Growth and Welfare Effects of Business Cycles In Economies with Idiosyncratic Human Capital Risk

First Version: September, 2001

This Draft: July, 2002

Tom Krebs*

Brown University†

Abstract

This paper develops a tractable macroeconomic model with incomplete markets and proves the existence of recursive equilibria that are simple in the sense that equilibrium asset returns (asset prices) only depend on the exogenous aggregate shock. Moreover, equilibrium allocations and asset returns can be found by solving a one-agent decision problem. This characterization result is used to show that a reduction in the variation in idiosyncratic human capital risk decreases the ratio of physical to human capital and increases the total investment return and welfare. If the degree of risk aversion is less than or equal to one, then economic growth is enhanced. Finally, this paper provides a quantitative assessment of the macroeconomic effects of business cycles based on a calibrated version of the model. Even for relatively small degrees of risk aversion (around one) the model implies that the elimination of business cycles has substantial effects on investment in physical and human capital, economic growth, and welfare.

---

*I would like to thank for helpful comments Oded Galor, Peter Howitt, Tomo Nakajima, Herakles Polemarchakis, Tony Smith, David Weil and seminar participants at Arizona State University, Brown University, Carnegie-Mellon University, the Economic Theory Conference, Ischia 2001, and the Incomplete Markets Workshop, Stony-Brook 2001. All errors are mine.

†Department of Economics, Box B, Brown University, Providence, RI 02912. E-mail: tom_krebs@brown.edu.

I. Introduction

Recent work on dynamic general equilibrium models with infinitely-lived agents and uninsurable idiosyncratic risk has provided important insights into the macroeconomic effects of market incompleteness (Ljungqvist and Sargent, 2000). One drawback of this incomplete-markets approach to macroeconomics is that recursive equilibria are in general complex in the sense that the minimal state space contains endogenous variables (Kehoe and Levine, 2001, Krebs, 2002). This tractability problem has two undesirable consequences. First, the theoretical literature still lacks a general existence proof.[1] Second, applied work has to rely on sophisticated numerical methods to compute recursive equilibria even for simple economic environments.[2] This paper develops a tractable incomplete-markets model for which there exist recursive equilibria that are simple in the sense that endogenous equilibrium prices (asset returns) only depend on exogenous aggregate shocks.[3] This simplicity of equilibrium means that issues of existence and comparative dynamics can be studied at a level of generality comparable to the complete-markets literature, and that many quantitative applications are computationally straightforward.

The model developed in this paper is an incomplete-markets version of the class of convex growth models analyzed by Jones and Manuelli (1990) and Rebelo (1991).[4] More specifically,

[1]Duffie, Geanakoplos, Mas-Colell, and McLennan (1994) show the existence of stationary recursive (Markov) equilibria for exchange economies, but they rule out short-sales (borrowing) by assumption and use a state space that includes endogenous variables in addition to the wealth distribution. Becker and Zilcha (1997) and Reffett, Mirman, and Morand (2002) prove the existence of recursive equilibria for neoclassical production economies with no assets except physical capital (no bonds).

[2]For applied work relying on computational methods, see, for example, Aiyagari (1994), denHaan (1997), Heaton and Lucas (1996), Huggett (1993), and Krusell and Smith (1998).

[3]See also Magill and Quinzii (2000) for a tractable incomplete-markets model with quadratic utility functions and Angeletos and Calvet (2001) and Davis and Willen (2001) for work based on exponential utility functions and normally distributed random variables. Krebs (2001) considers a simplified version of the current model with log-utility preferences and no aggregate shocks.

[4]See also Alvarez and Stokey (1998), Caballe and Santos (1993), and Jones et al.(1999).

households have identical CRRA-preferences, production displays constant returns to scale with respect to reproducible input factors (physical and human capital), and all markets are competitive. There are aggregate productivity shocks that affect the return to physical and human capital investment (stock returns and wages), and there are idiosyncratic human capital shocks that only affect human capital returns.[5] These idiosyncratic human capital shocks are unpredictable, although their distribution may vary with the aggregate state. Finally, the financial market structure is incomplete in the sense that there are no assets with payoffs that depend on idiosyncratic shocks. However, households have the opportunity to trade stocks (accumulate physical capital) and any asset in zero net supply with payoffs that depend on the aggregate shock variable (bonds). In particular, all households can borrow and lend at the common risk-free rate. Moreover, households' ability to trade existing assets is only limited by their ability to repay their debt in the future. In short, the only market imperfection is the lack of explicit insurance markets for idiosyncratic human capital risk.[6]

This paper shows that there exist simple recursive equilibria in which endogenous asset returns (prices) only depend on the exogenous aggregate state. In particular, neither the endogenous wealth distribution nor idiosyncratic shocks affect equilibrium returns. Moreover, equilibrium allocations and returns can be found by solving a one-agent decision problem. Thus, the incomplete-markets model analyzed in this paper is as tractable as its

[5] A negative human capital shock (depreciation shocks) might occur when specific skills are either destroyed or made obsolete in the event of a job loss. Jovanovic (1979) and Ljungvist and Sargent (1998) analyze search models with specific human capital, but they assume risk-neutral workers and do not model the accumulation of physical capital.

[6] More formally, this paper follows Hernandez and Santos (1996), Levine and Zame (1996), and Magill and Quinzii (1994) in limiting the set of tradable assets, but not limiting the extent to which existing assets may be traded beyond what is required to rule out Ponzi schemes. In contrast, Alvarez and Jermann (2000), Kehoe and Levine (1993, 2001), and Kocherlakota (1996) do not limit the set of tradable assets, but constrain the extent to which existing assets may be traded by introducing additional solvency constraints that arise from commitment/enforcement problems. Zhang (1997) and Perri and Krueger (2001) analyze models that feature both incomplete markets and solvency constraints. Notice that for the incomplete-markets model considered here, the equilibrium allocation does not change if the trading of existing assets is restricted by additional solvency constraints.

complete-markets counterpart. However, whereas idiosyncratic risk does not affect the equilibrium allocation when markets are complete, it does affect the equilibrium allocation in the incomplete-markets model. Consequently, the two models may lead to very different policy conclusions.

Two properties of the model are essential in deriving the characterization and existence result. First, in equilibrium the ratio of physical to human capital (capital-to-labor ratio) is identical across households regardless of their current wealth or current idiosyncratic shock realization, which implies the existence of a reduced-form production function that is linear. Second, households choose not to trade the assets in zero-net-supply. This no-trade result extends the work by Constantinides and Duffie (1996) to production economies. In accordance with Constantinides and Duffie (1996), the current paper emphasizes the importance of permanent income shocks in the sense that income follows (approximately) a logarithmic random walk. Thus, neither borrowing nor running down savings (self-insurance) is an optimal response to negative income shocks. However, in contrast to Constantinides and Duffie (1996), this paper derives the random walk property of income as an endogenous outcome.

In a highly influential contribution, Lucas (1987) argues that the welfare cost of business cycle fluctuations might be very small. His argument is based on a calibrated version of a complete-markets (representative-agent) model without production, that is, Lucas (1987) assumes that there is no uninsurable idiosyncratic risk and that aggregate output is exogenous. In contrast, the model developed in this paper allows for uninsurable idiosyncratic human capital risk and for an endogenous output response. The second part of this paper analyzes to what extent these deviations from the Lucas-framework change the conclusions regarding the welfare cost of business cycles. The economic motivation for this part of the paper comes from recent empirical work (Storesletten, Telmer, and Yaron, 2001, and Meghir and Pistaferri, 2001) that has documented strong counter-cyclical variations in uninsured idiosyncratic labor income (human capital) risk, and the idea that the elimination of business cycles might lead to the elimination of these variations in idiosyncratic risk (Atkeson and Phelan, 1994, Imrohoroglu, 1989, Krusell and Smith, 1999, and Storesletten et al., 2000).

A qualitative analysis shows that business cycles have the following general effects on growth and welfare. First, a reduction in the variation in idiosyncratic human capital risk makes human capital investment less risky, and therefore induces households to invest more in high-return human capital. Thus, economic growth increases if total investment in physical and human capital does not decrease, which is the case if the degree of relative risk aversion is less than or equal to one. However, if the degree of risk aversion is larger than one, then economic growth may decrease due to the reduction in total investment. Second, although the growth effect of eliminating business cycles might be negative for high degrees of risk aversion, the welfare effect is always positive since the equilibrium allocation is the solution to a one-agent decision problem.

In addition to the qualitative analysis, this paper also provides a quantitative assessment of the growth and welfare consequences of business cycle fluctuations. The quantitative analysis assumes that the elimination of business cycles eliminates both the fluctuations in aggregate productivity and the variation in idiosyncratic labor income risk, and the model economy is calibrated so that the variation in idiosyncratic labor income risk in the economy with business cycles matches the estimates obtained by recent empirical studies (Storesletten et al., 2001, and Meghir and Pistaferri, 2001). The general finding is that even for moderate degrees of risk aversion (around one), business cycle fluctuations have a substantial impact on resource allocation (investment in physical capital versus investment in human capital), growth, and welfare. In particular, the welfare gain of eliminating business cycles is at least one order of magnitude, and perhaps even two orders of magnitude, larger than what Lucas (1987) has found for the complete-markets exchange economy. Almost all of the welfare gain is due to the elimination of the variation in uninsurable idiosyncratic human capital risk, which improves welfare for two reasons: it eliminates the variation in the volatility of individual consumption growth and, for moderate degrees of risk aversion, it increases average consumption growth. Both of these welfare effects are substantial, but the volatility effect is somewhat stronger. In short, even without an endogenous output response (exchange economy) the welfare costs of business cycles are quite large once market incompleteness is introduced, but taking into account the endogeneity of output (production economy) increases the cost even further.

The welfare costs of business cycles found in this paper are significantly larger than the welfare costs found by the previous literature on incomplete-market economies (Atkeson and Phelan, 1994, Imrohoroglu, 1989, Krusell and Smith, 1999, and Storesletten et al., 2000). There are three reasons for this difference. First, in this paper and in Storesletten et al. (2000), idiosyncratic income shocks matter to households because they are permanent (random walk!). In contrast, Imrohoroglu (1989) and Krusell and Smith (1999) only consider income shocks with moderate degree of persistence, which implies that self-insurance becomes a very effective means to insulate consumption from income shocks. Second, in this paper the level of aggregate economic activity affects the magnitude of idiosyncratic income losses. That is, recessions are not only times in which more workers become unemployed, but also times in which the cost of unemployment increases. In contrast, the previous literature has assumed that the level of aggregate economic activity only affects the probability of becoming unemployed, but not the magnitude of the income loss experienced by displaced workers.[7] As has been pointed out by Atkeson and Phelan (1994) and Krusell and Smith (1999), in this case the elimination of business cycles only affects the equilibrium outcome

[7] Clearly, the probability of becoming unemployed increases during an economic downturn. However, there are also good reasons to expect the size of the income loss to be affected: the distribution of employment opportunities worsens inducing the unemployed worker to accept lower offers and to increase the average time of search (increase in unemployment duration).

if there are indirect price effects, which are absent in the current model and relatively small in Krusell and Smith (1999).[8]

The final reason for the diverging estimates of the welfare costs of business cycles is related to the growth effect of business cycles. More specifically, the previous incomplete-markets literature has relied on a framework that either disregards the output response to changes in business cycle activity (Imrohoroglu, 1989) or implies that the elimination of business cycles decreases output (Krusell and Smith, 1999, and Storesletten et al., 2000). In contrast, in the current model the elimination of business cycles enhances economic growth, at least for moderate degrees of risk aversion. This positive relationship between business cycles and economic growth finds support in cross-country data (Ramey and Ramey, 1995)[9], and the magnitude of this growth effect implied by the calibrated model economy is broadly consistent with the estimates obtained by Ramey and Ramey (1995).[10]

II. Model

Time is indexed by $t = 0, 1, \ldots$ and individual households by $i = 1, \ldots, I$. A complete description of the exogenous state of the economy in period t is a vector $(s_{1t}, \ldots, s_{It}, S_t)$, where we interpret s_{it} as a household-specific (idiosyncratic) shock and S_t as an economy-wide

[8] Storesletten et al. (2000) find larger indirect price effects than Krusell and Smith (1999) because they assume a higher degree of relative risk aversion (4 versus 1) and use an individual income process with highly persistent, counter-cyclical component.

[9] But see also Kormendi and Meguire (1985) for the opposite finding. Acemoglu and Zilbotti (1997) and Quah (1993) find a strong negative link between per capita income and volatility of growth.

[10] Jones, Manuelli, and Stachetti (1999) study this relationship within the context of the complete-markets version of the human capital model, in which case business cycles may affect total investment, but not the capital-to-labor ratio. Barlevy (2000) provides an explanation based on a representative-agent model with endogenous technological innovation (Aghion and Howitt, 1992), and Acemoglu and Zilbotti (1997) discuss a model in which non-diversifiable entrepreneurial risk affects aggregate volatility and economic growth. Finally, Boldrin and Rustichini (1994) discuss the relationship between growth and fluctuations using a model with deterministic fundamentals and indeterminate equilibria.

(aggregate) shock. We assume that s_{it} is an element of a time- and household-independent set $\mathbf{s}$, and that S_t is an element of a time-invariant set $\mathbf{S}$. To avoid mathematical technicalities, the formal proofs also assume that the two sets $\mathbf{s}$ and $\mathbf{S}$ are finite. We denote the vector of idiosyncratic shocks by $s_t = (s_{1t}, \ldots, s_{It})$. A (partial) history of idiosyncratic, respectively aggregate, shocks is denoted by $s^t = (s_0, \ldots, s_t)$, respectively $S^t = (S_0, \ldots, S_t)$. Clearly, the ordered set of all histories defines an event tree with date-events (nodes) (s^t, S^t).

The process of exogenous shocks, $\{s_t, S_t\}$, is a Markov process with stationary transition probabilities denoted by $\pi(s_{t+1}, S_{t+1}|s_t, S_t)$ or $\pi(s', S'|s, S)$. We make two assumptions on these transition probabilities. First, idiosyncratic shocks have no predictive power: $\pi(s_{t+1}, S_{t+1}|s_t, S_t) = \pi(s_{t+1}, S_{t+1}|S_t)$. Second, households are ex-ante identical in the sense that $\pi(\ldots, s_{i,t+1}, \ldots, s_{i',t+1}, \ldots, S_{t+1}|S_t) = \pi(\ldots, s_{i',t+1}, \ldots, s_{i,t+1}, \ldots, S_{t+1}|S_t)$. For simplicity, we also assume $\pi(s', S'|s, S) > 0$ for all $(s, S) \in \mathbf{s} \times \mathbf{S}$. The transition probabilities in conjunction with the initial distribution define in the canonical way the node-probabilities $\pi(s^t, S^t)$ and the conditional node-probabilities $\pi(s^{t+n}, S^{t+n}|s^t, S^t)$.

Economic variables at time t are often defined by functions $x_t : (\mathbf{s})^{t+1} \times (\mathbf{S})^{t+1} \to \mathbb{R}^n$, $x_t = x_t(s^t, S^t)$. Any function x_t defines a random variable in the canonical way. For this random variable, we denote the unconditional expectation by $E[x_t] = \sum_{s^t,S^t} \pi(s^t, S^t)x_t(s^t, S^t)$ and the conditional expectation by $E[x_{t+n}|s^t, S^t] = \sum_{s^t,S^t} \pi(s^{t+n}, S^{t+n}|s^t, S^t)x_{t+n}(s^{t+n}, S^{t+n})$.

There is one firm that produces an "all-purpose" good which can be used for consumption, investment in physical capital, and investment in human capital. If the firm employs K_t units of physical capital and H_t units of human capital in period t, then it produces $Y_t = A_t F(K_t, H_t)$ units of the good in period t. Here F is a standard neoclassical production function. More specifically, we assume that F displays constant-returns-to-scale, is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $F(0, H) = F(K, 0) = 0$ as well as $\lim_{K\to 0} F_k(K, H) = \lim_{H\to 0} F_h(K, H) = +\infty$

and $\lim_{K \to \infty} F_k(K,H) = \lim_{H \to \infty} F_h(K,H) = 0$. Total factor productivity is a function $A : \mathbf{S} \to \mathbb{R}_{++}$ that assigns to each aggregate state S_t a (strictly positive) productivity level $A_t = A(S_t)$. The firm rents input factors (physical and human capital) in competitive markets. We denote the rental rate of physical capital by $\tilde{r}_{kt}$ and the rental rate of human capital (the wage rate per efficiency unit of labor) by $\tilde{r}_{ht}$. In each period, the firm hires capital and labor up to the point where current profit is maximized. Thus, the firm solves the following static maximization problem:

$$max_{K_t,H_t} \{ A_t F(K_t,H_t) - \tilde{r}_{kt} K_t - \tilde{r}_{ht} H_t \} \tag{1}$$

Let k_{it} and h_{it} stand for the stock of physical and human capital owned by household i at the beginning of period t, and denote the corresponding investment levels by x_{kit} and x_{hit}. If we denote household $i's$ consumption by c_{it}, then the sequential budget constraint reads:

$$\begin{aligned} c_{it} + x_{kit} + x_{hit} &= \tilde{r}_{kt} k_{it} + \tilde{r}_{ht} h_{it} \qquad (2) \\ k_{i,t+1} &= (1-\delta_{kt}) k_{it} + x_{kit} \ , \quad k_{it} \geq 0 \\ h_{i,t+1} &= (1-\delta_{ht} + \eta_{it}) h_{it} + x_{hit} \ , \quad h_{it} \geq 0 \\ & (k_{i0}, h_{i0}) \ given \ . \end{aligned}$$

In (2) δ_{kt} and δ_{ht} denote the average depreciation rate of human and physical capital, respectively. These average depreciation rates are defined by functions $\delta_k : \mathbf{S} \to \mathbb{R}_+$ and $\delta_h : \mathbf{S} \to \mathbb{R}_+$ assigning to each aggregate shock S_t a deprecation rate $\delta_{kt} = \delta_k(S_t)$, respectively $\delta_{ht} = \delta_h(S_t)$. The term η_{it} denotes a household-specific shock to the stock of human capital and is defined by a function $\eta : \mathbf{s} \times \mathbf{S} \to \mathbb{R}_+$ assigning to each (s,S) a realization $\eta_{it} = \eta(s_{it}, S_t)$. Since $\tilde{r}_{ht}\eta_{it}$ is labor income of household i, the random variable η_{it} determines the nature of idiosyncratic labor income risk.

The budget constraint makes three implicit assumptions. First, it rules out extended periods of unemployment (the household receives wage payments in each period). Second, it does not impose a non-negativity constraint on human capital investment. Third, it neglects the labor-leisure choice of workers.[11]

The formulation (2) does not distinguish between general and specific human capital. Similarly, the shocks to human capital, η_{it}, can be either shocks to general or shocks to specific human capital. The particular application of the model to business cycles (Section IV) emphasizes negative human capital shocks, $\eta_{it} < 0$, that occur when workers loose firm- or sector-specific human capital subsequent to job termination (worker displacement). This interpretation is consistent with the idea that workers choose the level of specific human capital by choosing from a larger number of firms with diverse training policies, and that they pay for this investment in terms of lower starting salaries (Lazear, 1990, and Topel, 1991). It also explains the large permanent wage losses of displaced workers found by the empirical literature (Jacobson, LaLonde, and Sullivan 1993, Neal 1995, and Topel 1991).

To simplify the analysis, we do not explicitly mention financial markets. However, the equilibrium allocation of the above economy in which households accumulate physical capital is also the equilibrium allocation of a stock market economy in which the firm is a stock company that makes the intertemporal investment decision.[12] If we normalize the number of outstanding shares to one, the stock price is $Q_t = K_{t+1}$, household $i's$ equity share is $\theta_{i,t+1}Q_t = k_{i,t+1}$, and the return to equity investment is $\tilde{r}_{kt} - \delta_{kt}$. Moreover, the equilibrium allocation is unchanged if households are given the opportunity to trade $j = 1, \ldots, J$ securities in zero net supply with payoffs $D_{jt} = D_j(S_t)$ (see proposition 1). In particular, the

[11] Human capital accumulation is often modeled as a time investment. This is equivalent to the formulation (2) if human capital production is linear in time.

[12] In general, this type of market arrangement might lead to conceptual problems when markets are incomplete because shareholders (households) do not agree on the optimal investment policy (Magill and Quinzii, 1996). This, however, is not the case for the economy analyzed in this paper since here we have agreement among shareholders in the sense that the equilibrium investment policy maximizes the expected present discounted value of one-period profits using any household's intertemporal marginal rate of substitution to discount future profits.

introduction of a risk-free asset in zero net supply (borrowing and lending at the risk-free rate) will not change the equilibrium allocation.

The budget constraint can be rewritten in a way that shows how the households's optimization problem is basically a standard intertemporal portfolio choice problem. To this end, define the following variables: $w_{it} \doteq k_{it} + h_{it}$ (total wealth) and $\tilde{k}_{it} \doteq k_{it}/h_{it}$ (the capital-to-labor ratio). With this new notation, the fraction of total wealth invested in physical capital is $\tilde{k}_{it}/(1+\tilde{k}_{it})$ and the fraction of total wealth invested in human capital is $1/(1+\tilde{k}_{it})$. Introduce further the following (average) rate of returns on the two investment opportunities: $r_{kt} \doteq \tilde{r}_{kt} - \delta_{kt}$ and $r_{ht} \doteq \tilde{r}_{ht} - \delta_{ht}$. Using this notation, the budget constraint reads:

$$\begin{aligned} w_{i,t+1} &= \left[1 + \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} r_{kt} + \frac{1}{1+\tilde{k}_{it}} \left(r_{ht} + \eta_{it}\right)\right] w_{it} - c_{it} \\ & w_{it} \geq 0 \ , \ \tilde{k}_{it} \geq 0 \ , \\ & (w_{i0}, \tilde{k}_{i0}) \ given \ . \end{aligned} \tag{3}$$

Households have identical preferences over consumption plans $\{c_{it}\}$. These preferences allow for a time-additive expected utility representation:

$$U(\{c_{it}\}) = E\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] \ . \tag{4}$$

Moreover, we assume that the one-period utility function, u, is given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma \neq 1$, or $u(c) = log\, c$, that is, preferences exhibit constant degree of relative risk aversion γ.

In general, a sequential equilibrium is a process of prices (returns) and actions defined by a sequence of functions mapping histories (date-events),(s^t, S^t), into current prices and actions. In this paper, however, we are only interested in sequential equilibria with a recursive (Markov) structure. Indeed, in this paper we focus attention on recursive equilibria that are simple in a sense to be defined next.

Introduce the aggregate capital-to-labor ratio $\tilde{K}_t \doteq K_t/H_t$ and the production function $f = f(\tilde{K})$ with $f(\tilde{K}) \doteq F(\tilde{K}, 1)$. Using the definitions $r_{kt} \doteq \tilde{r}_{kt} - \delta_{kt}$ and $r_{ht} \doteq \tilde{r}_{ht} - \delta_{ht}$, the first-order conditions associated with the firm's static maximization problem define two functions $r_k : \mathbb{R}_+ \times \mathbf{S} \rightarrow \mathbb{R}_+$ and $r_h : \mathbb{R}_+ \times \mathbf{S} \rightarrow \mathbb{R}_+$ as follows

$$
\begin{aligned}
r_k(\tilde{K}_t, S_t) &= A(S_t) f'(\tilde{K}_t) - \delta_k(S_t) \\
r_h(\tilde{K}_t, S_t) &= A(S_t) f(\tilde{K}_t) - \tilde{K}_t f'(\tilde{K}_t) - \delta_h(S_t) \ .
\end{aligned} \tag{5}
$$

Below we show that there is an equilibrium in which the capital-to-labor ratio is determined by a function $\tilde{K} : \mathbf{S} \rightarrow \mathbb{R}_+$ assigning to each aggregate state S_{t-1} a capital-to-labor ratio $\tilde{K}_t = \tilde{K}(S_{t-1})$. Thus, we have $r_{kt} = r_k(\tilde{K}(S_{t-1}), S_t)$ and $r_{ht} = r_h(\tilde{K}(S_{t-1}), S_t)$, and endogenous returns (prices) in period t therefore depend on (S_{t-1}, S_t) only. From the budget constraint (3) it then immediately follows that in this equilibrium any individually optimal plan is generated by a policy function $g : \mathbb{R}_+ \times \mathbf{s} \times \mathbf{S^2} \rightarrow \mathbb{R}^3_+$ that assigns to each state $(w_{it}, s_{it}, S_{t-1}, S_t)$ an action $(c_{it}, w_{i,t+1}, \tilde{k}_{i,t+1})$. Notice that we do not index the policy function, g, by i, that is, we confine attention to symmetric recursive equilibria.

Definition *A simple recursive equilibrium is a list of functions $\tilde{K} : \mathbf{S} \rightarrow \mathbb{R}_+$, $r_k : \mathbf{S} \times \mathbf{S} \rightarrow \mathbb{R}_+$, $r_h : \mathbf{S} \times \mathbf{S} \rightarrow \mathbb{R}_+$, and $g : \mathbb{R}_+ \times \mathbf{s} \times \mathbf{S} \times \mathbf{S} \rightarrow \mathbb{R}^3_+$ satisfying the following conditions.*
i) *Firms maximize: the functions r_k and r_h are defined by (5).*
ii) *Households maximize: the policy function g generates a plan $\{c_{it}, w_{it}, \tilde{k}_{it}\}$ that maximizes expected lifetime utility (4) subject to the budget constraint (3).*
iii) *Markets clear:*

$$
\frac{\sum_i \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} w_{it}}{\sum_i \frac{1}{1+\tilde{k}_{it}} w_{it}} = \tilde{K}_t
$$

The above definition of equilibrium does not explicitly mention the stock of human and physical capital since the equilibrium values of these variables are determined by $k_{it} = \tilde{k}_{it} w_{it}/(1+\tilde{k}_{it})$ and $h_{it} = w_{it}/(1+\tilde{k}_{it})$. Notice also that the above market clearing condition simply states that $\sum_i k_{it}/\sum_i h_{it} = K_t/H_t$. This condition in conjunction with the individual budget constraints imply that $\sum_i k_{it} = K_t$ and $\sum_i h_{it} = H_t$. Similarly, summing over the individual budget constraints implies $Y_t = C_t + X_{kt} + X_{ht}$.

III. Existence and Characterization of Equilibrium

Consider the decision problem of a household i who has direct access to the production technology F. In this case, household i chooses a plan $\{c_{it}, k_{it}, h_{it}, x_{kit}, x_{hit}\}$ solving

$$
\begin{aligned}
& max \sum_{t=0}^{\infty} E\left[\beta^t u(c_{it})\right] && (6)\\
s.t.\ :\ c_{it} + x_{kit} + x_{hit} &= A_t F(k_{it}, h_{it})\\
k_{i,t+1} &= (1-\delta_{kt})k_{it} + x_{kit}\ \ ,\ \ k_{it} \geq 0\\
h_{i,t+1} &= (1-\delta_{ht} + \eta_{it})h_{it} + x_{iht}\ \ ,\ \ h_{it} \geq 0\\
& (k_{i0}, h_{i0})\, given\ ,
\end{aligned}
$$

with $u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$, $\gamma \neq 1$, or $u(c_{it}) = log\, c_{it}$. The stochastic productivity and depreciation parameters in (6) are again defined by functions $A_t = A(S_t)$, $\delta_{kt} = \delta_k(S_t)$, $\delta_{ht} = \delta_h(S_t)$, and $\eta_{it} = \eta(s_{it}, S_t)$. The transition probabilities $\pi(s_{i,t+1}, S_{t+1}|s_{it}, S_t) = \pi(s_{i,t+1}, S_{t+1}|S_t)$ of the Markov process $\{s_{it}, S_t\}$ are given by the formula $\pi(s_{i,t+1}, S_{t+1}|S_t) = \sum_{-i} \pi(s_{1,t+1}, \ldots, s_{I,t+1}, S_{t+1}|S_t)$. Because of our previous assumption that the transition probabilities are symmetric with respect to households, these marginal transition probabilities are the same for all households.

Let w_{it} and $\tilde{k}_{it}$ be defined as before. Because of the constant-returns-to-scale assumption, the maximization problem (6) can be rewritten as

$$
\begin{aligned}
& maxE\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] && (7)\\
s.t.\ :\ \ w_{i,t+1} &= \left[1 + r(\tilde{k}_{it}, s_{it}, S_t)\right] w_{it} - c_{it}
\end{aligned}
$$

$$w_{it} \geq 0 \ , \ \tilde{k}_{it} \geq 0 \ ,$$

$$(w_{i0}, \tilde{k}_{i0}) \ given \ ,$$

where we introduced the total return on investment (in physical and human capital)

$$r(\tilde{k}_{it}, s_{it}, S_t) = \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} r_k(\tilde{k}_{it}, S_t) + \frac{1}{1+\tilde{k}_{it}} \left(r_h(\tilde{k}_{it}, S_t) + \eta(s_{it}, S_t) \right) \ . \tag{8}$$

In (8) the investment return functions r_k and r_h are defined as in (5).

As in the previous section, it is straightforward to show that a plan solving (7) is generated by a policy function $g : \mathbb{R}_+ \times \mathbf{s} \times \mathbf{S} \rightarrow \mathbb{R}^3_+$ assigning to each state (w_{it}, s_{it}, S_t) an action $(c_{it}, w_{i,t+1}, \tilde{k}_{i,t+1})$. Define the consumption-to-wealth ratio , $\tilde{c}_{it} = \frac{c_{it}}{[1+r(\tilde{k}_{it}, s_{it}, S_t)]w_{it}}$. We have the following result:

Proposition 1. *Suppose a solution g to the one-agent decision problem (7) exists and has the property that $\tilde{k}_{it} = \tilde{k}(S_{t-1})$, $\tilde{k} : \mathbf{S} \rightarrow \mathbb{R}_+$, and $\tilde{c}_{it} = \tilde{c}(S_t)$, $\tilde{c} : \mathbf{S} \rightarrow [\epsilon, 1-\epsilon]$, for some $\epsilon > 0$. Then a simple recursive equilibrium exists. In this equilibrium the firm chooses the capital-to-labor ratio $\tilde{K}(S_{t-1}) = \tilde{k}(S_{t-1})$ and households choose the policy g. Moreover, this equilibrium is also an equilibrium for an economy in which households have the opportunity to trade short-lived assets $j = 1, \ldots, J$ in zero net supply with payoffs $D_{jt} = D_j(S_t)$ (no trade in securities $j = 1, \ldots, J$).*

Proof: See appendix.

There is a simple intuition for the result that households will not trade assets in zero net supply if their payoffs only depend on the aggregate state. Because of the joint assumption of homothetic preferences and no exogenous source of income (labor income is generated through human capital accumulation), the relative share of wealth invested in any security is independent of the wealth level. Further, these portfolio shares do not depend on s^t because idiosyncratic shocks have no predictive power. Thus, the (relative) excess demand for any security whose payoffs do not depend on s_{t+1} is the same for all households (independent of w_{it} and s_{it}), and the only way to clear markets is to have zero excess demand for each household.

Proposition 1 assumes that a solution to the one-agent decision problem (7) exists. If $\gamma < 1$ and capital returns are too high or if $\gamma > 1$ and capital returns are too low (too negative), then a solution to (7) will not exist. However, if the condition

$$sup_{S,\tilde{k}} \ \beta E\left[\left(1 + r(\tilde{k}, s_i', S')\right)^{1-\gamma} \mid S\right] < 1 \tag{9}$$

is satisfied, then a solution exists (proposition 2 below). Notice that for $\gamma = 1$ (log-utility), (9) reduces to $\beta < 1$. Condition (9) extends the condition appearing in Jones and Manuelli (1990) to the case of uncertainty. Jones, Manuelli, and Stachetti (1999) consider an economy with uncertainty similar to the one analyzed here. They, however, confine attention to the linear Markov case with Cobb-Douglas production function and no depreciation shocks, but allow for random variables with uncountable support. For linear Markov processes with Cobb-Douglas production function and no depreciation shocks, condition (9) is the finite-state-space analog of the existence condition in Jones, Manuelli, and Stachetti (1999).

Proposition 2. *Suppose condition (9) is satisfied. Then there exists a solution to the maximization problem (7) with $\tilde{k}_{it} = \tilde{k}(S_{t-1})$ and $\tilde{c}_{it} = \tilde{c}(S_t)$, where $\tilde{k} : \mathbf{S} \rightarrow \mathbb{R}_+$ and $\tilde{c} : \mathbf{S} \rightarrow [\epsilon, 1-\epsilon]$. The functions (Euclidian vectors) $\tilde{k}$ and $\tilde{c}$ are the unique solution to the following equation system:*

$$\forall S : \tilde{c}(S) \ = \ \frac{1}{1 + \left(\beta E\left[\left(1 + r(\tilde{k}(S), s_i', S')\right)^{1-\gamma} \tilde{c}^{-\gamma}(S')|S\right]\right)^{1/\gamma}} \tag{10}$$

$$E\left[\frac{r_h(\tilde{k}(S), S') + \eta(s_i', S') - r_k(\tilde{k}(S), S')}{\left(1 + r(\tilde{k}(S), s_i', S')\right)^{\gamma} \tilde{c}(S')^{\gamma}}|S\right] \ = \ 0 \ .$$

In particular, if $\gamma = 1$ (log-utility), we have $\tilde{c} = 1 - \beta$.

Proof: See appendix.

The intuition for the result that the optimal $\tilde{k}_{it}$ only depends on S_{t-1} and the optimal $\tilde{c}_{it}$ only on S_t is similar to the intuition for the no-trade result. First, the portfolio share $1/(1+\tilde{k}_{it})$ and the consumption-to-wealth ratio $\tilde{c}_{it}$ do not depend on w_{it} because of the joint assumption of homothetic preferences and no exogenous source of income (labor income is generated through human capital accumulation). Second, these variables do not depend on s_{it}, or more generally on s^t, because idiosyncratic shocks have no predictive power. Past values of aggregate shocks, $S_0, S_1, \ldots, S_{t-1}$, do not matter because of the Markov assumption.

Combining proposition 1 and 2, we have:

Corollary *Suppose condition (9) is satisfied. Then there exists a simple recursive equilibrium with equilibrium allocation*

$$\tilde{k}_{it} = \tilde{k}(S_{t-1}) \quad ; \quad \tilde{c}_{it} = \tilde{c}(S_t) \quad ; \quad c_{it} = \tilde{c}(S_t)\left[1 + r(\tilde{k}(S_{t-1}), s_{it}, S_t)\right] w_{it}$$

$$k_{it} = \frac{\tilde{k}(S_{t-1})}{1+\tilde{k}(S_{t-1})} w_{it} \quad ; \quad h_{it} = \frac{1}{1+\tilde{k}(S_{t-1})} w_{it} \quad ; \quad w_{i,t+1} = [1-\tilde{c}(S_t)]\left[1 + r(\tilde{k}(S_{t-1}), s_{it}, S_t)\right] w_{it}$$

and aggregate asset returns

$$r_{kt} = r_k(\tilde{k}(S_{t-1}), S_t) \quad ; \quad r_{ht} = r_h(\tilde{k}(S_{t-1}), S_t) \ ,$$

where $\tilde{k}$ and $\tilde{c}$ are the solution to the equation system (10) and r, r_k, and r_h are the functions defined in (5) and (8). Moreover, the above allocation and asset returns are also the equilibrium allocation and asset returns for an economy in which households have the opportunity to trade short-lived securities $j = 1, \cdots, J$ in zero net supply with payoffs $D_{jt} = D_j(S_t)$. In this recursive equilibrium, there is no trade of securities, and security prices are

$$Q_j(S_t) = E\left[M(S_t, S_{t+1}) D_j(S_{t+1}) | S_t\right] \ ,$$

where the pricing kernel, M, is given by

$$M(S_t, S_{t+1}) = \beta E\left[\left(\frac{(1 + r(\tilde{k}(S_t), s_{i,t+1}, S_{t+1})\tilde{c}(S_{t+1})(1-\tilde{c}(S_t))}{\tilde{c}(S_t)}\right)^{-\gamma} | S_t, S_{t+1}\right] \ .$$

It immediately follows from the corollary that

$$\frac{y_{hi,t+1}}{y_{hit}} = \varphi(S_{t-1}, S_t, S_{t+1}) \left[1 + r(\tilde{k}(S_t), s_{i,t+1}, S_{t+1})\right] \tag{11}$$

where $y_{hit} = \tilde{r}_{ht} h_{it}$ is labor income of household i in period t. Thus, conditional on the history of aggregate shocks, the growth rate of labor income is unpredictable (recall that $s_{i,t+1}$ is unpredictable), and the idiosyncratic component of individual labor income therefore follows approximately a logarithmic random walk (see equation (14) for details). In this sense, income shocks are permanent, which provides yet another intuition for the no-trade result.

IV. Macroeconomic Effects of Business Cycles

IV.A. Qualitative Analysis

The subsequent analysis assumes that aggregate shocks are unpredictable, that is, the underlying state process $\{s_t, S_t\}$ and the corresponding marginal process $\{s_{it}, S_t\}$ are i.i.d. with (stationary) distribution $\pi(s_t, S_t)$, respectively $\pi(s_{it}, S_t)$. In the case of i.i.d. shocks, the Euler equations (10) reduce to

$$\begin{aligned} E\left[\frac{r_h(\tilde{k}, S') + \eta(s'_i, S') - r_k(\tilde{k}, S')}{\left(1 + r(\tilde{k}, s'_i, S')\right)^{\gamma}}\right] &= 0 \\ \tilde{c} = 1 - \left(\beta E\left[\left(1 + r(\tilde{k}, s'_i, S')\right)^{1-\gamma}\right]\right)^{1/\gamma} & \quad . \end{aligned} \tag{12}$$

The first equation in (12) implicitly determines a unique value for the state-independent capital-to-labor ratio, $\tilde{k}$. Given this value of $\tilde{k}$, the second equation determines the value of the state-independent consumption-to-wealth ratio, $\tilde{c}$.

In this section we are interested in the change in $\tilde{k}$ and $\tilde{c}$, and the corresponding changes in growth and welfare, when we move from an economy with business cycles to an economy without business cycles. This amounts to "removing" the aggregate shocks S, that is, replacing the joint probabilities $\pi(s_i, S)$ with probabilities $\pi'(s_i)$ and any function with values $f(s_i, S)$ (describing an exogenous economic variable in the economy with business cycles) by a function with values $f'(s_i)$ (describing the same exogenous economic variable in the economy without business cycles). The question that arises is what relationship holds between π and π' as well as f and f'. It seems natural to compute the probabilities in the economy without business cycles as the simple marginals of the probabilities in the economy with business cycles: $\pi'(s_i) = \sum_S \pi(s_i, S)$. Similarly, for the function f' the following principle suggests itself: $f'(s_i) = \sum_S f(s_i, S)\pi(S|s_i)$. In words: the conditional mean of any exogenous variables in the economy with business cycles is equal to the value of this variable in the economy without business cycles. This procedure was first suggested by Krusell and Smith (1999) as a general principle guiding the analysis of business cycle effects in incomplete-markets economies, and is a generalization of the principle adopted by Atkeson and Phelan (1994). Krusell and Smith (1999) call it the integration principle, a terminology we will follow in this paper. Notice that this procedure ensures that the random variable f is a mean-preserving spread of the random variable f', and in this sense the economy with business cycle fluctuations exhibits more risk with no change in the mean of exogenous economic variables.

The integration principle determines in an unambiguous way how to move from an economy with business cycles to an economy without business cycles. However, any conclusion regarding the effects of eliminating business cycles still depends on the way aggregate shocks S affect $\pi(s_i, S)$ and $f(s_i, S)$ in the economy with business cycles. The appendix discusses in more details the different approaches that can be adopted to model the interaction between aggregate shocks and idiosyncratic risk. However, regardless of the particular approach taken, we have the following general result.

Proposition 3. *Assume that $\{s_t, S_t\}$ is an i.i.d. process and $A(S_t) = A$, $\delta_k(S_t) = \delta_k$,* *and* $\delta_h(S_t) = \delta_h$. *Consider two economies, one economy with joint probabilities* $\pi(s_i, S)$ *and idiosyncratic human capital shocks* $\eta = \eta(s_i, S)$ *(the economy with business cycles) and another economy with probabilities* $\pi'(s_i) = \sum_S \pi(s_i, S)$ *and idiosyncratic human capital shocks* $\eta'(s_i) = \sum_S \eta(s_i, S)\pi(S|s_i)$ *(the economy without business cycles). Then the following comparative dynamics results hold, where all inequalities are strict if the function* $\eta = \eta(s_i, S)$ *depends in a non-trivial way on* S.

- Capital-to-labor ratio: $\tilde{k} \geq \tilde{k}'$
- Average investment returns:

$$\begin{aligned} r_k(\tilde{k}) &\leq r_k(\tilde{k}') \\ r_h(\tilde{k}) &\geq r_h(\tilde{k}') \\ E\left[r(\tilde{k}, s_i, S)\right] &\leq E\left[r(\tilde{k}', s_i, S)\right] \end{aligned}$$

- Consumption-to-wealth ratio:

$$\begin{aligned} \tilde{c} \geq \tilde{c}' \quad if \quad \gamma < 1 \\ \tilde{c} = \tilde{c}' = 1 - \beta \quad if \quad \gamma = 1 \\ \tilde{c} \leq \tilde{c}' \quad if \quad \gamma > 1 \end{aligned}$$

- Growth rates:

$$E\left[\frac{c_{i,t+1}}{c_{it}}\right] \leq E\left[\frac{c'_{i,t+1}}{c'_{it}}\right] \quad if \quad \gamma \leq 1 \ ,$$

- Welfare:

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] \leq E\left[\sum_{t=0}^{\infty} \beta^t u(c'_{it})\right]$$

Proof: See appendix.

IV.B. Quantitative Analysis

We continue to assume that the aggregate shock process is a sequence of i.i.d. random variables. As shown above, this implies that the two ratios $\tilde{k}$ and $\tilde{c}$ are time-independent, which in turn yields the following expression for aggregate consumption growth (invoking the law of large numbers)

$$\begin{aligned} \frac{C_{t+1}}{C_t} &= E\left[\frac{c_{i,t+1}}{c_{it}}|S^{t+1}\right] \\ &= \beta\left(1+E[r(\tilde{k},s_{i,t+1},S_{t+1})|S_{t+1}]\right) . \end{aligned} \tag{13}$$

Thus, aggregate consumption growth rates are i.i.d., that is, aggregate consumption follows (approximately) a logarithmic random walk and the risk-free rate is constant. Annual data on real short-term interest rates and consumption show only small deviations from these two properties (Campbell and Cochrane, 1999). Given that this paper uses annual data to calibrate the model economy (see below), the assumption of an i.i.d. aggregate process seems therefore a reasonable first approximation.

Model Specification

We consider a two-state aggregate shock process, $S \in \{L,H\}$, with $prob(S=L)=prob(S=H)=1/2$, where L stands for low aggregate economic activity and H stands for high aggregate economic activity. We assume that $s_{it} \sim N(0,1)$ regardless of the aggregate shock, S. That is, we assume $\pi(s_i,S)=\pi(s_i)\pi(S)$. We further specify that $\eta(s_i,L)=\sigma(L)s_i$ and $\eta(s_i,H)=\sigma(H)s_i$. Thus, we have $\eta_{it} \sim N(0,\sigma^2(L))$ if $S_t=L$ and $\eta_{it} \sim N(0,\sigma^2(H))$ if $S_t=H$. The assumption of normally distributed human capital shocks with $S-$dependent variance allows us to relate the current model to recent empirical studies of idiosyncratic labor income risk (see below). Moreover, the assumption that the aggregate state, S, enters in a non-trivial manner into the function $\eta=\eta(s_i,S)$ ensures that business cycles impact growth and welfare through their effect on idiosyncratic risk. See Appendix IV for more on this issue.

For the baseline economy, we assume log-utility preferences: $\gamma = 1$. In this case, $\tilde{c} = 1 - \beta$ always solves the intertemporal Euler equation. We further assume a Cobb-Douglas production function:$f(\tilde{K}) = A(S)\tilde{K}^{\alpha}$. Aggregate depreciation rate of physical and human capital are equal and may depend on the aggregate state: $\delta_{kt} = \delta_{ht} = \delta(S_t)$. The introduction of aggregate depreciation shocks allows us to match the volatility of both aggregate output and aggregate consumption. Without these aggregate depreciation shocks, the simple human capital model analyzed here generates an excessively smooth aggregate consumption series (Jones et al. 1999). An increase in the aggregate depreciation rate might be the result of an increase in the rate of business failure if plant and business closure destroys specific physical and human capital.

Calibration

We calibrate the model economy as follows. We assume that the period length is one year (annual data). This choice is made to ensure consistency with a number of empirical studies of labor income risk that use annual PSID data (see below). We choose $\alpha = .36$ to match capital's share in income. The remaining parameters $A(L), A(H), \delta(L), \delta(H), \sigma_{\eta}(L), \sigma_{\eta}(H)$, and β are determined in conjunction with the equilibrium value for $\tilde{k}$ by the following restrictions:

- The remaining Euler equation holds
- $E[\delta_t] = .06$
- $E\left[Y_{t+1}/Y_t - 1\right] = .02$
- $E\left[X_{kt}/Y_t\right] = .25$
- $\sigma\left[Y_{t+1}/Y_t\right] = .0237$
- $\sigma\left[C_{t+1}/C_t\right] = .0168$

- $\sigma\left[y_{hi,t+1}/y_{hit}|S_t = L\right] = .24 \quad , \quad \sigma\left[y_{hi,t+1}/y_{hit}|S_t = H\right] = .12$

The first restriction ensures that the portfolio choice $\tilde{k}$ is an equilibrium outcome. The next restriction pins down the average depreciation rate. The value .06 is a compromise between the probably higher depreciation rate of physical capital[13] and the probably lower depreciation rate of human capital. This value is also assume in Jones et al. (2000). The next two restrictions ensure that the model generates a realistic average growth rate and average saving rate, and the following two restrictions imply that the model matches the observed volatility of aggregate consumption and output growth for the period 1949-1999. The last restriction requires the model's process for idiosyncratic labor income risk to be consistent with the evidence form microeconomic data, to which we now turn.

The time series behavior of individual labor income is determined by (11). Using the approximation $log(1+r) \approx r$, this yields for the case $\tilde{c} = 1 - \beta$ (log-utility) the formula

$$log y_{hi,t+1} = \varphi(\tilde{k}, S_t, S_{t+1}) + log y_{hit} + \tilde{\eta}_{it} \ , \tag{14}$$

where $\tilde{\eta}_{it} = \frac{1}{1+\tilde{k}}\eta_{it}$. In other words, the idiosyncratic component of individual labor income approximately follows a logarithmic random walk with error term $\tilde{\eta}_{it} \sim N(0, \sigma_y^2(S_t))$, where $\sigma_y(S_t) = \sigma_\eta(S_t)/(1+\tilde{k})$.[14] The random walk specification is often used by the empirical literature to model the permanent component of idiosyncratic labor income risk (Carroll and Samwick, 1997, Hubbard, Skinner, and Zeldes, 1995, Meghir and Pistaferri, 2001, and Storesletten et.al., 2001). Thus, their estimate of the standard deviation of the error term for the random walk component of annual labor income corresponds to the value of $\sigma_y(S)$.

[13] A common choice is $\delta_k = .10$. However, Cooley and Prescott (1995) argue that $\delta_k = .05$ is more realistic.

[14] We have $\tilde{\eta}_{it}$ instead of $\tilde{\eta}_{i,t+1}$ in equation (14), and the latter is the common specification for a random walk. However, this is not a problem if the econometrician observes the idiosyncratic depreciation shocks with a one-period lag. In this case, (14) is the correct equation from the household's point of view, but a modified version of (14) with $\tilde{\eta}_{i,t+1}$ replacing $\tilde{\eta}_{it}$ is the specification estimated by the econometrician.

For the average standard deviation, $E[\sigma_y(S)]$, Carroll and Samwick (1997) and Hubbard et al.(1995) estimate a value of .15, Meghir and Pistaferri (2001) find an estimate of .19, and Storesletten et al.(2001) have .25. For the baseline economy, we choose $E[\sigma_y(S)] = .18$. Meghir and Pistaferri (2001) and Storesletten et al.(2001) are the only studies so far that allow σ_y to vary with the aggregate state S. Meghir and Pistaferri (2001) estimate that the variation in σ_y, measured by $\sigma[\sigma_y(S)]$, is equal to .05. Storesletten et al.(2001) find $\sigma[\sigma_y(S)] = .10$. For our baseline economy, we choose $\sigma[\sigma_y(S)] = .06$, but we also consider and economy with $\sigma[\sigma_y(S)] = .10$. For the two-state aggregate process used in this section, the two restrictions $E[\sigma_y(S)] = .18$ and $\sigma[\sigma_y(S)] = .06$ translate into $\sigma_y(L) = .24$ and $\sigma_y(H) = .12$.

The above approach might underestimate human capital risk if the actual distribution of idiosyncratic income shocks has a lower tail that is fatter than suggested by the normal distribution framework. For strong evidence for such a deviation from the normal-distribution framework, see Brav, Constantinides, and Greczy (2002) and Geweke and Keane (2000). There is, however, also an argument that the current approach might overestimate human capital risk because it assumes that all of labor income is return to human capital investment. If some component of labor income is independent of human capital investment, as argued in Mankiw, Romer, and Weil (1992), and if this component is random (random endowment of genetic skills), then some part of the variance of labor income is not human capital risk.

Equilibria are computed using a simple two-state approximation of the normally distributed random variable η. More specifically, we replace the random variable $\eta \sim N(0, \sigma_\eta(S))$ by a discrete random variable with two possible realizations, $-\sigma_\eta(S)$ and $+\sigma_\eta(S)$, which occur with equal probability. This approximation procedure will be used throughout this section to compute equilibria. The implied parameter valued are $A(L) = .2869$, $A(H) = .3001$, $\delta(L) = .0756$, $\delta(H) = .0444$, $\beta = .9364$. Notice that the results are quantitatively very similar if a four-state approximation based on Gauss-Hermite quadrature (Judd, 1988) is used.

The above calibration procedure ensures that the model economy matches as many features of the US economy as there are free parameters. It is also interesting to investigate how the calibrated model performs in matching additional features of the US economy. For example, the implied value for the average return on physical capital is $E[r_k(S)] = 5.55\%$. This is higher than the observed real interest rate on short-term US government bonds, but lower than the observed real return on US equity.[15] The model's equity premium is .06 percent, which is a substantial improvement over the value found in Mehra and Prescott (1985) for logarithmic utility (.003 percent), but still very far away from the observed value for the U.S. stock market (7 percent). The model's Sharpe ratio, however, comes much closer to its observed counterpart (3.5 versus 35). Put differently, equity returns are excessively smooth in the model economy, and the implied equity premium is therefore quite small.

The implied average return on investment in human capital, $E[r_h(S)] = 11.92\%$, is in line with the estimates of rate of returns to schooling.[16] Notice that the implied excess return on human capital investment is $E[r_h(S)] - E[r_k(S)] = 6.37\%$. Thus, the model generates a substantial "human capital premium".

Finally, notice that conditional on the aggregate shock, individual consumption growth is normally distributed, $g_{i,t+1} = c_{i,t+1}/c_{it} - 1 \sim N(\mu_g(S_{t+1}), \sigma_g^2(S_{t+1}))$. The calibrated model yields an average standard deviation of consumption growth of $\sigma_g = .5\,\sigma_g(L) + .5\,\sigma_g(H) = 16.83\%$. This amount of consumption volatility is somewhat lower than what is found in the data. For example, using CEX data on consumption of non-durables and services Brav

[15]The RBC literature usually strikes a compromise and chooses the parameter values so that the implied return on capital is 4%, which is somewhat lower than the value used here.

[16]The estimates vary considerably across households and studies, with an average of about 10% (Krueger and Lindhal, 2001).

et al.(2002) find that the standard deviation of quarterly consumption growth ranges from 6 percent to 12 percent for different household groups with an average of about 9 percent. If quarterly consumption growth is i.i.d., then this corresponds to a standard deviation of annual consumption growth of 18 percent.

Results

In accordance with the general principles discussed in Section IV.A, we assume that the elimination of business cycles amounts to replacing the functions $A = A(S), \delta = \delta(S)$, and $\eta = \eta(s_i, S)$ by the $S-$independent functions $A' = .5\, A(L) + .5\, A(H)$, $\delta' = .5\, \delta(L) + .5\, \delta(H)$, and $\eta'(s_i) = .5\, \eta(s_i, L) + .5\, \eta(s_i, H)$. In particular, this means that the elimination of business cycles amounts to replacing the process of human capital risk, $\{\eta_{it}\}$, defined by $\eta_{it} \sim N(0, \sigma_\eta^2(S_t))$, with a process of human capital risk, $\{\eta'_{it}\}$, defined by $\eta'_{it} \sim N(0, \sigma_\eta^{2\,\prime})$ with constant standard deviation $\sigma'_\eta = .5\, \sigma_\eta(L) + .5\, \sigma_\eta(H)$.[17]

The first and second row in table 1 reports the growth and welfare gains from eliminating business cycles, which are calculated as follows. Let $\{c_{it}\}$ stand for the individual consumption process in the equilibrium with business cycles. This process is defined by an initial consumption level, c_{i0}, and an i.i.d. sequence of individual consumption growth rates, $\{g_{it}\}$, that are distributed according to $g_{it} \sim N(\mu_g(S_t), \sigma_g^2(S_t))$ with $S_t = L, H$. When business cycles are removed, a new equilibrium is established with individual consumption process $\{c'_{it}\}$ defined by an initial consumption level c'_{i0} and an i.i.d. sequence of growth rates, $\{g'_{it}\}$, that are distributed according to $g'_{it} \sim N(\mu'_g, \sigma_g^{2\,\prime})$. The first row in table 1 shows the effect of eliminating business cycles on average per capita consumption growth, that is, it reports the

[17] Storesletten et al.(2000) also assume that idiosyncratic shocks are normally distributed in both economies (with and without business cycles),and that the respective standard deviations are related through $\sigma'_\eta = .5\, \sigma_\eta(L) + .5\, \sigma_\eta(H)$. However, they do not discuss how this approach relates to the general integration principle. Moreover, in their computation of equilibria they use a finite-state-space approximation of the normal distribution which minimizes the effect of business cycles since it assumes that S only affects probabilities.

difference $\Delta\mu_g = \mu'_g - \mu_g$, where $\mu_g = .5\,\mu_g(L) + .5\,\mu_g(H)$. The second row in table 1 reports the welfare gains from eliminating business cycles expressed in equivalent growth-rate terms, which are calculated as follows.

Expected lifetime utility associated with the consumption plan $\{c_{it}\}$ is given by

$$
\begin{aligned}
E_0\left[\sum_{t=0}^{\infty}\beta^t \frac{c_{it}^{1-\gamma}}{1-\gamma}\right] &= \frac{c_{i0}^{1-\gamma}}{(1-\gamma)\,(1-\beta E[(1+g)^{1-\gamma}])} \quad \gamma \neq 1 \\
E_0\left[\sum_{t=0}^{\infty}\beta^t log c_{it}\right] &= \frac{1}{1-\beta}\, log c_{i0} + \frac{\beta}{(1-\beta)^2} E[log(1+g)]\ ,
\end{aligned}
\tag{15}
$$

where we dropped the indexes i and t on the consumption growth rate g. The expectation in (15) extends over aggregate and idiosyncratic risk. Define the risk-adjusted consumption growth rate, $\tilde{\mu}_g$, as that certain consumption growth rate which yields a level of expected lifetime utility equal to the level associated with the random consumption growth rate, g, keeping the initial level of consumption, c_{i0}, constant. Using the welfare formula (15), we find

$$
\begin{aligned}
\tilde{\mu}_g &= \left(E[(1+g)^{1-\gamma}]\right)^{1/(1-\gamma)} - 1 \quad \gamma \neq 1 \\
\tilde{\mu}_g &= e^{E[log(1+g)]-1}\ .
\end{aligned}
\tag{16}
$$

The risk-adjusted growth rate allows us to express the welfare cost of business cycles in economically meaningful terms. More precisely, the difference $\Delta\tilde{\mu}_g = \tilde{\mu}_g{}' - \tilde{\mu}_g$ is the total welfare gain from removing business cycles expressed in growth-rate units assuming that $c_{i0} = c'_{i0}$. This total welfare gain is denoted by ΔEU_1 in table 1. Moreover, the difference between the second and first row in table 1, $\Delta\tilde{\mu}_g - \Delta\mu_g = \mu_g - \tilde{\mu}_g$, is the welfare gain from removing business cycles for fixed average consumption growth, that is, the welfare gain that is implied if the calculations are based on an exchange economy.

Table 1 shows that the growth effect is positive and substantial: .07 percent for the baseline economy and .21 percent for the economy with large variations in idiosyncratic risk. Most of the increase in growth is due to the elimination of variation in idiosyncratic risk. However, in contrast to the complete-markets model, in the model with uninsurable idiosyncratic risk the elimination of aggregate productivity and depreciation shocks has an impact on growth. Finally, we observe that the growth effect of business cycles is tremendous if we consider a partial equilibrium model with fixed asset returns: .49 percent for the baseline economy and 1.54 percent for the economy with large variations in idiosyncratic risk.

The second row in table 1 shows that the welfare costs of business cycles are substantial: .22 percent for the baseline economy and .56 percent for the economy with large variations in idiosyncratic risk. Moreover, the welfare cost are at least 10 times larger (baseline economy), and perhaps even 25 times larger (strong variation in idiosyncratic risk), than than the welfare cost in the corresponding complete-markets economy. A comparison of the first and second row in table 1 shows that roughly 1/4 of the total welfare change is due to an increase in average consumption growth, μ_g, and 3/4 is due to the elimination in the variation of individual consumption volatility, $\sigma_g(S)$. In contrast, for the partial equilibrium model the welfare cost of business cycles are all due to the change in average growth.

It is instructive to express the welfare changes in terms of equivalent changes in initial consumption levels. These changes are reported in the third row of table 1. For the log-utility case, using (15) leads to $\Delta log c_{i0} = \beta/(1-\beta)\Delta log(1+\tilde{\mu}_g)$, that is, $\Delta c_{i0}/c_{i0} \approx \Delta\tilde{\mu}_g\beta/(1-\beta)$. Thus, in order to find the welfare changes in terms of equivalent changes in consumption levels, we simply multiply $\Delta\tilde{\mu}_g$ by $\beta/(1-\beta) = 14.72$. For example, for the baseline economy with moderate variation in idiosyncratic risk, the welfare gain from eliminating business cycles is equivalent to the gain from increasing initial consumption by 3.24, and for the economy with strong variations in idiosyncratic risk this gain is 8.24 percent. In contrast, Lucas (1987) finds a welfare gain of only .1 percent for a complete-markets economy with log-utility preferences.[18]

Finally, table 1 also shows the effect of business cycles on asset returns and investment. Notice again that these effects are quite substantial. The strong asset return effect is another indication that general equilibrium effects seem to be very important.

V. Conclusion

This paper makes two contributions. First, it develops a tractable macro model with incomplete markets and provides a convenient characterization of simple recursive equilibria. Second, it uses the model to discuss the qualitative and quantitative effects of business cycles on growth and welfare.

The current paper does not address the issue of why certain insurance markets for idiosyncratic human capital risk are missing. One possible explanation for this lack of insurance might be the asymmetry of information with respect to idiosyncratic human capital shocks. An interesting question for future research is to investigate under what conditions the equilibrium allocation of the incomplete-markets economy is also the constrained efficient allocation of an economy with asymmetric/private information. The results by Atkeson and Lucas (1992) and Cole and Kocherlakota (2001) taken together show that the relationship between competitive allocations and constrained-efficient allocations crucially depends on the nature of private information.

[18] The welfare gains reported in table 1 for the complete-markets economy (last column) are higher than the values found in Lucas (1987) because this paper assumes that aggregate consumption follows a logarithmic random walk, whereas Lucas (1987) assumes that aggregate consumption is trend-stationary.

Appendix 1: Proof of Proposition 1

The proof of proposition 1 splits into two parts. First, it is shown that the solution to the one-agent decision problem (7) is also the equilibrium allocation of a one-agent market economy with supporting prices given by (5) (second welfare theorem for the one-agent economy). The proof offered in the appendix uses Euler equations and transversality condition, but a dynamic programming approach along the lines of Prescott and Mehra (1980) would also work.[19] Second, it is argued that equilibrium choices and prices for the one-agent market economy are also the equilibrium choices and prices for the I-agent economy.

For this proof, denote the solution to the one-agent decision problem (7) by $\{w_{it}^*, \tilde{k}_{it}^*, c_{it}^*\}$ (the optimal plan), respectively g^* (the optimal policy generating the optimal plan). We can think of (7) as the social planner problem for a one-agent economy. There is a market problem corresponding to this social planner problem (7), which is to maximize (4) subject to the budget constraint (3) with (given) market returns $r_{kt} = r_k(\tilde{k}_{it}^*, S_t)$ and $r_{ht} = r_h(\tilde{k}_{it}^*, S_t)$. Denote the solution to this market problem (if it exists) by $\{w_{it}^{**}, \tilde{k}_{it}^{**}, c_{it}^{**}\}$, respectively g^{**}. We first show that if g^* exists and satisfies the properties stated in the proposition, then $g^{**} = g^*$ is also a solution to the market problem.

Notice first that $\{w_{it}^*, \tilde{k}_{it}^*, c_{it}^*\}$ must satisfy the Euler equations associated with (7), which read

$$\begin{aligned} c_{it}^{-\gamma} &= \beta E\left[(1 + r_{i,t+1})c_{i,t+1}^{-\gamma} | s^t, S^t\right] \qquad \text{(A1)} \\ E\left[c_{i,t+1}^{-\gamma}\left(r_{h,t+1} + \eta_{i,t+1} - r_{k,t+1}\right) | s^t, S^t\right] &= 0\ . \end{aligned}$$

Moreover, below we show that the existence of a solution to (7) in conjunction with the

[19] The results in Prescott and Mehra (1980) are not directly applicable because of the unboundedness of decision variables.

condition that $\tilde{k}^*_{it}$ and $\tilde{c}^*_{it}$ are bounded away from zero imply that the transversality condition

$$\beta^t E\left[c_{it}^{-\gamma}(1+r_{it})w_{it}\right] \to 0 \ . \qquad \text{(A2)}$$

holds (the transversality condition is necessary). Moreover, (A1) and (A2) are also the Euler equations and transversality condition associated with the market problem of maximizing (4) subject to the budget constraint (3) (straightforward calculation). Since Euler equations and transversality condition together are sufficient conditions for utility maximization[20] and because $\{w^*_{it}, \tilde{k}^*_{it}, c^*_{it}\}$ is budget-feasible, this plan is also the solution to the market problem.

Consider now the I-agent market economy. From the above argument we conclude that the policy g^* is also individually optimal in the I-agent market economy when returns are given by $r_{kt} = r_k(\tilde{k}^*_{it}, S_t)$ and $r_{ht} = r_h(\tilde{k}^*_{it}, S_t)$. Thus, it suffices to show that market clearing holds. But with a common capital-to-labor ratio, $\tilde{k}^*_{it} = \tilde{k}^*(S_{t-1})$, market clearing automatically holds (for any possible wealth distribution).[21]

It is left to show that utility maximization implies the transversality condition (A2). Suppose therefore that (7) has a solution $\{w^*_{it}, \tilde{k}^*_{it}, c^*_{it}\}$. Then for $\{c^*_{it}\}$ expected lifetime utility exists and is finite, which for $\gamma \neq 1$ implies that the series $\sum_{t=0}^{\infty} a_t \doteq \lim_{T\to\infty} \sum_{t=0}^{T} a_t$ with $a_t = \beta^t E[(c^*_{it})^{1-\gamma}/(1-\gamma)]$ converges. Thus, we must have $a_T \to 0$, that is,

$$\beta^t E\left[\frac{(c^*_{iT})^{1-\gamma}}{1-\gamma}\right] \to 0 \ . \qquad \text{(A3)}$$

Using $w^*_{it} = \frac{c^*_{it}}{(1+r_{it})\tilde{c}^*_{it}}$ and $\epsilon \leq \tilde{c}^*_{it} \leq 1-\epsilon$ and $r_{min} \leq r_{it} \leq r_{max}$ for some $\epsilon > 0$, $r_{min} > -1$, and $r_{max} < \infty$, we find that the transversality condition (A2) holds iff (A3) holds. The existence of r_{min} and r_{max} follows from the maintained assumptions on the production process. For $\gamma = 1$ (log-utility), an analogous argument shows that (A2) holds.

[20] See, for example, Stokey and Lucas (1989). With our finite-state-space assumption, their proof of sufficiency extends to the uncertainty case in a straightforward way.

[21] Put differently, with a common $\tilde{k}$, the technology is basically linear, and joint production in one firm is equivalent to production in I individual firms (one firm per household).

Suppose now that households have the opportunity to trade $j = 1, \ldots, J$ securities in zero net supply with payoffs $D_{jt} = D_j(S_t)$. To render the individual optimization problem well-defined (to rule out Ponzi-schemes), let us impose the constraints $\theta_{ijt} \geq -B$ for some $B > 0$. Here η_{ijt} denotes the portfolio holding of security j by households i. Fix the policy $g^* = g^{**}$, respectively the plan $\{w^*_{it}, \tilde{k}^*_{it}, c^*_{it}\}$, and define a security price function $Q^*_t = Q^*(S_t)$ by

$$Q^*_j(S_t) \;=\; E\left[M^*(s_{i,t+1}, S_t, S_{t+1}) D_j(S_{t+1}|S_t\right] \qquad \text{(A4)}$$

$$M^*(s_{i,t+1}, S_t, S_{t+1}) \;=\; \beta \left(\frac{(1 + r(\tilde{k}^*(S_t), s_{i,t+1}, S_{t+1})\tilde{c}^*(S_{t+1})(1 - \tilde{c}^*(S_t))}{\tilde{c}^*(S_t)} \right)^{-\gamma} .$$

The pricing kernel M^* is simply the intertemporal marginal rate of substitution, $\beta \left(c^*_{i,t+1}/c^*_{it} \right)^{-\gamma}$. Clearly, the assumption that $\tilde{k}^*_{i,t+1} = \tilde{k}(S_t)$ and $\tilde{c}^*_{it} = \tilde{c}^*(S_t)$ is essential for ensuring that the pricing kernel in period $t+1$, $M^*_{t+1} = M^*(\tilde{k}^*(S_t), s_{i,t+1}, S_{t+1})$, does not depend on s_{it}, which in turn ensures that the expression on the right-hand-side of (A4) is the same for all households $i = 1, \ldots, I$. Moreover, the unpredictability of $s_{i,t+1}$ implies that the right-hand-side of (A4) is unchanged if we include s_{it} or $s_t = (s_{1t}, \ldots, s_{It})$ in the information set when calculating the conditional expectation. Thus, if household i is given the opportunity to trade the securities at prices (A4), then his Euler equations regarding security trade will be satisfied at $\theta^*_{it} = 0$ (no trade). Since an extended version of the transversality condition still holds, the choice of $\{w^*_{it}, \tilde{k}^*_{it}, c^*_{it}\}$ together with $\theta^*_{it} = 0$ is individually optimal. Since by construction all markets clear, we have found a (recursive) equilibrium for the extended economy in which households have the opportunity to trade the securities $j = 1, \ldots, J$.

Appendix 2: Proof of Proposition 2

The proof runs as follows. First, we show that there is a solution to the Euler equations which has the stated properties. Second, we show that any solution to the Euler equations also satisfies a transversality condition. Since in our case Euler equations and transversality condition are sufficient conditions for an optimum, we have proved that a solution to the maximization problem (7) with the stated properties exists. Uniqueness of the solution immediately follows from the strict concavity of the objective function in conjunction with the convexity of the choice set.[22]

We will prove the proposition for $\gamma \neq 1$. For $\gamma = 1$ (log-utility) the proof follows similar lines. Using $c_{it} = \tilde{c}_{it}(1+r_{it})w_{it}$ and $w_{i,t+1} = (1+r_{it})w_{it} - c_{it}$, we find that the Euler equations (A1) are satisfied if the equation system (10) in proposition 2 has a solution $0 < \tilde{c}(S) \leq 1$ and $\tilde{k}(S) \geq 0$. Denote the number of elements of $\mathbf{S}$ by $|\mathbf{S}|$. Since the state space, $\mathbf{S}$, is finite, the functions $\tilde{c}$ and $\tilde{k}$ can be identified with finite-dimensional vectors $\tilde{c} \in \mathbb{R}_+^{|\mathbf{S}|}$ and $\tilde{k} \in \mathbb{R}_+^{|\mathbf{S}|}$. Let $x = (\tilde{c}, \tilde{k}) \in \mathbb{R}_+^{2|\mathbf{S}|}$. Finding a solution to the equation system (10) amounts to finding a fixed point, $x = Tx$, for the operator $T : \mathbf{X} \to \mathbf{X}$, $\mathbf{X} \subset \mathbb{R}_+^{2|\mathbf{S}|}$, defined as follows. Let $x' = Tx$ and $x' = (\tilde{c}', \tilde{k}')$. Then $\tilde{c}'$ is given by the right-hand-side of the first set of Euler equations in (10) and $\tilde{k}'$ is determined as the solution of the second set of Euler equations. Notice that for any $\tilde{c} >> 0$, the solution,$\tilde{k}'$, to the second set of Euler equations exists and is unique. This immediately follows from the properties of r_h, r_k, r that are an implication of the assumptions on the production function. To prove the existence of a solution to (10), we apply Brower's fixed point theorem. Thus, we need to show the existence of a non-empty, convex, and compact set $\mathbf{X}$ for which T is continuous.

We choose $\mathbf{X} \equiv ([\epsilon, 1])^{|\mathbf{S}|} \times ([0, B])^{|\mathbf{S}|}$ for some $0 < \epsilon < 1$ and $B < \infty$ (below we show that we can bound $\tilde{c}$ away from one). Clearly, this set is non-empty, convex, and compact.

[22] There is an alternative way of proving proposition 2. First, extend the argument in Becker and Boyd (1997) and Jones and Manuelli (1990) to show that a solution to (7) exists, that is, show that the objective function is semi-continuous and the choice set is compact in the product topology. Since the solution to (7) is unique (strict concavity of the utility function in conjunction with convexity of the choice set) and Euler equations are necessary, it then suffices to show that a unique solution to the Euler equations exists (contraction mapping theorem). Jones, Manuelli, and Staccetti (1999) provide a proof along those lines for economies with Cobb-Douglas production function, linear Markov shocks, and no depreciation shocks.

Moreover, it is straightforward to show the continuity of T on $\mathbf{X}$. Thus, it is left to show that the two numbers B and ϵ exist. Notice the importance of bounding $\tilde{c}$ away from zero, since T is not even defined if $\tilde{c}(S) = 0$ for some $S \in \mathbf{S}$.

We begin with the existence of a strictly positive number ϵ. We want to show that if $\tilde{c} \in ([\epsilon, 1])^{|\mathbf{S}|}$, then $\tilde{c} = T\tilde{c} \in ([\epsilon, 1])^{|\mathbf{S}|}$. Since $T\tilde{c}(S) \leq 1$ obviously holds, we only need to show that $\forall S : T\tilde{c}(S) \geq \epsilon$ if $\epsilon \leq \tilde{c} \leq 1$. Suppose therefore that $\tilde{c}(S) \geq \epsilon$ for all S. In this case the inequality $T\tilde{c}(S) \geq \epsilon$ holds for all S if

$$\forall S \; : \; \frac{1}{1 + \epsilon^{-1}\left(\beta E\left[\left(1 + r(\tilde{k}(S), s_i', S')\right)^{1-\gamma} |S\right]\right)^{1/\gamma}} \; \geq \; \epsilon \; . \qquad \text{(A5)}$$

Condition (9) ensures that the term in the brackets is strictly less than one, which implies that for each S we can find a small enough $\epsilon(S) > 0$ so that (A5) holds. Since there are only finitely many states S, we can choose $\epsilon \doteq min_S \; \epsilon(S)$. Notice that in general $B = B(\epsilon)$, but that the number ϵ can be found independently of $\tilde{k}$ and therefore B.

Finally, we show that the transversality condition (A2) holds. Using $c_{iT} = \tilde{c}_{iT}(1+r_{iT})w_{iT}$ and the fact that $\epsilon \leq \tilde{c}_{iT} \leq 1 - \epsilon$ and $r_{min} \leq r_{iT} \leq r_{max}$ for some $\epsilon > 0$, $r_{min} > -1$, and $r_{max} < \infty$, we find that (A2) holds iff the following holds

$$\beta^T E\left[w_{iT}^{1-\gamma}\right] \to 0 \; . \qquad \text{(A6)}$$

Repeated substitution of $w_{i,t+1} = (1 - \tilde{c}_{it})(1 + r_{it})w_{it}$, $t = 0, 1, \ldots, T-1$, shows that (A6) holds if

$$\beta E\left[(1 - \tilde{c}_{i,t+1})^{1-\gamma}(1 + r_{i,t+1})^{1-\gamma}|S_t\right] \; < \; 0 \; . \qquad \text{(A7)}$$

In (A7) we condition the expectation on S_t only because of the Markov assumption and the properties of $c_{i,t+1}$ and $r_{i,t+1}$. Clearly, if $(1 - \tilde{c}_{it})^{1-\gamma} < 1$, then condition (9) ensures that (A7) holds. Since $0 < (1 - \tilde{c}_{it}) < 1$, for $\gamma < 1$ the inequality $(1 - \tilde{c}_{it})^{1-\gamma} < 1$ is satisfied. It is left to show that (A7) also holds for the case $\gamma > 1$.

Let $\tilde{c}_{max} = max_S \tilde{c}(S)$. The Euler equations (10) imply

$$\forall S: \ \tilde{c}(S) \leq \frac{1}{1 + \tilde{c}_{max}^{-1} \left(\beta E \left[(1 + r_{i,t+1})^{1-\gamma} | S\right]\right)^{1/\gamma}} \ .$$

Taking the maximum and rearranging terms yields

$$\tilde{c}_{max} \leq 1 - \left(\beta E \left[(1 + r_{i,t+1})^{1-\gamma} | S\right]\right)^{1/\gamma} \ . \qquad \text{(A8)}$$

Inequality (A8) establishes an upper bound on $\tilde{c}$ that is strictly smaller than one. Using this inequality, condition (9) in the main text, and $\gamma > 1$, we find

$$\begin{aligned} \beta E \left[(1 - \tilde{c}_{i,t+1})^{1-\gamma}(1 + r_{i,t+1})^{1-\gamma} | S_t\right] &\leq (1 - \tilde{c}_{max})^{1-\gamma} \beta E \left[(1 + r_{i,t+1})^{1-\gamma} | S_t\right] \\ &\leq \left(\beta E \left[(1 + r_{i,t+1})^{1-\gamma} | S_t\right]\right)^{\frac{1-\gamma}{\gamma}} \beta E \left[(1 + r_{i,t+1})^{1-\gamma} | S_t\right] \\ &< 1 \ . \end{aligned}$$

This completes the proof of proposition 2.

Appendix 3: Proof of Proposition 3

Define

$$\varphi(\tilde{k}, \eta) \doteq \frac{r_h(\tilde{k}) + \eta - r_k(\tilde{k})}{\left(1 + r(\tilde{k}, \eta)\right)^{\gamma}} \ .$$

The Euler equation (12) in proposition 2 determining $\tilde{k}$ then reads $E[\varphi(\tilde{k}, \eta(s_i, S))] = 0$. Let $\tilde{k}$ be the solution of this equation for η and $\tilde{k}'$ for η', that is, $\tilde{k}$ and $\tilde{k}'$ solve

$$\begin{aligned} E\left[\varphi(\tilde{k}, \eta(s_i, S))\right] &= 0 \qquad \text{(A9)} \\ E\left[\varphi(\tilde{k}', \eta'(s_i))\right] &= 0 \ . \end{aligned}$$

It is straightforward to show that $r_h(\tilde{k}) > r_k(\tilde{k})$ (agents are risk averse and human capital investment is riskier than physical capital investment). Using this and that r_h is increasing and r_k is decreasing in $\tilde{k}$, we find that the function φ is increasing in $\tilde{k}$ and convex in η. Recalling that $\eta'(s_i) = E[\eta(s_i, S)|s_i]$, we derive

$$\begin{aligned} E\left[\varphi\left(\tilde{k}, \eta(s_i, S)\right)\right] &= E\left[E\left[\varphi\left(\tilde{k}, \eta(s_i, S)\right)|s_i\right]\right] \\ &\geq E\left[\varphi\left(\tilde{k}, E\left[\eta(s_i, S)|s_i\right]\right)\right] \\ &= E\left[\varphi\left(\tilde{k}, \eta'(s_i)\right)\right] . \end{aligned} \tag{A10}$$

Combining (A9) and (A10) we therefore conclude that $E[\varphi(\tilde{k}, \eta'(s_i))] \leq E[\varphi(\tilde{k}', \eta'(s_i))]$, which implies that $\tilde{k} \geq \tilde{k}'$ because φ is decreasing in $\tilde{k}$. This proves the first part of the proposition. The statements about returns and investment then follow immediately.

It is left to show that the statement about $\tilde{c}$ is true (the growth-rate effect then immediately follows). From the formula for $\tilde{c}$ we infer that we need to discuss the effect on $E\left[(1 + r(\tilde{k}, \eta(s_i, S)))^{1-\gamma}\right]$. There are two effects: a direct effect (change in η) and an indirect effect due to the change in $\tilde{k}$. Recall that the movement from η to η' decreases $\tilde{k}$ and that r is decreasing in $\tilde{k}$. For $\gamma < 1$, both effects decrease the expression because in this case $(1 + r)^{1-\gamma}$ is a concave and increasing function of r. For $\gamma > 1$, both effects increase the expression because in this case $(1+r)^{1-\gamma}$ is a convex and decreasing function of r. From this the statement about $\tilde{c}$ follows. This completes the proof of proposition 3.

Appendix 4: Aggregate Shocks and Idiosyncratic Risk

As in proposition 3, assume $A(S) = A, \delta_k(S) = \delta_k$, and $\delta_h(S) = \delta_h$. One approach to modeling the interaction between business cycles and idiosyncratic risk (approach 1), which is the one adopted by Atkeson and Phelan (1994) and Krusell and Smith (1999), is to assume that S only affects probabilities of idiosyncratic shocks (the probability of becoming unemployed), but not the severity of idiosyncratic shocks (the magnitude of the income loss when unemployed). Formally, this amounts to the assumption that η is constant across different

S-realizations. For the economy analyzed in this paper, the equilibrium allocation is the solution to the one-agent decision problem (5), which in turn is determined by the equation system (12). However, the solution to (11), $\tilde{k}$ and $\tilde{c}$, is the same for all joint probability distributions $\pi(s_i, S)$ satisfying $\sum_S \pi(s_i, S) = \pi'(s_i)$ for fixed marginal probabilities $\pi'(s_i)$ since none of the functions r_k, r_h, η, and r that enter into (12) depends on S. Hence, the elimination of business cycles has no effect on the allocation since it simply amounts to replacing the joint probabilities $\pi(s_i, S)$ by the marginal probabilities $\pi'(s_i) = \sum_S \pi(s_i, S)$. It also has no effect on welfare because expected lifetime utility is unchanged.[23]

Suppose now that business cycles also affect the realizations of idiosyncratic human capital shocks: $\eta(s_i, S) \neq \eta(s_i, S')$ for some s_i, S, S'. Clearly, in this case the neutrality result does not hold (proposition 3). In the most extreme case, we can imagine that $\pi(s_i, S) = \pi_1(s_i)\pi_2(S)$, and that any "observed" S-dependence of the distribution of human capital shocks is entirely generated through the function $\eta(s_i, S)$. In words: the level of aggregate economic does not affect the probability of receiving a bad human capital shock, but it determines the severity of a bad shock. This approach (approach 2) is the one taken in the quantitative section of this paper. It is quite easy to see that both approaches (1 and 2) are flexible enough to generate any "observed" S-dependent family of distributions of human capital shocks. We next demonstrate this claim for the particular (but important) example of normally distributed idiosyncratic shocks. This discussion will also help clarify the difference between the two approaches.

[23]This neutrality result was first pointed out by Atkeson and Phelan (1994) and Krusell and Smith (1999). Its proof still goes through if the state process follows a general Markov process as long as S_t is not useful in predicting $s_{i,t+1}$ (an assumption that is implicit in the set-up of the current model because we assume that idiosyncratic shocks are unpredictable). However, if S_t contains information about $s_{i,t+1}$, then the optimal investment decision in general depends on S_t, and changes in the joint probabilities will affect the equilibrium outcome even for fixed marginal probabilities. Notice also that the proof of the neutrality result hinges on the assumption that preferences allow for an expected utility representation (independence axiom) and that S does not directly enter into the utility function (no taste shocks). Finally, we note that the neutrality result does not hold if the equilibrium allocation is not the solution to a one-agent decision problem (indirect price effects).

For the sake of concreteness, suppose that there are two aggregate states, $S \in \{L, H\}$, that occur with equal probability. Assume further that in the economy with business cycles idiosyncratic human capital shocks are conditionally normally distributed: $\eta_L \sim N(0, \sigma_L^2)$ and $\eta_H \sim N(0, \sigma_H^2)$, where η_L is the idiosyncratic shock variable if $S = L$ and η_H is the idiosyncratic shock variable if $S = H$. We show next how this "observed" family of distributions can be generated by the two approaches (approach 1 and 2).

Let us begin with approach 1. More specifically, assume $\eta_h(s_i, S) = \eta_h'(s_i) = s_i$. In order to match the observed distribution of human capital shocks, we further assume that $s_i \sim N(0, \sigma_L^2)$ if $S = L$ and $s_i \sim N(0, \sigma_H^2)$ if $S = H$. When we now remove business cycles, then in the new economy the probabilities are determined as $\pi'(s_i) = 1/2\,\pi(s_i|L) + 1/2\,\pi(s_i|H)$, where $\pi(s_i|L)$, respectively $\pi(s_i|H)$, is the density function of a normal distribution with zero mean and standard deviation σ_L, respectively σ_H. Thus, the distribution of human capital shocks in the economy without business cycles is the mixture of two normal distributions with common mean but different variances, and is therefore not normally distributed. By construction, the allocation and welfare in this new economy (without business cycles) is identical to the allocation and welfare in the old economy (with business cycles).

Let us now turn to approach 2. More specifically, suppose that probabilities are independent of business cycle activity: $\pi(s_i, S) = \pi_1(s_i)\pi_2(S)$. For the particular case of interest with two aggregate states and normally distributed idiosyncratic states, this means that that $\pi_1(s_i)$ is the density function of a normal distribution and that $\pi_2(L) = 1/2$. Choose $s_i \sim N(0, 1)$. To ensure that the consistency with the observed distribution of idiosyncratic human capital shocks, we specify $\eta(s_i, L) = \sigma_L s_i$ and $\eta(s_i, H) = \sigma_H s_i$. The integration principle now requires that $\eta'(s_i) = 1/2\sigma_L\, s_i + 1/2\sigma_H\, s_i$, which implies that we have $\eta' \sim N(0, \sigma^2)$ with $\sigma = 1/2\sigma_L + 1/2\sigma_H$. In words: idiosyncratic human capital shocks in the economy without business cycles are normally distributed with a standard deviation that is equal to the mean of the two standard deviations in the economy with business cycles. It is straightforward to show that any risk averse decision maker prefers to face uncertainty described by one normal distribution with standard deviation σ to facing uncertainty which is described by two normal distributions with fluctuating standard deviations σ_L and σ_H (the latter is equivalent to the fat-tailed mixture distribution discussed in the previous paragraph). Put differently, the elimination of business cycles affects the equilibrium allocation and increases welfare. This example forms the basis of the quantitative analysis conducted in the next section.

References

Acemoglu, D. and F. Zilibotti (1997) “Was Prometheus Unbound by Chance? Risk, Diversification, and Growth,” *Journal of Political Economy*, 105: 709-751.

Aiyagari, R. (1994) “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics* 109, 659-84.

Aghion,P. and P. Howitt (1982) “A Model of Growth Through Creative Destruction,” *Econometrica* 60: 323-352.

Alvarez, F. and N. Stokey (1998) “Dynamic Programming with Homogeneous Functions,” *Journal of Economic Theory*

Alvarez, F. and U. Jermann (2002) “Efficiency, Equilibrium, and Asset Pricing with Risk of Default,” *Econometrica* 68: 775-799.

Angeletos, G. and L. Calvet (2001) “Incomplete Markets, Growth, and the Business Cycle,” Working Paper, Harvard University.

Atkeson, A. and R. Lucas (1992) “On Efficient Distribution With Private Information,” *Review of Economic Studies*, 59: 427-453.

Barlevy, G. (2000) “Evaluating the Costs of Business Cycles in Models of Endogenous Growth,” Working Paper, Northwestern University.

Becker, R. and J. Boyd, *Capital Theory, Equilibrium Analysis, and Recursive Utility*, Blackwell Publishers, 1997.

Becker, R. and I. Zilcha (1997) “Stationary Ramsey Equilibria Under Uncertainty,” *Journal of Economic Theory*, 75: 122-141.

Boldrin, M. and A. Rustichini (1994) “Growth and Indeterminacy in Dynamic Models with Externalities,” *Econometrica*, 62: 332-392.

Brav, A. Constantinides, G. and C. Gescy (2002) “Asset Pricing with Heterogenous Consumers and Limited Participation: Empirical Evidence,” *Journal of Political Economy* forthcoming.

Caballe, J. and M. Santos (1993) “On Endogenous Growth with Physical and Human Capital,” *Journal of Political Economy*, 101: 1042-67.

Campbell, J., and J. Cochrane (1999) “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock-Market Behavior,” *Journal of Political Economy*, 107: 205-251.

Carroll, C. and A. Samwick (1997) “The Nature of Precautionary Saving,” *Journal of Monetary Economics*, 40: 41-72.

Cole, H. and N. Kocherlakota (2001) “Efficient Allocations with Hidden Income and

Hidden Storage," *Review of Economic Studies*, 68: 523-542.

Cooley, T. and E. Prescott (1995) "Economic Growth and Business Cycles," in Cooley, T. (ed) *Frontiers in Business Cycle Research*, Princeton University Press, Princeton, New Jersey.

Constantinides, G. and D. Duffie (1996) "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy*, 104: 219-240.

Davis, S. and P. Willen (2001) "Using Financial Assets to Hedge Labor Income Risk: Estimating the Benefits," Working Paper, University of Chicago.

Den Haan, W. (1997) "Solving Dynamic Models with Aggregate Shocks and Heterogenous Agents," *Macroeconomic Dynamics* 1: 355-386.

Duffie, D., J. Geanakoplos, A. Mas-Colell, and A. Mc Lennan (1994) "Stationary Markov Equilibria," *Econometrica*, 62: 745-781.

Geweke, J. and M. Keane (2000) "An Empirical Analysis of Male Income Dynamics in the PSID: 1968-1989," *Journal of Econometrics* 96: 293-356.

Hernandez, A. and M. Santos (1996) "Competitive Equilibria for Infinite Horizon Economies with Incomplete Markets," *Journal of Economic Theory* 71: 102-130.

Hubbard, R., Skinner, J. and S. Zeldes (1995) "Precautionary Saving and Social Insurance", *Journal of Political Economy*, 103: 360-399.

Huggett, M. (1993) "The Risk-Free Rate in Heterogenous-Agent Incomplete-Market Economies," *Journal of Economic Dynamics and Control* 17: 953-969.

Jacobson, L., LaLonde, R., and D. Sullivan (1993) "Earnings Losses of Displaced Workers," *American Economic Review* 83: 685-709.

Imrohoroglu, A. (1989) "Costs of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, 1364-83.

Jones, L. and Manuelli,R. "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy*, 98: 1008-1038.

Jones, L., Manuelli, R. and E. Stacchetti (1999) "Technology and Policy Shocks in Models of Endogenous Growth," NBER Working Paper # 7063.

Jovanovic, B. "Firm-Specific Capital and Turnover," *Journal of Political Economy* 87: 1246-60.

Kehoe, T. and D. Levine (1993) "Debt Constrained Asset Markets," *Review of Economic Studies* 60: 865-888.

Kehoe, T. and D. Levine (2001) "Liquidity Constrained Markets versus Debt Constrained

Markets," *Econometrica*, 69: 575-599.

Kocherlakota, N. (1996) "Implications of Efficient Risk Sharing Without Commitment," *Review of Economic Studies* 63: 595-609.

Kormendi, R. and P. Meguire (1985) "Macroeconomic Determinants of Growth: Cross-Country Evidence," *Journal of Monetary Economics* 16: 141-163.

Krebs, T. (2001) "Idiosyncratic Risk, Aggregate Saving, and Economic Growth," Working Paper, Brown University.

Krebs, T. (2002) "Non-Existence of Recursive Equilibria on Compact State Spaces When Markets Are Incomplete," Working Paper, Brown University.

Krueger, A. and M. Lindhal (2001) "Education for Growth: Why and for Whom?," *Journal of Economic Literature* 39: 1101-1136.

Krueger, D. and F. Perri (2001) "Does Income Inequality Lead to Consumption Inequality? Empirical Findings and a Theoretical Explanation," Working Paper, Stanford University.

Krusell, P. and A. Smith (1998) "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106: 867-896.

Krusell, P. and A. Smith (1999) "On the Welfare Effects of Business Cycles," *Review of Economic Dynamics*, 2: 247-272.

Lazear, E. (1990) "Job Security Provisions and Employment," *Quarterly Journal of Economics* 105: 699-726.

Levine, D. and B. Zame (1996) "Debt Constraints and Equilibrium in Infinite-Horizon Economies with Incomplete Markets," *Journal of Mathematical Economics* 26: 103-131.

Ljungqvist, L. and T. Sargent (1998) "The European Unemployment Dilemma," *Journal of Political Economy* 106: 514-550.

Ljungqvist, L. and T. Sargent, *Recursive Macroeconomic Theory*, MIT Press, Cambridge, 2000.

Lucas, R. *Models of Business Cycles*. New York: Blackwell, 1987.

Mankiw, G., Romer, D. and D. Weil (1992) "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics* 107: 407-437.

Magill, M. and M. Quinzii (1996) "Infinite Horizon Incomplete Markets," *Econometrica* 62: 853-880.

Magill, M. and M. Quinzii (1997) *Theory of Incomplete Markets*. MIT Press, Cambridge, Massachusetts.

Magill, M. and M. Quinzii (2000) “Infinite-Horizon CAPM Equilibrium,” *Economic Theory* 15: 103-138.

Meghir, C. and L. Pistaferri (2001) “Income Variance Dynamics and Heterogeneity,” Working Paper, Stanford University.

Neal, D. (1995) “Industry-Specific Human Capital: Evidence from Displaced Workers,” *Journal of Labor Economics* 13: 653-677.

Prescott, E. and R. Mehra (1980) “Recursive Competitive Equilibrium: The Case of Homogenous Households,” *Econometrica*, 48: 1365-79.

Quah, D. (1993) “Empirical Cross-Section Dynamics in Economic Growth,” *European Economic Review* 37: 426-34.

Ramey, G. and V. Ramey (1995) “Cross-Country Evidence on the Link Between Volatility and Growth,” *American Economic Review*, 85: 1138-51.

Rebelo, S. (1991) “Long-Run Policy Analysis and Long-Run Growth,” *Journal of Political Economy* 99: 500-521.

Reffett, K., Mirman, L. and O. Morand (2002) ”A Qualitative Theory of Markovian Equilibria in Infinite-Horizon Economies with Capital,” Working Paper, Arizona State University.

Stokey, N. and R. Lucas *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989.

Storesletten, K., Telmer, C., and A. Yaron (2000) “The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variations in Idiosyncratic Risk,” *NBER Working Paper 8040*.

Storesletten, K., Telmer, C. and A. Yaron (2001) “Asset Pricing with Idiosyncratic Risk and Overlapping Generations,” Working Paper, Carnegie-Mellon University.

Topel, R. (1991) “Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority,” *Journal of Political Economy* 99: 145-176.

Zhang, H. (1997) “Endogenous Borrowing Constraints with Incomplete Markets,” *Journal of Finance* 52: 2187-2209.

TABLE 1. Business Cycle Effects

Economy	$1a$	$1b$	$1c$	$2a$	$2b$	$2c$	3
$\Delta\mu_c$	.012 %	.071 %	.49 %	.024 %	.210 %	1.54 %	0 %
ΔEU_1	.047 %	.22 %	.23 %	.069 %	.558 %	.72 %	.015 %
ΔEU_2	.69 %	3.24 %	3.38 %	.93 %	7.48 %	9.65 %	.435 %
$\Delta X_k/Y$	-.090 %	-.58 %	-3.57 %	-1.37 %	-2.02 %	-7.18 %	0 %
$\Delta X_h/Y$	.13 %	.79 %	5.61 %	.20 %	1.97 %	-19.25 %	0 %
Δr_k	.06 %	.38 %	0 %	.10 %	.99 %	0 %	0 %
Δr_h	-.05 %	-.32 %	0 %	-.09 %	-.88 %	0 %	0 %

1: incomplete-markets economy with $\sigma_y(L) = .24$ and $\sigma_y(H) = .12$.
2: incomplete-markets economy with $\sigma_y(L) = .28$ and $\sigma_y(H) = .08$.
3: complete-markets economy

a: elimination of variation in aggregate productivity and depreciation
b: elimination of variation in idiosyncratic risk and variation in aggregate productivity and depreciation
c: partial-equilibrium effect (fixed asset returns) of eliminating variations in idiosyncratic risk and variations in aggregate productivity and depreciation

$\Delta\mu_c$: change in aggregate consumption growth
ΔEU_1: change in welfare expressed in equivalent consumption growth units
ΔEU_2: change in welfare expressed in equivalent consumption level units
$\Delta X_k/Y$: change in the rate of saving in physical capital
$\Delta X_h/Y$: change in the rate of saving in human capital
Δr_k: change in interest rate
Δr_h: change in wage rate