a(T_n), T_n\right] - \xi(T_n)a(T_n)$, implying the investor's first-order condition. For the "if" part, we consider a plan a that satisfies the first-order conditions and compare it to some other plan a'. We find

$$\begin{aligned}
& \mathbb{E}[V_0^\infty(a) - V_0^\infty(a')] \\
= \ & \mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n}\left(U(a(T_n),T_n) - U(a'(T_n),T_n) - \xi(T_n)\left(a(T_n)-a'(T_n)\right)\right)\right] \\
\geq \ & \mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n}\Big(U_a(a(T_n),T_n) - \xi(T_n)\Big)\Big(a(T_n)-a'(T_n)\Big)\right] \geq 0,
\end{aligned}$$

where the first inequality follows because of concavity, and the second inequality follows because of the first-order condition in the two Lemma.

A.2.3 The expression for $U_i(a)$ and $\xi(t)$ along the recovery path

The flow inter-contact time utility is $(r+\kappa)^{-1}U\left[a(T_n),T_n\right] = (r+\kappa)^{-1}U_{i(T_n)}\left[a(T_n)\right]$, where $U_i(a)$ is defined in equation (9) of the lemma. To see why, let

$$\tilde{V}_i(a,t) = \mathbb{E}_i\left[\int_0^{\tilde{T}} e^{-rs}u_{k(t+s)}(a')\, ds \middle| k(t) = i\right].$$

By the Markovian nature of the process $k(t)$, $\tilde{V}_i(a,t)$ only depends on t through the condition $k(t) = i$ which is already captured by the subscript i. Therefore, hereafter we will slightly abuse notation and write $\tilde{V}_i\left(a\right)$ for $\tilde{V}_i\left(a,t\right)$. Denote $\hat{T}$ the length of the period of time before the investor receives a preference shock. By definition, $\hat{T}$ is exponentially distributed with mean $1/\delta$. The value of an investor can then be written recursively as follows,

$$\tilde{V}_i(a) = \mathbb{E}\left[\int_0^{\tilde{T}\wedge\hat{T}} e^{-rs}u_i(a)ds + \mathbb{I}_{\left\{\hat{T}<\tilde{T}\right\}}e^{-r\hat{T}}\tilde{V}_{k(\hat{T})}(a)\right], \tag{40}$$

where $k(\hat{T})$ indicates the new realization of the preference shock at time $\hat{T}$. Using the fact that $\tilde{T}$ and $\hat{T}$ are independent random variables, one can rewrite the first term on the right-hand side of (40) as

$$\begin{aligned}
\mathbb{E}\left[\int_0^{\tilde{T}\wedge\hat{T}} e^{-rs}u_i(a)ds\right] &= \mathbb{E}\left[\int_0^{\infty} \mathbb{I}_{\{s\leq\tilde{T}\wedge\hat{T}\}}e^{-rs}u_i(a)\, ds\right] = u_i(a)\int_0^\infty \mathbb{E}\left[\mathbb{I}_{\{s\leq\tilde{T}\wedge\hat{T}\}}\right]e^{-rs}\, ds \\
&= u_i(a)\int_0^\infty e^{-(r+\kappa+\delta)s}\, ds = \frac{u_i(a)}{r+\kappa+\delta}.
\end{aligned} \tag{41}$$

The second equality follows because $u_i(a)$ is constant over the interval of integration, and by interchanging the integral and expectation sign. The third equality follows because $\tilde{T}$ and $\hat{T}$ are independent exponential random variables with respective parameter κ and δ: thus $\tilde{T}\wedge\hat{T}$ is exponential as well with parameter $\kappa+\delta$.

Turning to the second term in (40), we first note that the realizations of the preference shocks are independent and identically distributed according to π_i. Thus, the distribution of $k(\hat{T})$ is given by $\{\pi_i\}_{i=1}^I$. Therefore,

$$\mathbb{E}\left[\mathbb{I}_{\left\{\hat{T}<\tilde{T}\right\}}e^{-r\hat{T}}\tilde{V}_{k(\hat{T})}(a)\right] = \mathbb{E}\left[\mathbb{I}_{\left\{\hat{T}<\tilde{T}\right\}}e^{-r\hat{T}}\right]\sum_{k=1}^{I}\pi_k\tilde{V}_k(a) = \frac{\delta}{\delta+r+\kappa}\sum_{k=1}^{I}\pi_k\tilde{V}_k(a). \tag{42}$$

Adding (41)and (42), one finds

$$\tilde{V}_i(a) = \frac{u_i(a)}{r+\kappa+\delta} + \frac{\delta}{r+\kappa+\delta}\sum_{k=1}^{I}\pi_k\tilde{V}_k(a), \tag{43}$$

for all $i\in\{1,\ldots,I\}$. One then easily verifies that this system of equation is solved by

$$\tilde{V}_i(a) = \frac{U_i\left(a\right)}{r+\kappa}, \tag{44}$$

where $U_i(a)$ is as in (9).

To derive expression (10), just note that the expected discounted price at the time the investor regains direct access to the asset market is:

$$\mathbb{E}[e^{-r\tilde{T}}p(t+\tilde{T})] = \kappa \int_0^{\infty} e^{-(r+\kappa)s} p(t+s)ds. \tag{45}$$

A.2.4 The expression for $U_i(a)$ and $\xi(t)$ during the crisis

We let

$$\begin{aligned}
\tilde{V}_i^C(a) &= \mathbb{E}\left[\int_0^{\tilde{T}} e^{-rs}\left(\theta + (1-\theta)\mathbb{I}_{\{s\geq T_\rho\}}\right) u_i(a)\, ds\right] \\
&= \mathbb{E}\left[\int_0^{\tilde{T}\wedge\hat{T}\wedge T_\rho} e^{-rs}\theta u_i(a)\, ds\right] + \mathbb{E}\left[\mathbb{I}_{\{\hat{T}<\tilde{T}\wedge T_\rho\}} e^{-r\hat{T}} \tilde{V}^C_{k(\hat{T})}(a)\right] + \mathbb{E}\left[\mathbb{I}_{\{T_\rho<\tilde{T}\wedge\hat{T}\}} e^{-rT_\rho}\tilde{V}_i(a)\right] \\
&= \frac{\theta u_i(a)}{r+\kappa+\delta+\rho} + \frac{\delta}{r+\kappa+\delta+\rho}\sum_{j=1}^{I}\tilde{V}_j^C(a) + \frac{\rho}{r+\kappa+\delta+\rho}\tilde{V}_i(a) \\
&= \frac{\theta u_i(a) + \rho\tilde{V}_i(a)}{r+\kappa+\delta+\rho} + \frac{\delta}{r+\kappa+\delta+\rho}\sum_{j=1}^{I}\tilde{V}_j^C(a).
\end{aligned}$$

where the second last equality follows from the exact same calculation as for $\tilde{V}_i(a)$ in the previous paragraph. One sees that this is exactly the same equation as (43), except that $u_i(a)$ is replaced by $\theta u_i(a) + \rho\tilde{V}_i(a)$ and κ is replaced by $\kappa+\rho$. Thus the result of the last section applies and we have that:

$$(r+\kappa+\rho)\tilde{V}_i^C(a) = \frac{(r+\kappa+\rho)\left(\theta u_i(a)+\rho\tilde{V}_i(a)\right) + \delta\sum_{j=1}^I\left(\theta u_j(a)+\rho\tilde{V}_j(a)\right)}{r+\kappa+\rho+\delta}$$

Letting $U_i^C(a) = (r+\kappa)\tilde{V}_i^C(a)$, we have

$$\begin{aligned}
U_i^C(a) &= \frac{r+\kappa}{r+\kappa+\rho}(r+\kappa+\rho)\tilde{V}_i^C(a) \\
&= \frac{r+\kappa}{r+\kappa+\rho}\frac{(r+\kappa+\rho)\left(\theta u_i(a)+\rho\tilde{V}_i(a)\right) + \delta\sum_{j=1}^I\left(\theta u_j(a)+\rho\tilde{V}_j(a)\right)}{r+\kappa+\rho+\delta} \\
&= \frac{(r+\kappa+\rho)\left[\frac{r+\kappa}{r+\kappa+\rho}\theta u_i(a) + \frac{\rho}{r+\kappa+\rho}U_i(a)\right] + \delta\sum_{j=1}^I \pi_j\left[\frac{r+\kappa}{r+\kappa+\rho}\theta u_j(a) + \frac{\rho}{r+\kappa+\rho}U_j(a)\right]}{r+\kappa+\rho+\delta},
\end{aligned}$$

keeping in mind that $U_i^C(a) = (r+\kappa)\tilde{V}_i(a)$. This is the formula stated in the Lemma.

To derive the expected value of the re-sale price, we use the fact that $\tilde{T}-t$ and $T_\rho - t$ are two independent exponentially distributed random variables:

$$\begin{aligned}
&\mathbb{E}\left[e^{-r(\tilde{T}-t)}\left[\mathbb{I}_{(\tilde{T}<T_\rho)}p^C(\tilde{T}) + \mathbb{I}_{(\tilde{T}\geq T_\rho)}p(T_\rho,\tilde{T})\right]\right] = \\
&\int_t^\infty\int_t^\infty e^{-r(\tau_\kappa-t)}\left[\mathbb{I}_{(\tau_\kappa<\tau_\rho)}p^C(\tau_\kappa) + \mathbb{I}_{(\tau_\kappa\geq\tau_\rho)}p(\tau_\rho,\tau_\kappa)\right]\kappa e^{-\kappa(\tau_\kappa-t)}\rho e^{-\rho(\tau_\rho-t)}\,d\tau_\rho\,d\tau_\kappa \\
&= \int_t^\infty e^{-r(\tau_\kappa-t)}\left[e^{-\rho(\tau_\kappa-t)}p^C(\tau_\kappa) + \int_t^{\tau_\kappa}\rho e^{-\rho(\tau_\rho-t)}p(\tau_\rho,\tau_\kappa)\right]\kappa e^{-\kappa(\tau_\kappa-lt)}\,d\tau_\rho\,d\tau_\kappa.
\end{aligned}$$

A.3 Proof of Proposition 1

Suppose $A_d(t) > 0$ for some $t \geq T_\rho$. Let $T = \inf\{s \geq t : A_d(s) = 0\}$. Since $A_d(s)$ is continuous, we have $A_d(T) = 0$ and so $T > t$. Now for $s \in [t,T)$, $A_d(s) > 0$ so $A_d(s)$ and $\xi(s)$ solve the system of ODEs given by

(14) and

$$\dot{\xi}(s) \quad = \quad (r+\kappa)\xi(s).$$

Integrating the second ODE gives $\xi(s) = \xi(t)e^{(r+\kappa)(s-t)}$. Plugging this back into the first ODE, (14), gives:

$$A_d(s) = e^{-\alpha(s-t)}A_d(t) + \alpha \int_t^s e^{-\alpha(s-u)} \left[A - \sum_{i=1}^{I} \pi_i U_i'^{-1}\left[\xi(u)\right] \right] du. \tag{46}$$

Equipped with this equation, we first prove:

Lemma 7. *If, for some $t \geq T_\rho$, $A_d(t) > 0$, then $\xi(t) < \bar{\xi}$ and $\dot{A}_d(t) < 0$.*

Suppose to the contrary that there were some $t \geq T_\rho$ such that $A_d(t) > 0$ and $\xi(t) \geq \bar{\xi}$. Then, the above calculation show that $\xi(u) \geq \bar{\xi}$ for all $u \in [t, T)$, and therefore:

$$A - \sum_{i=1}^{I} \pi_i U_i'^{-1}\left[\xi(u)\right] \geq A - \sum_{i=1}^{I} \pi_i U_i'^{-1}\left[\bar{\xi}\right] = 0.$$

It thus follow that $A_d(s) > e^{-\alpha(s-t)}A_d(t)$. Since $A_d(T) = 0$ we must have that $T = \infty$. But this means that $\dot{\xi}(s) = (r+\kappa)\xi(s)$ for all $s \geq t$ and, because of equation (12), that $rp(s) = \dot{p}(s)$ for all $s \geq t$. But then the only way the no-bubble condition holds is if $p(t) = \xi(t) = 0$, which is impossible given that $\xi(t) \geq \bar{\xi}$. That $\dot{A}_d(t) < 0$ follows from substituting $\xi(t) < \bar{\xi}$ in ODE (14).

We are now ready to solve for an equilibrium path. We start at T_ρ with some positive inventory $A_d(T_\rho) > 0$. Let T be the first time greater than T_ρ such that $A_d(T) = 0$. If $T = \infty$, then as before the no-bubble condition would be violated. So $T < \infty$. Since $A_d(t) > 0$ for all $t < T$, it follows by Lemma (7) and the continuity of $\xi(t)$ that $\xi(T) \leq \bar{\xi}$. But if $\xi(T) < \bar{\xi}$ then ODE (14) implies that $\dot{A}_d(T) < 0$. Moreover,since $\xi(s)$ is continuous, it follows from ODE (14) that $A_d(t)$ is continuously differentiable. Thus, we must have that $\dot{A}_d(s) < 0$ for some $s > T$, which would violate the short-selling constraint. Therefore, $\xi(T) = \bar{\xi}$. Next, we show that $A_d(s) = 0$ for all $s \geq T$. Suppose to the contrary that there is some $s > T$ such that $A_d(s) > 0$. Since $A_d(t)$ is continuously differentiable, we can apply Taylor Theorem and find some $s' \in [T, s]$ such that

$$\dot{A}_d(s') = \frac{A_d(s) - A_d(T)}{s - T} > 0.$$

The contrapositive of Lemma 7 then implies that $A_d(s') = 0$. Now, since $A_d(s')$ is continuously differentiable, there must be some $u > s'$ such that $A_d(u) > 0$ and $\dot{A}_d(u) > 0$, which contradicts Lemma 7. Thus $A_d(s) = 0$ for all $s \geq T$. Plugging this back into equation (14), it follows that $\xi(s) = \bar{\xi}$ for all $s \geq T$.

To solve for T, we plug $\xi(t) = \bar{\xi}e^{-(r+\kappa)(T-t)}$ back into equation (46) and solve for the unique solution of $A_d(T) = 0$, given the initial condition $A_d(T_\rho)$. That is, one has to solve the equation:

$$\begin{aligned}
& 0 = e^{-\alpha(T-T_\rho)}A_d(T_\rho) + \alpha \int_{T_\rho}^{T} e^{-\alpha(T-u)} \left[A - D(\bar{\xi}e^{-(r+\kappa)(T-u)}) \right] \\
\Leftrightarrow \quad & 0 = A_d(T_\rho) + \alpha \int_{T_\rho}^{T} e^{-\alpha(T_\rho-u)} \left[A - D(\bar{\xi}e^{-(r+\kappa)(T-u)}) \right] \\
\Leftrightarrow \quad & 0 = A_d(T_\rho) + \alpha \int_{0}^{\Delta T} e^{\alpha s} \left[A - D(\bar{\xi}e^{-(r+\kappa)(\Delta T-s)}) \right],
\end{aligned} \tag{47}$$

where

$$D(\xi) \equiv \sum_{i=1}^{I} \pi_i U_i'^{-1}\left[\xi(u)\right], \tag{48}$$

where the first equivalence follows from multiplying through by $e^{\alpha(T-T_\rho)}$, and the second one from the change of variable $\Delta T \equiv T - T_\rho$ and $s = u - T_\rho$. Since the function $D(\xi)$ is decreasing and since $D(\bar{\xi}e^{-(r+\kappa)(\Delta T-s)}) >$

$D(\bar{\xi}) = A$ for all $s < \Delta T$, it follows that the right-hand side of (47) is a strictly increasing function of $A_d(T_\rho)$ and a strictly decreasing function of ΔT. Since $A_d(T_\rho) > 0$, it is clearly strictly positive at $\Delta T = 0$. Moreover, since $D(\xi)$ is strictly decreasing and since $A - D(\bar{\xi} e^{-(r+\kappa)(\Delta T - s)}$ is negative, we have

$$\begin{aligned} & \alpha \int_0^{\Delta T} e^{\alpha s} \left[A - D(\bar{\xi} e^{-(r+\kappa)(\Delta T - s)}) \right] \\ \leq \; & \alpha \int_0^{\Delta T > \varepsilon} e^{\alpha s} \left[A - D(\bar{\xi} e^{-(r+\kappa)\varepsilon}) \right] \\ = \; & e^{\alpha(\Delta T - \varepsilon)} \left[A - D(\bar{\xi} e^{-(r+\kappa)\varepsilon}) \right] \to -\infty, \end{aligned}$$

as ΔT goes to infinity. Thus, equation (47) has a unique solution $\Delta T > 0$ and an application of the Implicit Function Theorem shows that it strictly increasing in $A_d(T_\rho)$ and twice continuously differentiable. Moreover, it goes to infinity as $A_d(T_\rho)$ goes to infinity. Indeed since it is a monotonic function, it must have a limit. This limit can't be finite: otherwise, the second term on the right-hand-side of (47) would go to some finite limit, which is impossible since the first term goes to infinity and the two terms must sum to zero. From ΔT we obtain the function $\psi(A_d) = \bar{\xi} e^{-(r+\kappa)\Delta T}$. Therefore:

Lemma 8. *The function $\psi(A_d)$ is strictly decreasing, twice continuously differentiable, and goes to zero as A_d goes to infinity.*

A.4 Proof of Lemma 3

We proceed as in the Proof of Lemma 1, but for the integration by part we break the interval of integration in two: $[0, T_\rho \wedge T]$ and $[T_\rho \wedge T, T]$. After using the no-bubble condition, we find that the value of the inventory path is:

$$\begin{aligned} & a_d(t) p^C(t) + \int_t^{T_\rho} e^{-r(s-t)} \left(\dot{p}^C(s) - r p^C(s) \right) a_d^C(s) \, ds \\ & + e^{-r(T_\rho - t)} \left(p(T_\rho, T_\rho) - p^C(t) \right) a_d^C(T_\rho) \\ & + \int_{T_\rho}^{\infty} e^{-r(s-t)} \left(\frac{\partial p}{\partial t}(T_\rho, t) - r p(T_\rho, t) \right) a_d(T_\rho, t) \, ds. \end{aligned}$$

Taking expectations, ignoring the initial condition $a_d(t) p^C(t)$ and the last term that only depends on the inventory plan $a_d(T_\rho, t)$ along the recovery path, we find that before T_ρ the dealer chooses $a_d^C(s)$ in order to maximize:

$$\begin{aligned} & \mathbb{E}_t \left[\int_t^{\infty} \mathbb{I}_{\{s \leq T_\rho\}} e^{-r(s-t)} \left(\dot{p}^C(s) - r p^C(s) \right) a_d^C(s) \, ds \right] \\ & + \mathbb{E}_t \left[e^{-r(T_\rho - t)} \left(p(T_\rho, T_\rho) - p^C(t) \right) a_d^C(T_\rho) \right]. \end{aligned}$$

Note that, in the first expectation, the only random variable is $\mathbb{I}_{\{t \leq T_\rho\}}$ and its expectation is $e^{\rho(s-t)}$ for each s. Next, write the second expectation as an integral against the exponential density $\rho e^{\rho(s-t)}$. After collecting terms, we find that the dealer's objective is:

$$\int_t^{\infty} e^{-(r+\rho)(s-t)} \left[\dot{p}^C(s) - r p^C(s) + \rho \left(p(T_\rho, T_\rho) - p^C(t) \right) \right] a_d^C(s) \, ds,$$

and we can apply the same argument as in Lemma 1.

A.5 Proof of Proposition 2

The system of ODE we seek to solve is:

$$\dot{\xi}^C(t) = (r+\rho+\kappa)\xi^C(t) - \rho\psi(A_d^C(t)) \tag{49}$$

$$\dot{A}_d^C(t) = \alpha\left[A - A_d^C(t) - D^C(\xi^C(t))\right] \tag{50}$$

where $D^C(\xi)$ is defined as in equation (48) but based on $U_i^C(a)$. Given Lemma 8 and under our maintained regularity assumptions on the utility functions, we can apply standard existence and uniqueness Theorems for ODEs (see, for example Theorem 6.2.3 in Hubbard and West (1995)) given the initial condition $A_d^C(0) = 0$ and $\xi^C(0) > 0$. As it is standard with forward looking rational expectations dynamics, the initial condition $\xi^C(0)$ is found by arguing that the economy has to evolve along a saddle path of the system (49)-(50). Precisely, we establish two things: in Section A.5.1, we show that there exists a unique saddle path extending from the steady state $(\bar{\xi}^C, \bar{A}_d^C)$ to some initial condition $A_d^C(0) = 0$ and $\bar{\xi}^C(0) > 0$. Second, in Section A.5.2, we argue that other paths can't be the basis of an equilibrium.

A.5.1 The unique saddle path

We already established in the text that there is a unique steady state. Next, we verify that the local saddle-point property: the Jacobian of the system of differential equation at $(\bar{A}_d^C, \bar{\xi}^C)$ has two real eigenvalue which have opposite sign. The Jacobian is

$$\begin{pmatrix} (r+\rho+\kappa) & -\rho\psi'(\bar{A}_d^C) \\ -\alpha D^{C\prime}(\bar{\xi}^C) & -\alpha \end{pmatrix}.$$

Clearly, the determinant of the Jacobian is strictly negative which for a 2-by-2 matrix means that the matrix has two real eigenvalue of opposite sign. We can then apply Theorem 8.3.2 in Hubbard and West (1995) to assert that there is a unique trajectory that tends to $(\bar{A}_d^C, \bar{\xi}^C)$ from the left. This saddle path is indicated by the plain curve with double arrow in Figure 7.

Next, we need to show that this saddle path can be extended back to the y-axis, delivering the initial condition $\xi^C(0)$. We proceed in two steps. First we argue that, as long as $A_d^C(t) \geq 0$, the saddle path has to remain trapped into the area denoted by K and shaded in the figure, i.e. the area delimited by the y-axis to the west, the isocline $\dot{\xi}^C(t) = 0$ to the north, and the isocline $\dot{A}_d^C(t) = 0$ to the south. We know that the saddle path must eventually lie in K. Let t_1 be the last time when the saddle path enters K from outside. After t_1, the saddle path stays in K and converges to the steady state $(\bar{A}_d^C, \bar{\xi}^C)$. When the saddle path is in K, $A_d^C(t)$ increases and $\xi^C(t)$ decreases. Therefore, we have $A_d^C(t_1) < \bar{A}_d^C$ and $\xi^C(t_1) > \bar{\xi}^C$. Suppose that, at t_1, the saddle path enters K from the north, crossing the isocline $\dot{\xi}^C(t_1) = 0$ from above. Differentiating ODE (49) yields:

$$\ddot{\xi}^C(t) = (r+\rho+\kappa)\dot{\xi}^C(t_1) - \rho\psi'(A_d^C(t_1))\dot{A}_d^C(t_1) = -\rho\psi'(A_d^C(t_1))\dot{A}_d^C(t_1) > 0.$$

since $\psi'(A) < 0$ and $\dot{A}_d^C(t_1) > 0$ because $A_d^C(t_1) < \bar{A}_d^C$ and lies above the isocline $\dot{A}_d^C(t) = 0$. Thus, just after t_1, $\dot{\xi}^C(t)$ is strictly positive. But this is a contradiction: since the saddle path enters K from the north, at time t_1 $\dot{\xi}^C(t)$ must move from being zero to being strictly negative. Alternatively the saddle path cannot enter K from the south, because i) at that time $\xi(t)$ would have a value less than the steady state and *ii*) once the saddle path enters K for the last time, $\xi(t)$ is decreasing.

Now let us start the system on the saddle path with an initial condition to the left of the steady state, say $\tilde{A}_d^C(t_0)$ and $\tilde{\xi}^C(t_0)$, and let us run the system backward in time, for $t_0 - s \leq t_0$ (formally, this means making the change of variable $u = t_0 - s$ in the system of ODEs (49) and (50)). Graphically, think of moving along the saddle path towards the northwest of Figure 7. Since the saddle path stays in K, we know that $\tilde{A}_d^C(t_0 - s)$ is decreasing in s. Moreover, note that $\tilde{\xi}^C(t_0 - s) > \tilde{\xi}^C(t_0)$ and that $\tilde{A}_d^C(t_0 - s) < \tilde{A}_d^C(t_0)$. Plugging this back into ODE (50), we find that

$$\frac{d\tilde{A}_d^C(t_0-s)}{ds} = -\dot{\tilde{A}}_d^C(t_0-s) = -\alpha\left(A - \tilde{A}_d^C(t_0-s) - D^C(\tilde{\xi}^C(t_0-s))\right) \leq -\alpha\left(A - \tilde{A}_d^C(t_0) - D(\tilde{\xi}^C(t_0))\right) = -\alpha\dot{\tilde{A}}_d^C(t_0) < 0.$$

So the derivative of $\tilde{A}_d^C(t_0-s)$ is negative and bounded away from zero, implying that $\tilde{A}_d^C(t_0-s)$ reaches zero in finite time, say at s_0. This proves that the saddle path extends to the y-axis, and delivers the initial condition $\xi^C(0) = \tilde{\xi}^C(t_0-s_0)$.

A.5.2 Ruling out other solutions

Next, we need to show that other solutions of the system (49)-(50) can't be the basis of an equilibrium.

Preliminary remarks. Let J be the region of the positive quadrant below both isoclines and such that $A_d^C < \bar{A}_d^C$. Similarly, let L be the region of the positive quadrant above both isocline. The argument that allowed us to conclude that the saddle path stays trapped in region K also shows that a solution of the system can only move from region K to region L, and not vice versa. Thus, once a solution leaves K to L, it never comes back to K. One easily shows (using the same argument) that a solution can never leave region L. Similarly, one can show that a solution can only move from region K to region J, and not vice versa.

Now consider alternative initial conditions for $\xi^C(0)$. We let ξ_1 (ξ_2) denote the intersection of the $\dot{A}_d^C(t) = 0$ ($\dot{\xi}^C(t) = 0$) isocline with the y-axis. The condition for the existence of a steady state with $\bar{A}_d^C > 0$ implies that $\xi_1 \leq \xi_2$.

An initial condition $\xi^C(0) < \xi^1$. This can't be the basis of an equilibrium because $\dot{A}_d^C(0) < 0$: given that $A_d^C(0) = 0$, this would violate the dealers' short-selling constraint.

An initial condition $\xi^C(0) \in [\xi_1, \xi_2]$. Suppose there is a candidate equilibrium path with an initial condition in $[\xi_1, \xi_2]$ that is different from that of the saddle path. Because solutions of ODEs never cross, this candidate equilibrium path remains different from the saddle path at all subsequent times. Then we claim that the equilibrium path would eventually leave region K. Suppose that it stayed in K: then $A_d^C(t)$ would be increasing and bounded above and $\xi^C(t)$ would be is decreasing, so this candidate equilibrium path would have a limit in K as $t \to \infty$. But the limit must be equal to the unique steady state of the model which is impossible because this candidate equilibrium path is different from the saddle path. Thus, this we have two possibilities.

If the equilibrium leaves region K for region J at some time t, then it is clear from the figure that $A_d^C(t) < \bar{A}_d$. We also know from the previous paragraph that it never re-enters region K. Given that $\dot{A}_d^C(t) < 0$ in J, and given the equilibrium restrictions that $A_d^C(t) \geq 0$ and $\xi^C(t) \geq 0$, we obtain that the equilibrium path must stay trapped in region J forever, with $A_d^C(s) \leq A_d^C(t) < \bar{A}_d^C$ for all $s \geq t$. Using the fact that $A_d^C(t)$ and $\xi^C(t)$ are decreasing in J and bounded below by zero, we obtain that the candidate equilibrium path has a finite limit as $t \to \infty$, with $A_d^C(\infty) < \bar{A}_d^C$. But this is impossible given that $(\bar{A}_d^C, \bar{\xi}^C)$ is the unique steady state of the system.

If the equilibrium leaves region K for region L then we know it remains in L forever after. Starting from some time t when the candidate equilibrium is away from the boundary of L, we can solve ODE (49) given the path for $A_d^C(t)$:

$$\begin{aligned}
\xi^C(s) &= \xi^C(t)e^{(r+\rho+\kappa)(s-t)} - \int_t^s \rho\psi(A_d^C(u))e^{-(r+\rho+\kappa)(u-s)}\,du \\
&\geq \xi^C(t)e^{(r+\rho+\kappa)(s-t)} - \rho\psi(A_d^C(t))\int_t^s e^{-(r+\rho+\kappa)(u-s)}\,du \\
&= \left(\xi^C(t) - \frac{\rho\psi(A_d^C(t))}{r+\rho+\kappa}\right)e^{(r+\rho+\kappa)(s-t)} + \frac{\rho\psi(A_d^C(t))}{r+\rho+\kappa}.
\end{aligned}$$

where the second inequality follows because $A_d^C(t)$ is increasing since the equilibrium stays in L, and the third equality from integrating. Because we start away from the boundary we have that $\dot{\xi}^C(t) = (r+\rho+\kappa)\xi^C(t) - \rho\psi(A_d^C(t)) > 0$, so it follows from the above formula that $\xi^C(s)$ grows towards infinity at rate $r+\rho+\kappa$. But

$$p^C(s)e^{-rs} \quad \geq \quad p^C(s)e^{-rs} - \mathbb{E}_s\left[p(\tilde{T})e^{-r\tilde{T}}\right] = \xi^C(s)e^{-rs}.$$

So $p^C(s)e^{-rs}$ goes to infinity given that $\xi^C(s)$ grows at a rate $r+\rho+\kappa > r$. Thus the no-bubble condition is violated, and this rule out this candidate equilibrium path.

An initial condition $\xi^C(0) > \xi_2$. Then the equilibrium path starts in region L so the reasoning of the previous paragraph implies that the no-bubble condition is violated.

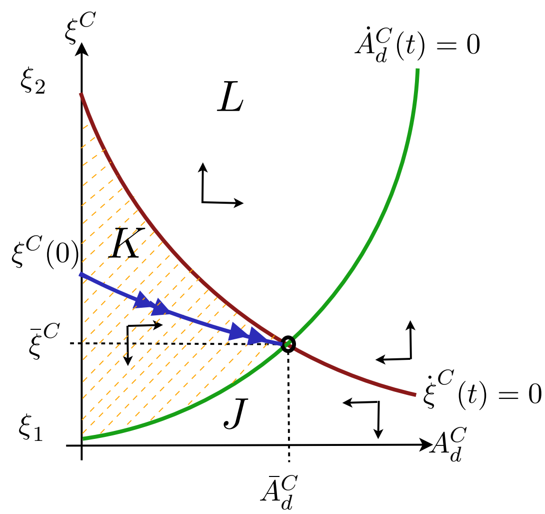

Figure 7: Phase diagram

A.6 Proof of Proposition 3

The price goes up during the recovery because it solves the ODE $\dot{p}(t) = rp(t)$ so it is equal to $p(t) = p(T_\rho)e^{r(t-T_\rho)}$, which is an increasing function of time. Before the recovery, the price solves the ODE:

$$\dot{p}^C(t) = (r+\rho)p^C(t) - \rho p(t,t). \tag{51}$$

Note that $p(t,t) = e^{-r(T-t)}\bar{\xi}/r$, where T denotes the time at which dealers have unwound their inventories and the price has reached its steady-state value. By definition of T and $\psi(A_d(t)^C)$, we have

$$\psi(A_d(t)^C) = e^{-(r+\kappa)(T-t)}\bar{\xi},$$

implying that:

$$p(t,t) = \left[\psi(A_d(t)^C)\right]^{\frac{r}{r+\kappa}} \bar{\xi}^{\frac{\kappa}{r+\kappa}}.$$

Since $A_d^C(t)$ is increasing and $\psi(A)$ is decreasing, it follows that $p(t,t)$ is decreasing. Now integrating (51) and using the no-bubble condition, it follows that:

$$p^C(t) = \int_t^\infty e^{-(r+\rho)(s-t)}\rho p(s,s)\, ds < \frac{\rho p(t,t)}{r+\rho},$$

because $p(s,s) < p(t,t)$. Note that this implies in particular that $p^C(t) < p(t,t)$: at the recovery time, the price jumps up. Rearranging this inequality gives $(r+\rho)p^C(t) - p(t,t) < 0$ and comparing with (51) yields $\dot{p}^C(t) < 0$.

A.7 Proof of Proposition 5

We first establish the equivalence between the two conditions of the Proposition. After plugging $p^C(t) = \bar{p}_0^C$, $\xi^C(t) = \bar{\xi}_0^C$ and $p(t,t) = \bar{p}$ in equation (26) one finds

$$
\begin{aligned}
& \frac{\rho\left(\bar{p} - \bar{p}_0^C\right)}{\bar{p}_0^C} > r \\
\Leftrightarrow \quad & \frac{\rho\left(\bar{\xi} - \bar{\xi}_0^C\right)}{r+\kappa} > \bar{\xi}_0^C \\
\Leftrightarrow \quad & \frac{\rho}{r+\kappa+\rho}\bar{\xi} > \bar{\xi}_0^C & (52) \\
\Leftrightarrow \quad & \sum_{i=1}^{I} \pi_i U_i^{C\prime -1}\left(\frac{\rho}{r+\kappa+\rho}\bar{\xi}\right) < \sum_{i=1}^{I} \pi_i U_i^{C\prime -1}\left(\bar{\xi}_0^C\right) & (53) \\
\Leftrightarrow \quad & \sum_{i=1}^{I} \pi_i U_i^{C\prime -1}\left(\frac{\rho}{r+\kappa+\rho}\bar{\xi}\right) < A, & (54) \\
& & (55)
\end{aligned}
$$

where we moved from (52) to (53) by applying the strictly decreasing function $\sum_{i=1}^{I} \pi_i U_i^{C\prime -1}(\xi)$ on both sides of (52); and we moved from (53) to (54) by using the market clearing condition (**??**).

Now inspecting (30) and (31) it is clear that condition (54) is necessary and sufficient for $\bar{A}_d^C > 0$, i.e. necessary and sufficient for dealers to accumulate inventories during the crisis.

A.8 Proof of Proposition 6

Using equation (19), normalizing T_ρ to 0 and assuming that $A_d(T_\rho) = \bar{A}_d^C$, we get

$$\bar{A}_d^C + \alpha \int_0^T e^{\alpha s}\left[A - \sum_{i=1}^{I} \pi_i U_i^{\prime -1}\left[\xi(s)\right]\right] ds = 0.$$

With the functional form $u_i(a) = \varepsilon_i a^{1-\sigma}/(1-\sigma)$ we have $U_i^{\prime -1}\left[\xi(s)\right] = \left[\bar{\varepsilon}_i/\xi(s)\right]^{1/\sigma}$ and $\xi(s) = \bar{\xi}e^{-(r+\kappa)(T-s)}$. Hence,

$$\bar{A}_d^C + \alpha \int_0^T e^{\alpha s}\left[A - \sum_{i=1}^{I} \pi_i \left[\frac{\bar{\varepsilon}_i}{\bar{\xi}}\right]^{1/\sigma} e^{\left(\frac{r+\kappa}{\sigma}\right)(T-s)}\right] ds = 0.$$

Notice steady-state market clearing after the recovery implies that $\sum_{i=1}^{I} \pi_i \left(\bar{\varepsilon}_i/\bar{\xi}\right)^{1/\sigma} = A$. Thus, after some calculations, we arrive at

$$\frac{\bar{A}_d^C - A}{A} + \frac{\gamma}{\gamma-\alpha}e^{\alpha T} - \frac{\alpha}{\gamma-\alpha}e^{\gamma T} = 0, \tag{56}$$

where $\gamma \equiv \frac{r+\kappa}{\sigma}$. On the other hand, steady state market clearing during the crisis – equation (31) – yields

$$A = \bar{A}_d^C + \sum_{i=1}^{I} \pi_i \left(\frac{\bar{\varepsilon}_i^C}{\bar{\xi}^C}\right)^{1/\sigma}.$$

Combined with (30), $\bar{\xi}^C = \frac{\rho}{r+\kappa+\rho}\bar{\xi}e^{-(r+\kappa)T}$,

$$A = \bar{A}_d^C + e^{\gamma T}\sum_{i=1}^{I} \pi_i \left[\frac{r+\kappa+\rho}{\rho}\frac{\bar{\varepsilon}_i^C}{\bar{\xi}}\right]^{1/\sigma}. \tag{57}$$

Using the fact that $\bar{\xi}^{\frac{1}{\sigma}} = \sum_{i=1}^{I} \pi_i \left(\bar{\varepsilon}_i\right)^{1/\sigma} / A$, (57) becomes

$$e^{\gamma T} \Omega = \frac{A - \bar{A}_d^C}{A}, \tag{58}$$

where $\Omega \equiv \left(\frac{r+\kappa+\rho}{\rho}\right)^{1/\sigma} \sum_{i=1}^{I} \pi_i \left(\bar{\varepsilon}_i^C\right)^{1/\sigma} / \sum_{i=1}^{I} \pi_i \left(\bar{\varepsilon}_i\right)^{1/\sigma}$. Substitute (58) into (56) to obtain

$$T = \frac{1}{\alpha - \gamma} \ln \left[\frac{\alpha}{\gamma} + \left(\frac{\gamma - \alpha}{\gamma} \right) \Omega \right]. \tag{59}$$

Finally, substitute the expression for T given by (59) into (58) to obtain

$$\bar{A}_d^C = A \left\{ 1 - \Omega \left[\frac{\alpha}{\gamma} + \left(\frac{\gamma - \alpha}{\gamma} \right) \Omega \right]^{\frac{\gamma}{\alpha - \gamma}} \right\}.$$

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