

Heckscher-Ohlin Business Cycles¹

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Abstract

This paper introduces Heckscher-Ohlin trade features into a two-country DSGE model, and studies how productivity shocks propagate through trade in goods. In comparison with standard models, the business cycle properties of our framework are broadly compatible with the empirical evidence. Moreover, the model yields some novel predictions that distinguish it from the existing international real business cycle literature: *(i)* transitory shocks to productivity have permanent effects on country-level aggregate variables; *(ii)* aggregate productivity shocks have relevant effects on the sectoral allocation of production factors; *(iii)* under complete asset markets, the international correlation of consumption is lower than that of output; *(iv)* the model's predictions on the correlation of the terms of trade with the main aggregate variables are compatible with the data.

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1 Introduction

This paper introduces Heckscher-Ohlin trade features into a two-country dynamic stochastic general equilibrium model, and studies the international transmission of productivity shocks through trade in goods. This framework generates business cycle properties that are broadly compatible with the empirical evidence, in the sense that it produces positive international correlations for almost all main aggregates. Our framework also yields some novel predictions that distinguish it from the existing international real business cycle (IRBC) literature:

1. A *transitory* shock to productivity in a country has *permanent* effects on most of the country-level aggregate variables, which are non-stationary from a stochastic point of view, and have ex-ante undetermined steady-state values. The world as a whole, however, behaves like a standard RBC model, and is characterized by stationary dynamics and a well-defined steady state.
2. *Aggregate* productivity shocks have cyclical and permanent effects on the *sectoral* allocation of production factors in line with the dynamics of the countries' relative factor endowments.
3. Under *complete asset markets*, the international correlation of consumption is lower than under financial autarky, and furthermore lower than the correlation of GDP.
4. The predictions on the sign of the correlation of the terms of trade with GDP and the current account vary across countries, depending on their comparative advantage.
5. The terms of trade predicted by the model are less volatile than the real exchange rate.

Starting with Backus and Kehoe (1992), the international properties of business cycles have attracted a large interest among the profession. The IRBC literature has developed basically two types of dynamic stochastic general equilibrium models to explain and reproduce the observed stylized facts:

Backus, Kehoe, and Kydland (1992), and Baxter and Crucini (1993) discuss variants of a two-country extension of the basic RBC model developed by Kydland and Prescott (1982) and Long and Plosser (1983). In this model, countries trade a homogeneous consumption good, and the main international transmission mechanism of country-specific productivity shocks is consumption smoothing under complete markets.¹ Two robust

¹Cole (1993) distinguishes between international consumption risk sharing, *i.e.* international trade in Arrow-Debreu state-contingent assets, and intertemporal trade, *i.e.* international trade in consumption

discrepancies between the properties of the data and those of the model have been documented: *(i)* the international correlation of consumption is lower than the correlation of income in the data, while the opposite happens in the model; *(ii)* income, investment, and labor are positively correlated across countries in the data, but not in the model.

Backus, Kehoe, and Kydland (1994) provide a more general model in which both intratemporal goods trade and consumption smoothing are at work as transmission mechanisms. In their model, countries specialize completely in the production of different intermediate goods that are internationally traded, and used to produce a final consumption good. The model is then used to study the dynamic properties of the trade balance and the terms of trade.² Three more discrepancies with the data have been highlighted here: *(iii)* the model does not generate enough volatility in the terms of trade; *(iv)* the terms of trade are positively correlated with income in the model, whereas in the data the sign of this correlation varies across countries; *(v)* the real exchange rate is necessarily less volatile than the terms of trade, as shown in Heathcote and Perri (2002).

Our paper is a natural follow-up on this second strand of the literature. Rather than imposing an exogenously given trade structure, we introduce comparative advantage elements that make the trade and production patterns of countries endogenous to the shocks and the dynamics of the model.³ More specifically, we assume that countries have access to the same production technologies, and that comparative advantage is motivated by cross-country differences in relative factor endowments. We focus on the Heckscher-Ohlin model's factor price equalization (FPE) case. However, we allow for cross-country differences in total factor productivity (TFP), which lead to a rather weak form of FPE.⁴

Our Heckscher-Ohlin setup underlies the results listed above: *(i)* Long-run FPE implies that the steady state interest rate is pinned down by the world capital-labor ratio, allowing for an infinite number of steady-state distributions of national capital-labor ratios; *(ii)* comparative advantage implies sectoral reallocation of factors in case the time paths of countries' capital-labor ratios are not parallel; *(iii)* in addition to the constraints imposed

loans or riskless bonds. These two mechanisms are however driven by the same economic force: consumption smoothing across time and states of the world. Baxter and Crucini (1995) show that intertemporal trade is the dominant international transmission mechanism if the stochastic process for productivity is stationary.

²Canova and Dellas (1993), Canova and Marrinan (1998), Boileau (1999), and Ambler, Cardia, and Zimmermann (2002) follow similar lines, focusing on trade interdependence as a channel of propagation. Backus and Smith (1993) and Stockman and Tesar (1995) focus instead on the role of non-traded goods.

³Standard IRBC models do not allow for comparative advantage considerations: identically parameterized countries are assumed to produce different goods with the same technology. Under these assumptions, it is hard to understand why countries need to trade with each other, but for the assumption that production of goods is country-specific.

⁴Kraay and Ventura (2000, 2001) also study the international transmission of business cycles in a Heckscher-Ohlin framework. Their work, however, is harder to relate to the standard IRBC models than ours due to their emphasis on how business cycles differ across rich and poor countries, and to the fact that they avoid capital accumulation in their analytical framework.

on economies trading in a complete asset market framework (the equalization of the marginal utilities of consumption), FPE imposes additional constraints on the marginal utilities of leisure; *(iv)* a country's terms of trade depend on the time path of the world's capital-labor ratio, and on its comparative advantage *vis-à-vis* the rest of the world.

The rest of the paper is structured as follows. In Section 2, we present a stochastic two-country model that combines Heckscher-Ohlin driven comparative advantage with the Ramsey model. To understand the role of international trade in the cross-country propagation of productivity shocks, in Section 3 we focus on the financial autarky case and shut down all international transmission mechanisms other than intratemporal goods trade. We subject countries to stochastic productivity shocks, and study the impulse response functions and the business cycle properties. Section 4 extends the analysis to the case when financial markets offer a complete set of Arrow-Debreu securities. Section 5 studies the model's dynamics and stochastic properties concerning the current account, the terms of trade, and the real exchange rate. Section 6 examines the sensitivity of our results to the choice of parameter values. Section 7 concludes.

2 The Model

2.1 Households

The world consists of two countries, denoted by $j = 1, 2$. Each country is inhabited by a *continuum* of identical and infinitely lived households that can be aggregated into a representative household. The representative household's preferences over consumption and leisure flows are summarized by the following intertemporal utility function:

$$U_{jt} = E_t \left[\sum_{s=t}^{\infty} \tilde{\beta}^{s-t} \frac{c_{js}^{1-\mu} (1 - n_{js})^{\tau(1-\mu)}}{1 - \mu} \right] \quad (1)$$

where $\tilde{\beta} \equiv \beta\gamma^{1-\mu}$; $\beta \in (0, 1)$ is the intertemporal discount factor; $\gamma > 0$ the exogenous growth factor; $\mu > 0$ the intertemporal rate of substitution; $\tau > 0$ the share of leisure in total utility; c_{jt} the per-capita consumption level in country j at date t ;⁵ n_{jt} the time share devoted to labor in country j at date t ; and E_t the expectation operator conditional on the information set available at date t .

Households own both factors of production, capital and labor, and sell their services in competitive spot markets. Factor income is then used to purchase a homogeneous final good in a competitive market. The final good can be consumed or invested. The

⁵We are implicitly normalizing the whole system with respect to an exogenous growth component, that can be interpreted as labor-augmenting technical progress.

representative households face therefore the following resource constraints:⁶

$$p_{jt}(c_{jt} + i_{jt}) = w_{jt}n_{jt} + r_{jt}k_{jt} \quad (2)$$

where p_{jt} is the price of the final good, i_{jt} the per-capita level of investment, w_{jt} the nominal wage rate,⁷ r_{jt} the nominal rental rate, and k_{jt} the per-capita stock of physical capital.

A set of critical assumptions are implicit in equation (2): (i) the final good is not traded internationally; (ii) neither the capital stocks nor the labor endowments are internationally mobile;⁸ (iii) countries are not allowed to pool their idiosyncratic risks internationally, *i.e.* international consumption risk sharing is ruled out; (iv) countries are not allowed to trade in riskless bonds, *i.e.* intertemporal trade is ruled out. The last two assumptions can be easily relaxed in our framework. However, since our goal is to study the propagation of productivity shocks through international trade in goods, we rule out any transmission mechanism other than intratemporal trade for the moment.⁹ In Section 4 we introduce complete Arrow-Debreu asset markets in the model and discuss their implications.

Investment drives the process of capital accumulation through the following dynamic constraint:

$$\gamma k_{jt+1} = (1 - \delta) k_{jt} + \varphi \left(\frac{i_{jt}}{k_{jt}} \right) k_{jt} \quad (3)$$

where $\delta \in (0, 1)$ is the exogenous depreciation rate. Following Baxter and Crucini (1993), we introduce a cost of adjusting capital in equation (3), such that $\varphi > 0$, $\varphi' > 0$, and $\varphi'' < 0$, and assume that it does not play any role in steady state, *i.e.* $\varphi' \left(\frac{i}{k} \right) = 1$. We define the elasticity of the adjustment cost near the steady state as $\xi_\varphi \equiv -\varphi'' \left(\frac{i}{k} \right) \left(\frac{i}{k} \right)$; the parameter ξ_φ is the only feature of the adjustment cost function that is relevant under our solution procedure.¹⁰

The representative households maximize equation (1) subject to equations (2) and (3),

⁶For the sake of notational simplicity, we avoid to make variables state-contingent explicitly.

⁷By *nominal* factor prices we mean factor prices expressed in the common unit of account; by *real* factor prices we mean instead the factor prices in purchasing power terms, *i.e.* expressed in terms of each country's consumption good units.

⁸With this assumption we are only ruling out the possibility that capital and/or labor may flow across countries instantly.

⁹Heathcote and Perri (2002) perform a similar exercise, and show that a version of the Backus, Kehoe, and Kydland (1994) model under financial autarky accounts better for some of the observed stylized facts. Backus, Kehoe, and Kydland (1992) show that a small trading cost reproduces most of the properties of financial autarky.

¹⁰The adjustment cost is actually redundant in the financial autarky version of our model, but will be essential in the complete asset markets version. We introduce the adjustment cost here for comparability reasons. Note that the adjustment cost applies only at the aggregate level: we assume that capital moves freely across sectors.

taking p_{jt} , w_{jt} , and r_{jt} as given. Under our assumptions, the first order conditions and the usual transversality conditions are necessary and sufficient for the households' dynamic optimization problem.

2.2 Firms

2.2.1 Final Goods

The final good is produced in each country by a continuum of competitive firms using two intermediate goods, x and z , with the following Cobb-Douglas production function:

$$Y_{jt} = a_{jt} z_{jt}^{\alpha} x_{jt}^{1-\alpha} \quad (4)$$

where $0 < \alpha < 1$; Y_{jt} is the per-capita output level of the final good;¹¹ a_{jt} the Total Factor Productivity (TFP) level;¹² z_{jt} and x_{jt} the amounts of intermediate goods used in the production of Y_{jt} .

We assume that TFP follows an exogenous stationary stochastic Markov process; in particular, we assume that the logarithm of $a_t \equiv [a_{1t}, a_{2t}]'$ is governed by a VAR(1):¹³

$$\ln a_{t+1} = \rho \ln a_t + \varepsilon_t \quad (5)$$

where ρ is the persistence matrix, and $\varepsilon_t \sim N(0, \Sigma)$ is an *iid* vector of innovations. We assume

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \quad (6)$$

where σ is the standard deviation of the shocks and ν their international correlation. Note that the current levels of TFP are known at date t .

2.2.2 Intermediate Goods

Although the final good is not traded internationally, intermediate goods are freely traded. We assume that the markets for intermediates are also competitive, and that firms in both

¹¹Under financial autarky, Y_{jt} corresponds also to real factor income and real GDP y_{jt} .

¹²We interpret the stochastic components a_{jt} literally as TFP levels, and in fact they represent the total productivity of factors used in the final good sector, *i.e.* the nontraded sector. However, productivity shocks in the consumption good sector can be interpreted also as demand shocks for the intermediate goods.

¹³The steady-state TFP level is implicitly normalized to one.

countries have access to the same technologies to produce them:

$$x_{jt} = k_{x,jt}^\phi n_{x,jt}^{1-\phi} \quad (7)$$

$$z_{jt} = k_{z,jt}^\psi n_{z,jt}^{1-\psi} \quad (8)$$

where $0 \leq \phi < \psi \leq 1$; $k_{x,jt}$ and $n_{x,jt}$ ($k_{z,jt}$ and $n_{z,jt}$) are respectively the amounts of capital and labor employed in sector x (z) in country i and date t . The constraints on ϕ and ψ imply that sector z is capital-intensive relative to sector x .¹⁴

2.3 Trade Equilibrium

We assume that countries are similar enough in their relative factor endowments for the trade equilibrium to yield factor price equalization. The FPE theorem implies that international trade in intermediate goods acts as a substitute for trade in factors; therefore, the wage and rental rates are equalized - here in nominal terms - across countries.¹⁵ FPE is linked to the concept of integrated equilibrium, which is defined as the resource allocation the world would have if both goods and factors were perfectly mobile internationally.¹⁶ The FPE set is the set of distributions of factors among economies that can achieve the resource allocation of the integrated equilibrium if we allow for free international trade, but no international factor mobility. Intuitively, the FPE set is the set of distributions of factors across economies that enable them to achieve full employment of resources while using the techniques implied by the integrated equilibrium. Thus, if the vector of production factors lies within the FPE set, the trading equilibrium will reproduce the integrated equilibrium's factor prices.¹⁷

Since the world's integrated equilibrium behaves like a closed economy, factor prices only depend on world aggregates. In terms of our model, the nominal wage rate w and the nominal rate of return to capital r depend, respectively, positively and negatively on the world's capital-labor ratio K/N . We choose the final good produced in country 2 as the numeraire, *i.e.* $p_{2t} = 1$. The relative price of the final good in country 1 in terms of the numeraire is $p_{1t} = a_{2t}/a_{1t}$. We define the real exchange rate as the ratio between

¹⁴Note that sectoral TFP levels are equal across countries, and normalized to one. In our framework, nominal FPE can hold only if the productivity shocks hit the consumption good sector and/or the productivity of capital and/or labor at the country level. We cannot allow productivity shocks to influence TFP differently at the sectoral level: if sectoral production functions are allowed to differ across countries, FPE will not hold.

¹⁵In our setup, FPE in real terms does not hold in the presence of cross-country differences in a_j . One can also introduce international differences in nominal factor prices by allowing the efficiency of labor and/or capital to vary across countries. See Trefler (1993).

¹⁶See Dixit and Norman (1980).

¹⁷The trade equilibrium and the FPE condition are fully worked out in a separate Technical Appendix available from the authors upon request.

the price of consumption in country 2 over the price of consumption in country 1, *i.e.* $e_t \equiv p_{2t}/p_{1t} = a_{1t}/a_{2t}$. Hence, productivity shocks in our framework can be interpreted as shocks to the real exchange rate.

It is easy to show that

$$w_t = a_{2t} \Gamma \left(\frac{s_N}{s_K} \right)^{s_K} \left(\frac{K_t}{N_t} \right)^{s_K} \quad (9)$$

$$r_t = a_{2t} \Gamma \left(\frac{s_K}{s_N} \right)^{s_N} \left(\frac{N_t}{K_t} \right)^{s_N} \quad (10)$$

where

$$\Gamma \equiv \alpha^\alpha (1-\alpha)^{1-\alpha} [\psi^\psi (1-\psi)^{1-\psi}]^\alpha [\phi^\phi (1-\phi)^{1-\phi}]^{1-\alpha} \quad (11)$$

$$s_K \equiv \frac{rK}{wN + rK} = (1-\alpha)\phi + \alpha\psi \quad (12)$$

and $s_N = 1 - s_K$ are all positive constants.

2.4 Dynamic Equilibrium

A dynamic recursive equilibrium under FPE can be summarized by the following set of equations (factor prices are expressed in real terms):

$$c_{jt}^{-\mu} (1 - n_{jt})^{\tau(1-\mu)} = \lambda_{jt} \varphi' \left(\frac{i_{jt}}{k_{jt}} \right) \quad (13)$$

$$\tau c_{jt}^{1-\mu} (1 - n_{jt})^{\tau(1-\mu)-1} = \lambda_{jt} \varphi' \left(\frac{i_{jt}}{k_{jt}} \right) \tilde{w}_{jt} \quad (14)$$

$$c_{jt} + i_{jt} = \tilde{w}_t n_{jt} + \tilde{r}_t k_{jt} \quad (15)$$

$$E_t \left\{ \lambda_{jt+1} \left[\varphi' \left(\frac{i_{jt+1}}{k_{jt+1}} \right) \tilde{r}_{t+1} + 1 - \delta + \Phi \left(\frac{i_{jt+1}}{k_{jt+1}} \right) \right] \right\} = \frac{\gamma}{\tilde{\beta}} \lambda_{jt} \quad (16)$$

$$\gamma k_{jt+1} = (1 - \delta) k_{jt} + \varphi \left(\frac{i_{jt}}{k_{jt}} \right) k_{jt} \quad (17)$$

where:

$$\Phi \left(\frac{i_t}{k_t} \right) \equiv \varphi' \left(\frac{i_t}{k_t} \right) \frac{i_t}{k_t} - \varphi \left(\frac{i_t}{k_t} \right) \quad (18)$$

$$\tilde{w}_{jt} \equiv \frac{w_t}{p_{jt}} = a_{jt} \Gamma \left(\frac{s_N}{s_K} \right)^{s_K} \left(\frac{K_t}{N_t} \right)^{s_K} \quad (19)$$

$$\tilde{r}_{jt} \equiv \frac{r_t}{p_{jt}} = a_{jt} \Gamma \left(\frac{s_K}{s_N} \right)^{s_N} \left(\frac{N_t}{K_t} \right)^{s_N} \quad (20)$$

λ_{jt} is the costate variable associated to the dynamic constraint, and can be interpreted as the shadow value of installed capital. The previous system of equations is valid if and only if the FPE condition is satisfied *ex-post* at all dates $t \in [0, \infty)$.¹⁸

2.5 Steady State

Consider the Euler equation (16). The system will be in steady state if and only if the following condition holds:

$$r = \frac{\gamma}{\beta} - 1 + \delta \quad (21)$$

Equation (21) pins down the steady-state rental rate of capital. Consider equation (20) evaluated at the steady state:

$$r = \Gamma \left(\frac{s_N}{s_K} \right)^{-s_N} \left(\frac{K}{N} \right)^{-s_N} \quad (22)$$

Equation (22) pins down the steady-state capital-labor ratio at the world level. It is easy to show that equations (21) and (22), together with the other first order conditions and resource constraints, characterize the world-level steady state of our integrated economy. However, any combination of k_j and n_j such that FPE holds and

$$\frac{k_1 + k_2}{n_1 + n_2} = \frac{K}{N} \quad (23)$$

is compatible with the steady state. Equations (21) and (22) are therefore unable to pin down the steady state at the *country level*. Evaluating equations (13), (14), (15), and (17) at the steady state, we can show that n_1 and n_2 are fully determined by k_1 and k_2 . Therefore, the model is compatible with a multiplicity of steady states at the country level, and these steady states are fully characterized by the cross-country distribution of capital stocks.¹⁹

The multiplicity of steady states does not imply the indeterminacy of the model's solution: once the initial conditions k_{j0} are exogenously given, the transitional dynamics leads the system to a unique and non-degenerate steady state. This can be understood

¹⁸More precisely, the first order conditions are valid only if the agents consider the possibility of leaving FPE as an event with probability zero.

¹⁹Giavazzi and Wyplosz (1984) show that a similar result holds in a two-country model under rational expectations and perfect capital mobility. In our set-up, international trade in goods is a substitute for perfect capital mobility. Krusell and Ríos-Rull (1999) obtain this result in a heterogenous-agents closed economy, due basically to the same economic mechanism: if all agents face the same rate of return, and the latter depends on the aggregate capital stock only, the steady-state wealth and income distributions are indeterminate. Lucas and Stokey (1984) discuss the issue in detail, and suggest that the multiplicity of stationary equilibria can be avoided by introducing the hypothesis of increasing marginal impatience.

as follows: (i) the world as a whole is a standard stationary Ramsey economy with a well specified steady state, characterized by a unique value of K/N ; (ii) given the initial conditions and our assumptions on the functional forms, the adjustment paths for all country-level variables are uniquely determined; (iii) equation (23) and the FPE condition imply that $(k/n)_{\min} \leq k_j/n_j \leq (k/n)_{\max}$ for some $(k/n)_{\max} > (k/n)_{\min} > 0$. In other words, the world reaches a steady state in which equations (21) and (22) hold, and both k_1 and k_2 are strictly positive. Such a steady state may be characterized by different values of consumption, hours worked, income, investment and capital across countries.

2.6 Calibration

To solve and simulate the model numerically, we adopt a standard parameterization: we set $\mu = 2$, $\beta = 0.987$, $\delta = 0.012$, and $\gamma = 1.004$, as in Cooley and Prescott (1995). The preference parameter τ is calibrated in order to make the model reproduce a time share devoted to labor equal to 0.31, and a capital share in income equal to 0.4 in a symmetric steady state; the implied value is $\tau = 1.65$.

Bentolila and Saint-Paul (1999) compute sectoral factor shares for 14 OECD countries over the 1973-93 time horizon, and show that factor shares seem more variable across industries than across countries: sectoral labor shares range from 0.20 in agriculture to 0.75 in machinery. To stress the different sectoral factor intensities in our model, we set $\phi = 0.2$ and $\psi = 0.8$. The parameter α is calibrated through equation (12) to make the model reproduce a capital share equal to 0.4 in a symmetric steady state; the implied value is $\alpha = 0.33$. Finally, we set $\xi_\varphi = 1/15$ as in Baxter and Crucini (1993).

In the RBC literature it is customary to approximate TFP with the standard Solow residual, defined as $\ln s_{jt} \equiv \ln y_{jt} - s_N \ln n_{jt} - s_K \ln k_{jt}$.²⁰ Since quarterly data for the capital stock are not directly available, the latter is simply dropped from the above expression, under the assumption that capital moves slowly over the cycle. Cooley and Prescott (1995) build a proxy for the US quarterly Solow residual by setting $s_N = 0.6$ and leaving the capital stock out of the empirical specification. They model this “simplified” Solow residual as an AR(1) process, and estimate a persistence parameter equal to 0.95 and a standard deviation of the shocks equal to 0.007.

In the IRBC literature, the joint stochastic properties of TFP are usually estimated by running a VAR(1) on the country-level proxies for the Solow residuals, as in Backus, Kehoe, and Kydland (1992) and others. Productivity shocks seem to be highly persistent and positively correlated across countries.²¹ To rule out any transmission mechanism

²⁰In our framework, the Solow residuals remains a good empirical proxy for TFP: $\ln s_{jt} = \ln a_{jt}$ up to a constant.

²¹Baxter and Crucini (1995) estimate an international correlation of shocks to the approximated Solow

other than international trade in goods, we exclude international spillovers of productivity shocks by setting the out-of-the-diagonal elements in the transition matrix ρ equal to zero.²² This leaves us with three parameters to pin down: the persistence parameter ρ_{jj} , the shocks' standard deviation σ , and the shocks' international correlation ν .

In the model, the variability of capital at higher frequencies is different from zero. To take this into account, we calibrate the stochastic process for TFP in order to reproduce the following properties of the “simplified” Solow residuals: $\hat{\rho}_{jj} = 0.95$, $\hat{\sigma} = 0.007$, and $\hat{\nu} = 0.4$. In other words, we simulate the model under the benchmark parameterization for 10,000 times over a 100 quarter horizon, drawing the shocks from a multivariate normal distribution. In each round, we estimate a symmetric VAR(1) model on the simulated “simplified” Solow residual.²³ We choose the values of ρ_{jj} , σ , and ν that generate the desired properties on average - across simulations. The calibrated parameters are $\rho_{jj} = 0.93$, $\sigma = 0.007$, and $\nu = 0.402$. These results can be interpreted as follows: using the Solow residual as a proxy of TFP and dropping the capital stock from the specification may lead to overestimating the shocks' persistence slightly, but has no relevant effects on the estimated standard deviation and international correlation of shocks.²⁴

To summarize, our benchmark parameterization is the following:

$$\begin{aligned} \mu &= 2, & \beta &= 0.987, & \tau &= 1.65 \\ \delta &= 0.012, & \xi_\varphi &= 1/15, & \gamma &= 1.004 \\ \phi &= 0.2, & \psi &= 0.8, & \alpha &= 0.33 \\ \rho_{jj} &= 0.93, & \sigma &= 0.007, & \nu &= 0.402 \end{aligned}$$

3 Results

The model is log-linearized around a *symmetric* steady state, and solved with the standard King, Plosser, and Rebelo (1988) procedure. We study the impulse response functions and the stochastic properties of the approximated model. In the Appendix we discuss the accuracy of the solution method.²⁵

residual equal to 0.434 between the US and Canada and to 0.258 between the US and Europe.

²²Reynolds (1992) shows that the off-diagonal elements of the persistence matrix are often not significantly different from zero.

²³Following Lütkepohl (1993, Sec. 5.2), we impose symmetry via linear constraints on the persistence matrix, and estimate the constrained VAR using EGLS.

²⁴The estimate of the spillover coefficient is slightly negative but not significant: the average value is -0.007 with a standard deviation of 0.029.

²⁵Giavazzi and Wyplosz (1985) show that linear dynamic systems characterized by the indeterminacy of the stationary equilibrium (*i.e.* singularity of the transition matrix) converge to a unique stationary equilibrium for any set of initial conditions, if the stability conditions are satisfied.

3.1 Impulse Response Functions

In this section we discuss the dynamic response to an unexpected positive shock to a_1 . We assume that a_1 suddenly increases by 1% at date 0, and solve for the corresponding impulse response functions over a 200 quarter time horizon. We start by describing how aggregate variables react to the shock. The impulse responses for the world's income, consumption, investment, and time share allocated to labor are plotted in Figure 1. The impulse responses for the country-level income, consumption, investment, capital, labor input, real wage rate, and real rental rate are plotted in Figures 2 and 3. The simulated series are expressed in percentage deviations from the initial symmetric steady state.

Figure 1 shows that the world reacts to the productivity shock as a closed-economy RBC model: income, consumption, investment, and the time share devoted to labor increase on impact and then converge slowly to their initial steady state values. Figures 2 to 4 tell quite a different story as far as the country-level variables are concerned. To understand the properties of our model, we need to study how the real and nominal variables react to the productivity shock on impact and during the transition to

For the sake of the discussion, let us assume for the moment that both the world capital stock K and the world labor supply N do not vary on impact, and consider the effect of the shock on prices. The only price affected on impact by the productivity shock is p_1 . An increase in a_1 implies a proportional decrease in p_1 . Since nominal factor prices remain unchanged, the fall in p_1 raises the real factor prices in country 1, leaving real factor prices in country 2 unaffected.

The increase in country 1's real wage and rental rates has the following effects: (i) the rise in the real wage raises labor supply, via the intertemporal labor/leisure substitution effect; (ii) the rise in the real interest rate tends to increase the slope of the consumption path, and - *ceteris paribus* - leads consumption to fall and investment to rise; (iii) the rise in the time share allocated to labor directly reduces the amount of leisure enjoyed by the representative household, and this tends - *ceteris paribus* - to reduce consumption; (iv) the increase in real factor prices and labor supply raises income, and therefore stimulates both consumption and investment. As a result, income, consumption, investment and the labor input in country 1 react positively on impact; consumption and investment react proportionally less and more than income, respectively.

The increase in n_1 raises world labor supply N . For a given world capital stock, this reduces the nominal wage rate and raises the nominal rental rate. The effects on real factor prices are straightforward: in country 1, the increase in the real wage rate generated directly by the fall in p_1 is partially dampened, while the real rental rate increases even further. In country 2, the real wage rate drops while the real rental rate rises.

The change in country 2's real factor prices has the following effects on impact: (i)

the fall in the real wage reduces labor supply; *(ii)* the rise in the real interest rate tends - *ceteris paribus* - to decrease consumption and increase investment; *(iii)* the rise in leisure tends - *ceteris paribus* - to increase consumption; *(iv)* the fall in labor income decreases total income, and therefore depresses both consumption and investment. As a result, income, investment and the labor input in country 2 react negatively on impact, while consumption remains basically unchanged. Note that the negative response of labor in country 2 dampens the reaction of the nominal factor prices to changes in the world labor supply. On impact, productivity shocks propagate across countries exclusively through variations in nominal factor prices generated by changes in world labor supply.

3.1.1 Transition

During the initial phase of the transition towards the final steady state, two main forces are at work: *(i)* the stochastic properties of TFP drive a_1 slowly back to its long-run value; hence, p_1 , converges back to unity; *(ii)* the process of capital accumulation raises the capital stock in country 1 and reduces the capital stock in country 2. Initially, the positive growth of capital in country 1 is higher in modulus than the negative growth rate of capital in country 2, and therefore the world capital stock increases. This reverses the impact variations of nominal factor prices gradually, and leads to a decrease in the labor supply in country 1, while the opposite happens in country 2. The negative growth rate of labor in country 1 dominates, and the world labor input converges to its initial steady state value. After a while, the world capital stock reverses its dynamics, and starts to converge to its initial steady state value. As a consequence, both nominal factor prices tend to converge to their original steady-state values.

Real factor prices are different across countries only as long as the productivity levels a_1 and a_2 differ. The convergence of a_1 equalizes real factor prices across countries, while the process of capital accumulation drives the world to its initial steady state.

3.1.2 Long-Run Effects

As soon as both countries share the same rental rate, their consumption paths become similar enough to prevent country-level capital stocks from converging to their initial steady-state levels. The capital stock k_1 remains permanently higher than before the shock, while k_2 remains permanently lower. Since the world capital stock must reach its initial steady state level, the increase in k_1 exactly offsets the decrease in k_2 . This permanent difference in the capital stocks implies permanent symmetric differences in consumption, investment, and labor. The labor share in country 1 converges to a permanently lower value, while the opposite happens in country 2. The investment share, the

capital-income ratio, and the capital-labor ratio converge to a higher value in country 1 and to a lower one in country 2.

The joint dynamics of capital and labor also have a permanent effect on the average productivity of labor, defined as the ratio between total income and the labor input. The transitory shock to productivity raises the labor productivity in country 1 and lowers it in country 2 permanently, since countries 1 and 2 become capital-abundant and capital-scarce, respectively.

Although the world as a whole is a stationary system, the country-level variables are non-stationary from a stochastic point of view. Each aggregate variable in country j is a unit-root process, and is cointegrated with the corresponding variable in country j : they are individually non-stationary, but their sum is actually stationary. This implies that the country-level steady states to which the system tends after a shock are different from the initial ones, but endogenously determined by the adjustment process itself.

In our model, investing in physical capital is the only way to accumulate wealth over time. Hence, productivity shocks have permanent effects on the cross-country wealth distribution, and indirectly on the income distribution. Baxter and Crucini (1995) show that in a standard IRBC model with restricted asset markets, in which only riskless bonds are internationally traded, the steady-state level of asset holdings - a sufficient statistic for the cross-country wealth distribution - is not invariant to productivity shocks. In a different framework, Obstfeld and Rogoff (1995) show that demand shocks can have permanent effects on the cross-country consumption differential and wealth distribution when again asset markets are restricted to riskless bonds. In both these contributions, the restricted financial markets introduce a more or less direct link between consumption growth rates, via the common interest rate. Similarly, in our model the consumption/leisure paths are eventually driven in both countries by the same interest rate, but this international link is generated by trade in goods rather than trade in bonds.²⁶

Given the non-stationarity of the country-level variables in our framework, dropping the capital stock from the specification of the Solow residuals may have misleading effects, as shown in Figure 3, where the approximated Solow residuals are plotted. Since the capital stocks end up to be permanently different, dropping them from the specification makes the approximated Solow residuals non-stationary. However, the variables' cross-country cointegration guarantees that the estimation of a VAR in levels is still statistically consistent.

²⁶Hall (1978) shows that in a partial equilibrium model of the life-cycle permanent income hypothesis the marginal utility of consumption follows a martingale - and the consumption level is therefore a unit root process - if the real interest rate is exogenous. Similar results emerge in a small open economy framework, as pointed out by Sen and Turnovsky (1989) and Correia, Neves, and Rebelo (1995). In our set-up, the equilibrium real interest rate is endogenous at the world level, but partially exogenous at the country level. Hence, Hall's result applies, to a limited extent, at the country level.

	Std.%	Rel. Std.	Auto.	Cor. y	Int. Cor.	
					Data	HO
y	1.18 (0.16)	-	0.66 (0.08)	-	0.42 (0.20)	0.25 (0.17)
c	-	0.45 (0.01)	0.67 (0.08)	0.99 (0.00)	0.23 (0.22)	0.40 (0.16)
i	-	3.33 (0.04)	0.66 (0.08)	1.00 (0.00)	0.31 (0.21)	0.16 (0.18)
n	-	0.46 (0.02)	0.66 (0.08)	0.97 (0.01)	0.35 (0.21)	-0.20 (0.17)

Table 1: Business cycle properties.

3.2 Stochastic Properties

To study the stochastic properties of our model, we simulate it for 10.000 times over a 100 quarter horizon, drawing the shocks from a multivariate normal distribution. At each round, we apply the Hodrick and Prescott (1980) filter²⁷ to the simulated series, and compute the standard business cycles statistics for the cyclical components in country 1: the volatility of income, measured by the standard deviation, the relative volatility of the remaining aggregate variables, their autocorrelation, their correlation with income, and their international correlations. Table 1 reports the averages and standard deviations of these statistics over the whole sample. We also report the average and standard deviation of the observed international correlations reported in Maffezzoli (2000, Tab. A.7).²⁸

As far as the national business cycle properties are concerned, the picture is very much like the standard results in the RBC literature. The relative volatility of consumption with respect to income is higher than in standard models, but still significantly lower than one. All series are highly autocorrelated, and positively correlated with income. Consumption, in particular, comoves almost perfectly with income. These results deserve

²⁷We Hodrick-Prescott filter the simulated series for two main reasons: (i) for comparison purposes with the existing literature; (ii) to extract the unit roots and obtain stationary cyclical components. Canova (1998) and others, however, show that applying the Hodrick-Prescott filter to integrated series is likely to induce spurious results. Furthermore, being the Hodrick-Prescott a univariate procedure, it ignores by construction the cointegrating relationships that link different variables in the same country and the same variables across countries. This issue is addressed in a companion paper. Note, however, that the same filtering procedure is used to extract the cyclical component from the data: in this sense, the model is compared to the data under the same conditions.

²⁸The data set regards ten OECD countries (Australia, Austria, Canada, France, Germany, Italy, Japan, Switzerland, UK and USA). The sample period is 1970:1-1997:4 (for Australia, Germany, and Switzerland the sample period is shorter for some variables); sources are the OECD's *Quarterly National Accounts* integrated by OECD's *Quarterly Labour Statistics*. Variables are GDP, private consumption, private fixed investment (all at constant prices), and civilian employment. All variables are expressed in logarithms, deseasonalized (for series not deseasonalized at the origin, the X-11 program was used), and Hodrick-Prescott filtered. The reported statistics are averages and standard deviations across all country pairs.

no further scrutiny.

Turning to the international business cycle properties, the model generates positive international correlations for all variables except the time share devoted to labor. However, only the correlation coefficients for consumption and income seem significantly positive. The international correlation of consumption is higher than the international correlation of income. Nevertheless, the size of the cross-country correlation coefficient for consumption is significantly smaller than in standard models, and not far from its empirical estimate. The overall impression is that the international propagation of productivity shocks through goods trade generates business cycle properties that are broadly compatible with the empirical evidence.

4 Complete Markets

So far we have assumed that countries are unable to pool idiosyncratic risks so as to rule out any transmission mechanism other than international trade. The standard IRBC model assumes instead the existence of a complete set of Arrow-Debreu securities; this allows for both international consumption risk sharing and intertemporal consumption smoothing. If asset markets are complete, we can solve for the Pareto optimal allocation, and hence for the Walrasian equilibrium, using the Negishi-Mantel algorithm. In other words, we maximize the following social welfare function²⁹

$$U_t = E_t \left[\sum_{s=t}^{\infty} \tilde{\beta}^{s-t} \sum_{j=1}^2 \frac{c_{jt}^{1-\mu} (1 - n_{jt})^{\tau(1-\mu)}}{1 - \mu} \right] \quad (24)$$

under the world-level budget constraint and the country-level accumulation equations:³⁰

$$p_{1t} (c_{1t} + i_{1t}) + p_{2t} (c_{2t} + i_{2t}) = w_t N_t + r_t K_t \quad (25)$$

$$\gamma k_{jt+1} = (1 - \delta) k_{jt} + \varphi \left(\frac{i_{jt}}{k_{jt}} \right) k_{jt} \quad (26)$$

In the integrated equilibrium, factor prices are still given by Equations (9) and (10),³¹

²⁹We are implicitly assuming that the population size is equal across countries, and hence use identical welfare weights.

³⁰Without adjustment costs, the actual international allocation of capital in the integrated equilibrium is undetermined: once the rate of returns are equalized across countries through trade in intermediate goods, the world capital stock becomes the only relevant state variable in the planner's dynamic problem. In the two-good model developed in Backus, Kehoe, and Kydland (1994) the cost of adjustment is not necessary, since the country specificity of each intermediate good is enough to identify consumption and investment at the country level.

³¹Given homotheticity in the production function of Y , world demand for intermediates does not depend on the way income is distributed across countries.

$p_{1t} = a_{2t}/a_{1t}$; and $p_{2t} = 1$. The budget constraint (25) can be rewritten as

$$p_{1t} (c_{1t} + i_{1t} + \pi_t) = y_{1t} \equiv w_t n_{1t} + r_t k_{1t} \quad (27)$$

$$p_{2t} \left(c_{2t} + i_{2t} - \frac{p_{1t}}{p_{2t}} \pi_t \right) = y_{2t} \equiv w_t n_{2t} + r_t k_{2t} \quad (28)$$

where π_t is the trade balance in country 1, expressed in units of the final good produced in the same country.

In the standard IRBC framework, international consumption risk sharing implies that the marginal utility of consumption is equalized across countries. Rearranging the first order conditions with respect to c_{jt} , i_{jt} , and n_{jt} under FPE *and* complete markets, we can show that both the marginal utility of consumption (corrected for the price of the final consumption good) and the marginal utility of leisure are equalized across countries:

$$c_{1t}^{-\mu} (1 - n_{1t})^{\tau(1-\mu)} \frac{p_{2t}}{p_{1t}} = c_{2t}^{-\mu} (1 - n_{2t})^{\tau(1-\mu)} \quad (29)$$

$$\tau c_{1t}^{1-\mu} (1 - n_{1t})^{\tau(1-\mu)-1} = \tau c_{2t}^{1-\mu} (1 - n_{2t})^{\tau(1-\mu)-1} \quad (30)$$

When income is pooled across countries, the planner can transform a unit of consumption in country 2 into p_2/p_1 units of consumption in country 1; under consumption risk sharing and along an optimal path, the marginal decrease in $U(c_2, l_2)$ generated by a decrease in c_2 has to exactly offset the marginal increase in $U(c_1, l_1)$ given by the corresponding rise in c_1 . (See equation (29).) Decreasing leisure by one unit generates the same reward in both countries, since the nominal wage rate is equalized under FPE; hence, being incomes pooled, the marginal utilities of leisure have to be equalized across countries. (See equation (30).)

Combining equations (29) and (30) yields

$$\frac{1 - n_{1t}}{1 - n_{2t}} = \left(\frac{a_{1t}}{a_{2t}} \right)^{\kappa-1} \quad (31)$$

$$\frac{c_{1t}}{c_{2t}} = \left(\frac{a_{1t}}{a_{2t}} \right)^{\kappa} \quad (32)$$

where $\kappa \equiv [\tau(1-\mu) - 1] / [\tau(1-\mu) - \mu]$.

The levels of consumption and leisure can be different across countries only as long as the TFP levels are also different. This implies that introducing trade in financial assets pins down the steady state levels of consumption and worked hours. However, this is not enough to pin down the steady state at the country level: equations (31) and (32) imply that in steady state $c_1 = c_2 = c$, and $n_1 = n_2 = n$. Equations (21) and (22) hold under complete markets, too. Hence, any combination of k_1 and k_2 such that FPE holds and

$\frac{k_1+k_2}{2n} = \frac{K}{N}$ is compatible with the steady state.

Combining equations (26), (27) and (28), evaluated at the steady state, with equation (21) leads to

$$\pi = \frac{1 - \tilde{\beta}}{2\tilde{\beta}} \gamma (k_1 - k_2) \quad (33)$$

The steady-state trade balance can be zero only if the steady state itself is symmetric. Otherwise, country 1 runs a permanent trade balance surplus if $k_1 > k_2$, or a permanent deficit if $k_1 < k_2$. Consumption and leisure levels have to be equalized in steady state, but capital stocks do not. The country with a higher capital stock therefore has to transfer a positive share of its income to the poorer country permanently.³²

Figures 5 and 6 plot the impulse response functions corresponding to an unexpected 1% positive shock to TFP in country 1. International consumption risk sharing and intertemporal consumption smoothing are now additional channels of propagation. Given that the final good is non-tradable, these new transmission mechanisms work through trade in intermediate goods.

4.1 Impulse Response Functions

The impulse responses for the country-level income, consumption, investment, capital, labor input, real wage rate, and real rental rate are plotted in Figures 5 and 6. In comparison with the financial autarky case (Figure 2), a positive shock to productivity in country 1 has on impact a similar positive effect on y_1 , a larger positive effect on c_1 and i_1 , and a lower positive effect on n_1 . Concerning country 2, in comparison with financial autarky, a positive shock to productivity in country 1 has a negative effect on consumption, a stronger negative effect on investment, and a weaker negative effect on n_2 .

The initial changes in n_1 and n_2 are more moderate despite the fact that factor prices change very similarly across financial autarky and complete asset markets. In comparison with financial autarky, the representative agent in country 1 finds it optimal to raise her labor supply by a smaller amount and her consumption level by a larger amount. She can afford to do this, because the representative agent in country 2 finds it optimal to reduce her labor supply by a smaller amount, reduce her consumption level, and finance investment and consumption in country 1. Notice that this implies that a move from financial autarky to complete asset markets reduces the international correlation of consumption.

³²Following the literature, we assume that international consumption risk sharing agreements are enforceable. This assumption is particularly strong in our framework, since permanent international income transfers can arise in equilibrium. Kehoe and Perri (2002) discuss how endogenous incomplete markets affect the international transmission of productivity shocks.

To understand these differences, recall that under financial autarky the productivity shock is transmitted across countries only via changes in factor prices: in particular, the consumption and investment paths depend mainly on the dynamics of the real interest rate. Under complete markets, two other mechanisms are at work: given the consumption risk sharing agreement, a positive, persistent, and transitory shock in one of the two countries generates: (i) a positive wealth effect in both countries, increasing - *ceteris paribus* - consumption and leisure; (ii) an incentive to transfer resources to the temporarily more productive country, that has strong effects on the dynamics of investment. The second mechanism clearly dominates on impact: investment drops dramatically in country 2 and rises sharply in country 1. For this flow of resources to take place, consumption in country 2 has to decrease slightly on impact:³³ the deviation from the initial steady state, however, becomes positive quickly, as soon as country 1 starts to pay back the initial inflow of resources, whose size was much larger than the wealth transfer implied by the risk sharing agreement.

The extreme reaction of investment has an obvious effect on capital accumulation: the capital stock in country 1 increases sharply, while the opposite happens in country 2. The interest rate reaches its steady-state value quickly, preventing the convergence of capital stocks. The steady-state difference between k_1 and k_2 is now three times larger than under financial autarky. This also shows in the steady-state differences in GDP and investment levels.

The consumption levels and the number of hours worked converge instead to their common steady-state values rapidly, as predicted by equations (31) and (32). Since GDP in country 2 ends up being permanently lower than in the initial steady state, a permanent inflow of resources from country 1 is needed to finance the optimal consumption level. In other words, country 1 runs a trade balance surplus in the new steady state: households in country 2 will only transfer resources to country 1 on impact if these resources will be paid back in the long run with a permanent wealth transfer.

4.2 Stochastic Properties

To evaluate the stochastic properties of our model under complete markets, we solve the model around a symmetric steady state, simulate it for 10.000 times, and calculate the usual Hodrick-Prescott filtered business cycle statistics. The results for country 1 are summarized in Table 2. As far as the national business cycle properties are concerned, the only quantitatively relevant effect of introducing complete markets is the sharp increase in the relative volatility of investment, that nearly doubles. Notice however that consumption still comoves almost perfectly with income. The trade balance is slightly

³³Country 1 incurs a trade deficit for the first 20 quarters, as shown in Figure 6.

	Std.%	Rel. Std.	Auto.	Cor. y	Int. Cor.	
					Data	HO
y	1.14	-	0.67	-	0.42	0.34
	(0.15)	-	(0.08)	-	(0.20)	(0.17)
c	-	0.50	0.67	0.99	0.23	0.24
	-	(0.02)	(0.08)	(0.01)	(0.22)	(0.18)
i	-	6.08	0.66	0.86	0.31	-0.61
	-	(0.57)	(0.08)	(0.05)	(0.21)	(0.12)
n	-	0.39	0.66	0.98	0.35	0.15
	-	(0.01)	(0.08)	(0.01)	(0.21)	(0.18)
π/y	-	0.75	0.66	-0.54	-	-
	-	(0.12)	(0.08)	(0.13)	-	-

Table 2: Business cycle properties (complete markets).

less volatile than GDP, and comoves negatively with it: these results are broadly in line with the empirical estimates for the US, equal to 0.28 for the relative volatility of the trade balance and to -0.46 for the contemporaneous correlation of the trade balance with GDP.³⁴

The international business cycle properties exhibit striking changes: the international correlation of GDP is now significantly positive, and *greater* than the correlation of consumption. The international correlation of employment is positive, although not significantly. However, the international correlation of investment is significantly negative, as we should expect from our previous discussion of the impulse response functions.³⁵ Canova and Ravn (1996) show that some testable restrictions implied by international consumption risk sharing in the standard IRBC framework are strongly rejected by the data. Lewis (1996) suggests that financial market restrictions can help explain the apparent lack of international consumption risk sharing in the data. In our model, consumption risk sharing actually generates a lower international correlation of consumption than financial autarky. Moreover, the international correlation of consumption is lower than the correlation of GDP under complete markets but not under financial autarky.³⁶

The key relationship to understand this result is equation (32): we can rewrite it in terms of deviations from the steady state as $\hat{c}_1 - \hat{c}_2 = \kappa (\hat{a}_1 - \hat{a}_2)$.³⁷ Note that, since $\tau > 0$

³⁴The trade balance has been computed as the ratio between net exports (exports minus imports) over GDP, both at current prices. We use again quarterly data for the 1973:1-2001:4, and Hodrick-Prescott filter the series without logging it.

³⁵The negative international correlation of investment is one of the most robust features of the standard IRBC model under complete markets.

³⁶Our findings are related to those in Stockman and Tesar (1995). In a different framework, they show that taste shocks, *i.e.* demand shocks, help explaining the observed properties of the data, while productivity shocks hitting the traded and non-traded sectors do not.

³⁷Equation (32) implies also that $\ln(c_j/c_{-j}) = \kappa \ln e_j$, where e_j is the real exchange rate. In other words, the model predicts a close relationship between consumption ratios and bilateral real exchange

and $\mu > 0$ by assumption, $\kappa < 0$ only if $(\tau - 1)/\tau < \mu < \tau/(\tau + 1)$ or $\tau/(\tau + 1) < \mu < (\tau - 1)/\tau$. Given that $\text{std}(\hat{a}_1) = \text{std}(\hat{a}_2)$ by assumption, and $\text{std}(\hat{c}_1) = \text{std}(\hat{c}_2)$ due to the symmetry of our set-up,³⁸ we can show that

$$\text{cor}(\hat{c}_j, \hat{c}_{-j}) = 1 - \kappa [\text{cor}(\hat{c}_j, \hat{a}_j) - \text{cor}(\hat{c}_j, \hat{a}_{-j})] \frac{\text{std}(\hat{a}_j)}{\text{std}(\hat{c}_j)} \quad (34)$$

The international correlation of consumption is smaller than one as long as: (i) if $\kappa > 0$, $\text{cor}(\hat{c}_j, \hat{a}_j) > \text{cor}(\hat{c}_j, \hat{a}_{-j})$; (ii) if $\kappa < 0$, $\text{cor}(\hat{c}_j, \hat{a}_j) < \text{cor}(\hat{c}_j, \hat{a}_{-j})$. It decreases - *ceteris paribus* - if: (i) κ increases; (ii) the correlation of consumption with TFP in the same country increases; (iii) the correlation of consumption with TFP in the other country decreases; (iv) the volatility of consumption decreases with respect to the volatility of TFP. Hence, since consumption levels are usually highly correlated with TFP in the same country and less volatile than TFP, the international correlation of consumption is likely to be significantly lower than one. Under our benchmark parameterization, it turns out to be lower than the correlation of GDP.

5 Net Exports, Terms of Trade, and the Real Exchange Rate

To study the model's predictions on net exports, the terms of trade and the real exchange rate, we solve its complete asset markets version around an asymmetric steady state, in which country 1 is endowed with 60% of the world capital stock.³⁹ The corresponding steady-state time shares allocated to labor have been obtained by solving the first order conditions evaluated in the steady state numerically.

5.1 Impulse Response Functions

The impulse response functions and the stochastic properties regarding all aggregate variables are different from those of the symmetric case, but these differences are neither qualitatively nor quantitatively significant. The productivity shock has important effects

rates as long as $\kappa \gg 0$. Backus and Smith (1993) find little empirical evidence for this relationship in OECD time series. Under our benchmark parameterization, κ is strictly positive; hence we are left with a puzzle that cannot be addressed in our framework.

³⁸Recall that consumption and TFP levels are stationary variables under complete markets.

³⁹Our preference for the complete markets version is motivated by our interest in studying the dynamics of the current account. The predictions of the model's financial autarky version on the real exchange rate and the terms of trade are almost identical to the ones we report below. Our preference for the asymmetric case is due to an undesirable property of the symmetric case: recall that after a one-time shock to a_1 , the ranking between k_1/n_1 and k_2/n_2 changed over time. This would complicate our analysis here, since it implies a reversal in the pattern of trade over time.

on the relative factor endowments of the two countries. On impact, it raises the labor supply in country 1 and reduces it in country 2, leaving the capital stocks unaltered. Our static Heckscher-Ohlin trade structure implies that in country 1 resources flow from the capital-intensive sector to the labor-intensive sector while the opposite happens in country 2. This is shown in Figure 8, which describes the sectoral reallocation of labor and capital after the productivity shock. In the initial asymmetric steady state, country 1 is relatively capital-abundant, while country 2 is labor-abundant. Therefore country 1 is a net exporter of the capital-intensive good, while country 2 is a net exporter of the labor-intensive good. The productivity shock in country 1 simply exacerbates this trade pattern in the new steady state. As a result, during the transition country 1 gradually reallocates both factors to the capital-intensive sector, while the opposite happens in country 2.

Figure 9 summarizes country 1's net trade flows and terms of trade, defined as the price of imports over the price of exports p_x/p_z . The pattern of trade is affected permanently by transitory shocks to productivity. For small deviations from the initial asymmetric steady state, country 1 remains a net exporter of the capital intensive good and a net importer of the labor intensive one. The p_x/p_z price ratio corresponds therefore to country 1's terms of trade. In the Technical Appendix we show that

$$\frac{p_{xt}}{p_{zt}} = \Xi \left(\frac{K_t}{N_t} \right)^{\psi-\phi} \quad (35)$$

where Ξ is a positive constant. Given $\psi > \phi$ by assumption, the initial rise in N_t has on impact a negative effect on the terms of trade in country 1; the process of capital accumulation, however, reverses this effect soon, raising the terms of trade. The latter deviate positively from the steady state during the remaining part of the transition. Being the steady-state capital-labor ratio uniquely pinned down at the world level, so should be the steady-state value of the terms of trade. Figure 9 confirms that the terms of trade worsen on impact, but then improve quite remarkably during the transition to the steady state. In the long run, the terms of trade are not affected by the productivity shock permanently.

5.2 Stochastic Properties

To analyze the stochastic properties of the real exchange rate, the terms of trade, and the trade balance, we solve the model around the same asymmetric steady state and simulate it for 10.000 times over a 100 quarter horizon. Table 3 reports the relative volatility, the autocorrelation, the correlation with income, and the correlation with π/y for the Hodrick-Prescott filtered series, as well as their estimated empirical counterparts for the

	Rel. Std.		Auto.		Cor. y		Cor. π/y	
	<i>US</i>	<i>HO</i>	<i>US</i>	<i>HO</i>	<i>US</i>	<i>HO</i>	<i>US</i>	<i>HO</i>
e	2.16	0.84	0.81	0.66	0.16	0.57	0.01	-0.83
	-	(0.06)	-	(0.08)	-	(0.12)	-	(0.08)
ToT	1.73	0.21	0.82	0.70	-0.17	-0.75	0.08	0.14
	-	(0.02)	-	(0.07)	-	(0.08)	-	(0.18)
π/y	0.28	1.14	0.78	0.62	-0.46	-0.53	-	-
	-	(0.01)	-	(0.11)	-	(0.12)	-	-

Table 3: The real exchange rate, the terms of trade, and the trade balance.

US.⁴⁰ The relative volatilities of both the terms of trade and the real exchange rate are much lower than their estimated values, but our model replicates their relative rank: the terms of trade are less volatile than the real exchange rate. Notice that in our model the volatility of the real exchange rate has a one-to-one relationship with the volatility of a_1/a_2 , whereas the volatility of p_x/p_z has a one-to-one relationship with the volatility of K/N . That is, the volatility of productivity shocks affects the volatility of a country's real exchange rate directly, while its effect on the volatility of a country's terms of trade is indirect and dampened by the fact that intermediate goods prices are formed at the world level. In contrast with this result, Heathcote and Perri (2002) show that the standard Backus, Kehoe, and Kydland (1994) model is unable to replicate neither the size nor the rank of these correlations.

The relative volatility of the trade balance is much larger than in the data. The correlations of the real exchange rate, the terms of trade, and the trade balance have the right signs, although the first two are larger in absolute terms than the observed value. Our model generates a significantly negative correlation of the trade balance with the real exchange rate, which is at odds with the almost zero correlation in the US data.

The real exchange rate is positively correlated with income, while the terms of trade are negatively correlated with income, as in the data. Heathcote and Perri (2002) show that the Backus, Kehoe, and Kydland (1994) model generates a significantly positive correlation between the terms of trade and income due to the complete specialization assumption. In that model, a positive shock to productivity in country j raises output, reducing the price of country j 's intermediate good. This implies that the terms of trade react positively on impact, being therefore highly correlated with income. In contrast, in our model productivity shocks have no direct effect on the terms of trade, since intermediate good prices depend only on the capital-labor ratio at the world level.

⁴⁰We use quarterly data for the 1973:1-2001:4 period, obtained from the Bureau of Economic Analysis and the Board of Governors. Real GDP is GDP at constant 1996 prices, the terms of trade are simply the ratio of the import deflator to the export deflator, while the real exchange rate is the inverse of the price-adjusted and trade-weighted Broad Index of the foreign exchange value of the U.S. dollar. All variables have been logged and Hodrick-Prescott filtered.

	y	c	i	n
$\rho_{ii} = 0.99$	0.24 (0.18)	0.48 (0.16)	-0.75 (0.13)	-0.50 (0.16)
$\rho_{ii} = 0.85$	0.38 (0.15)	0.11 (0.17)	-0.58 (0.12)	0.30 (0.16)
$\rho_{ij} = 0.05$	0.29 (0.17)	0.44 (0.15)	-0.78 (0.07)	-0.26 (0.17)
$\rho_{ii} = 0.88, \rho_{ij} = 0.05$	0.39 (0.15)	0.28 (0.16)	-0.63 (0.11)	0.19 (0.17)
$\phi = 0.30, \psi = 0.60$	0.35 (0.17)	0.25 (0.18)	-0.61 (0.12)	0.15 (0.19)
$\phi = 0.11, \psi = 0.99$	0.35 (0.16)	0.23 (0.17)	-0.62 (0.12)	0.14 (0.18)
$\xi_{\varphi} = 1/5$	0.29 (0.17)	0.40 (0.16)	0.10 (0.18)	-0.22 (0.17)

Table 4: Sensitivity analysis: international correlations (complete markets).

Backus, Kehoe, and Kydland (1994) yields a significantly negative correlation between the terms of trade and the trade balance: the terms of trade react positively to the shock, while the trade balance response is negative. In our model, both the terms of trade and the trade balance react negatively on impact to a positive productivity shock in the capital-abundant country, and are therefore positively correlated: a positive shock to a_1 leads country 1 to borrow from abroad, and it causes n_1 and subsequently N to rise on impact. This triggers a fall in both K/N and country 1's terms of trade p_x/p_z . The correlation between the trade balance and the terms of trade is negative instead for the labor-abundant country: a positive shock to a_2 leads country 2 to borrow from abroad and to a fall in K/N , but to a rise in country 2's terms of trade p_z/p_x . This implies a positive correlation between income and the terms of trade, and a negative correlation between the trade balance and the terms of trade. In this respect, it is interesting to note that Backus, Kehoe and Kydland (1994) report a positive correlation between net exports and the terms of trade for the US, and negative correlations for most of the other countries in their sample. In fact, in their sample, most of the countries exhibiting a remarkable negative correlation between the terms of trade and the trade balance also exhibit a positive correlation between the terms of trade and income.

6 Sensitivity Analysis

In this section we change parameter values to assess the robustness of the stochastic properties generated by our model under the assumption of complete asset markets. Except where noted, we change one parameter at a time with respect to our benchmark parameterization.

6.1 Cross-Country Correlations

Table 4 reports the international correlations of aggregate variables, obtained from the complete asset market model of Section 4 under alternative parameterizations. In general,

the model generates low international correlations of consumption. The downside is that the model never generates positive international correlations for both investment and employment under the same parameterization.

Persistence The first two rows of Table 4 report how our results change when we allow for more or less persistence in the stochastic process of the technology shock. A highly persistent productivity shock ($\rho_{jj} = 0.99$) in one country translates into sizable cross-country positively correlated variations of consumption on impact via the risk sharing agreement. The shock's high persistence enhances the incentive to transfer investment across countries, strengthening the negative international investment correlation. Notice that a more persistent productivity shock also causes more persistent real wage differences across countries, leading to a negative international correlation of hours worked. The negative international correlations of investment and labor reduce the positive international correlation of GDP with respect to our benchmark case. A low persistence ($\rho_{jj} = 0.85$) yields results comparable to our benchmark case.

Spillovers The third row of 4 reports the predictions of our model in the presence of technological spillovers ($\rho_{ji} = 0.05$). These results are comparable to those reported in the first row: introducing a positive out-of-the-diagonal element in the persistence matrix does not only introduce international spillovers of productivity shocks, but also increases the overall persistence of the stochastic process. In this case, the spectral radius of ρ is 0.98, not far from the value of 0.99 implicit in the high persistence experiment reported in the first row of Table 4. To isolate the effect of technological spillovers from the effects of higher persistence, we simulate our model with $\rho_{ii} = 0.88$ and $\rho_{ij} = 0.05$. Under this parameterization, the spectral radius of ρ is 0.93, as in our benchmark parameterization. The corresponding results are reported in the fourth row of Table 4. They are comparable to those reported in Section 4.

Factor intensities The fifth and sixth rows of Table 4 show that changes in the dispersion of ϕ and ψ that leave α unchanged have no major effects on the international correlations of aggregates with respect to our benchmark. Changes in the dispersion of ϕ and ψ do have stronger implications for the sectoral reallocation pattern of production factors over the transition (not reported here). It is easy to show that the more similar ϕ and ψ , the more important the reallocation process.

Adjustment Costs The last row of Table 4 reports the predictions of the model when adjustment costs to investment are very high. High adjustment costs reduce the volatility of investment substantially, and therefore raise the volatility of consumption and

	$\mu = \frac{1}{2}$		$\mu = 4$	
	FA	CM	FA	CM
y	-0.21 (0.18)	-0.25 (0.18)	0.29 (0.17)	0.26 (0.17)
c	0.25 (0.20)	0.08 (0.20)	0.34 (0.16)	0.37 (0.16)
i	-0.47 (0.15)	-0.44 (0.15)	0.25 (0.17)	-0.55 (0.13)
n	-0.73 (0.09)	-0.76 (0.08)	-0.10 (0.18)	-0.19 (0.18)

Table 5: Sensitivity analysis: financial autarky vs complete markets

the international correlation of investment. Equation (34) implies that - *ceteris paribus* - an increase in the standard deviation of consumption tends to raise its international correlation. In our benchmark case, the possibility of investing in the country subject to a positive shock mitigates the shock's initial negative effect on the other country's labor supply. With high adjustment costs to investment, this is no longer the case: on impact, the negative international correlation of real wages translates into a negative correlation of hours worked.

6.2 Financial Autarky vs Complete Markets

Table 5 reports the international correlations of aggregate variables for both the financial autarky model and the complete markets model under a low ($\mu = 1/2$) and a high ($\mu = 4$) elasticity of intertemporal substitution. The two models yield quite unrealistic correlations for these parameter values. However, the results reported in Table 5 illustrate the fact that in our framework a move from financial autarky to complete asset markets does not necessarily raise the international correlation of consumption significantly.

6.3 Net Exports, Terms of Trade, and the Real Exchange Rate

Table 6 reports the stochastic properties of the terms of trade, the real exchange rate, and the current account, obtained from the complete asset market model of Section 5 under alternative parameterizations. The results we obtained in our benchmark case are quite robust to changes in parameter values. The stochastic properties of technology shocks do not seem to play a major role here. Concerning the real exchange rate, this is due to the fact that most of the action takes place on impact through the initial change in a_j . As for the terms of trade, recall that technology shocks just have an indirect effect on this variable through the world's capital-labor ratio. The robustness of the behavior of the current account follows directly from the robustness of the behavior of aggregate

	Rel. Vol.			Cor. y			Cor. π/y		
	π/y	e	ToT	π/y	e	ToT	e	ToT	
$\rho_{ii} = 0.99$	1.06	0.86	0.11	-0.51	0.57	-0.66	-0.76	0.16	
	(1.10)	(0.26)	(0.03)	(0.18)	(0.19)	(0.19)	(0.23)	(0.18)	
$\rho_{ii} = 0.85$	0.98	0.83	0.23	-0.56	0.55	-0.77	-0.89	0.15	
	(0.30)	(0.12)	(0.02)	(0.11)	(0.12)	(0.06)	(0.05)	(0.16)	
$\rho_{ij} = 0.05$	1.08	0.88	0.15	-0.55	0.57	-0.72	-0.87	0.15	
	(0.47)	(0.14)	(0.02)	(0.12)	(0.12)	(0.08)	(0.06)	(0.17)	
$\rho_{ii} = 0.88, \rho_{ij} = 0.05$	0.97	0.82	0.21	-0.55	0.54	-0.75	-0.89	0.14	
	(0.28)	(0.12)	(0.02)	(0.11)	(0.12)	(0.07)	(0.04)	(0.16)	
$\phi = 0.30, \psi = 0.60$	0.98	0.79	0.10	-0.50	0.51	-0.69	-0.78	0.13	
	(0.51)	(0.29)	(0.03)	(0.18)	(0.19)	(0.20)	(0.23)	(0.18)	
$\phi = 0.11, \psi = 0.99$	1.13	0.85	0.31	-0.53	0.56	-0.75	-0.83	0.14	
	(0.64)	(0.13)	(0.03)	(0.13)	(0.13)	(0.08)	(0.08)	(0.18)	
$\xi_{\varphi} = 1/5$	0.07	0.89	0.15	-0.88	0.60	-0.73	-0.28	0.83	
	(0.01)	(0.14)	(0.02)	(0.04)	(0.12)	(0.08)	(0.16)	(0.06)	

Table 6: Sensitivity analysis: trade balance, real exchange rate, and terms of trade.

variables.

Persistence The first two rows of Table 6 report how our results change when we allow for more or less persistence in the stochastic process of the technology shock. Results are comparable to those reported in Section 5.

Spillovers The third and fourth rows show that introducing spillovers in the stochastic process of the technology shocks does not change the predictions of the model substantially. Results are again comparable to those reported in Section 5.

Factor intensities Changes in the dispersion of ϕ and ψ have a moderate effect on the volatility of the terms of trade. Equation (35) is self-explanatory in this respect.

Adjustment Costs The last row of Table 6 shows that a higher adjustment cost to investment only seems to reduce the volatility of π/y . This is a direct consequence of the reduction in the volatility of investment.

7 Concluding Remarks

In future research we plan to consider other types of shocks, such as government expenditure. On the empirical side, it would be interesting to assess the extent to which the model's predictions on sectoral reallocation are consistent with the data.

Our FPE scenario is just one of many possible ways to model international interactions through comparative advantage driven commodity trade. More realistic (yet less tractable) comparative advantage models may help produce additional insights on the way business cycles are propagated across countries.

One weakness in our analysis, common to a great share of the IRBC literature, is the theoretical prediction that investment is negatively correlated across countries. In this respect, current research stresses the importance of frictions in goods trade and asset trade. We see our frictionless comparative advantage model as a complement to the former: introducing frictions into our setup looks a promising avenue to better understand the international transmission of business cycles, and to produce models that yield predictions more in accordance with the empirical evidence.

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9 Appendix

The solution procedure log-linearizes the model around a particular steady state. The discussion of the impulse response functions made clear that the system does not return to its initial steady state after a country-specific shock. Therefore the quality of the approximation tends to decrease with the length of the simulation horizon. To check the importance of this issue, we solved the model under a certainty equivalence assumption using a Galerkin projection method. We found that for small deviations from the steady state, the King, Plosser, and Rebelo (1988) procedure obtains almost identical results with substantially greater computational efficiency.

Turning to the stochastic properties of the model, we concluded in the previous sections that all aggregate variables are cross-country cointegrated. The shocks to TFP are, by assumption, jointly normally distributed *iid* innovations. These two properties of the model imply that over the simulation horizon the system should remain near the initial steady state, even if the aggregate variables are in principle unit-root process. To verify this, in Table 7 we report the average value and the standard deviation - over the 10.000

	Heckscher-Ohlin				Baxter-Crucini	
	Fin. Autarky		Com. Markets		Med.	Std.
	Med.	Std.	Med.	Std.		
y	2.25	0.85	2.80	1.54	2.40	1.14
c	1.24	0.58	1.25	0.56	0.93	0.68
i	6.89	2.31	12.93	5.16	6.34	1.95
n	0.82	0.22	0.69	0.18	1.40	0.60
k	2.06	1.44	4.09	3.06	1.71	1.42

Table 7: Mean absolute percentage deviations.

simulations - of the mean absolute percentage deviation of each country-level variables from its initial steady-state value during the simulation horizon. We compare these figures with the corresponding statistics for the standard Baxter and Crucini (1993) IRBC model under our benchmark parameterization, and using the same sequence of innovations for simulating the models in each round.

As we can see, on average, and during the simulation horizon, our model does not worse than a standard, completely stationary model in terms of deviations from the initial steady state. It is well known in the literature that log-linearization methods generate accurate approximated solutions for standard models: this accuracy seems to be preserved in our framework.

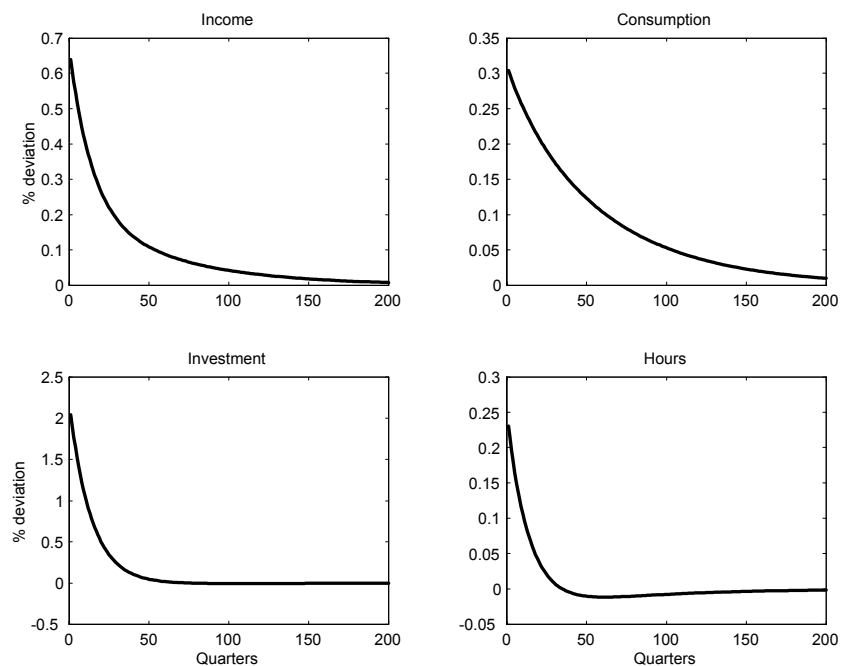


Figure 1: World aggregates.

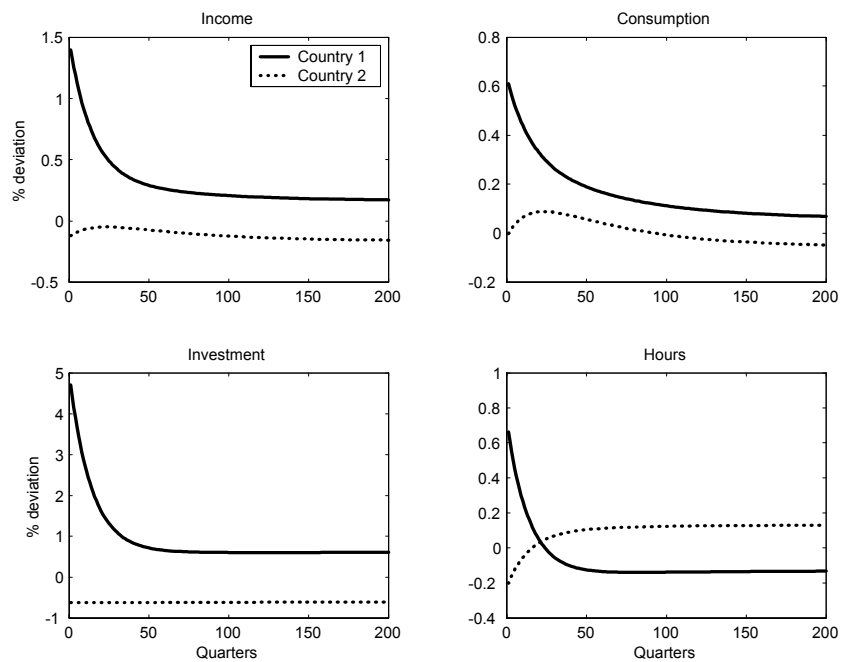


Figure 2: Income components and labor.

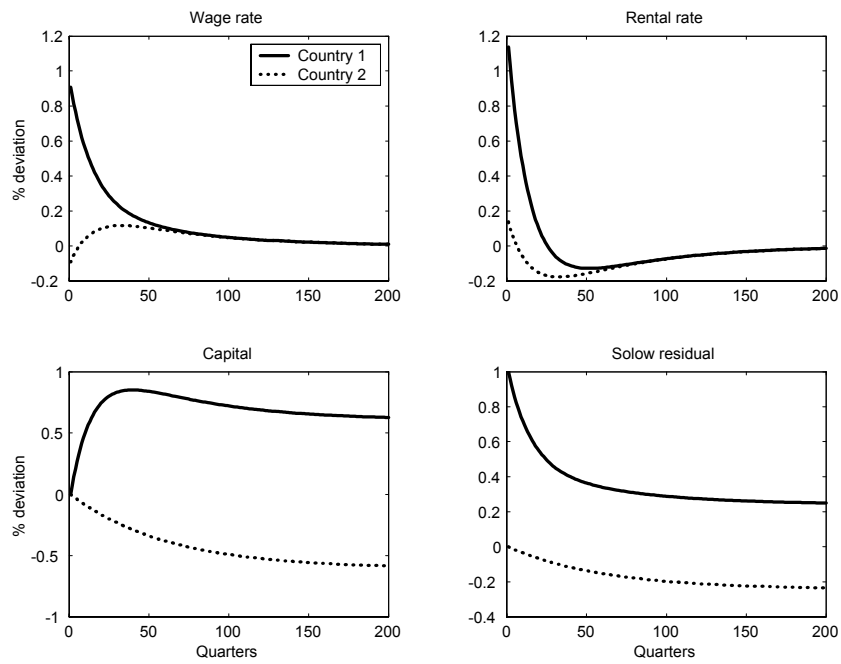


Figure 3: Factor prices, capital, and the Solow residual.

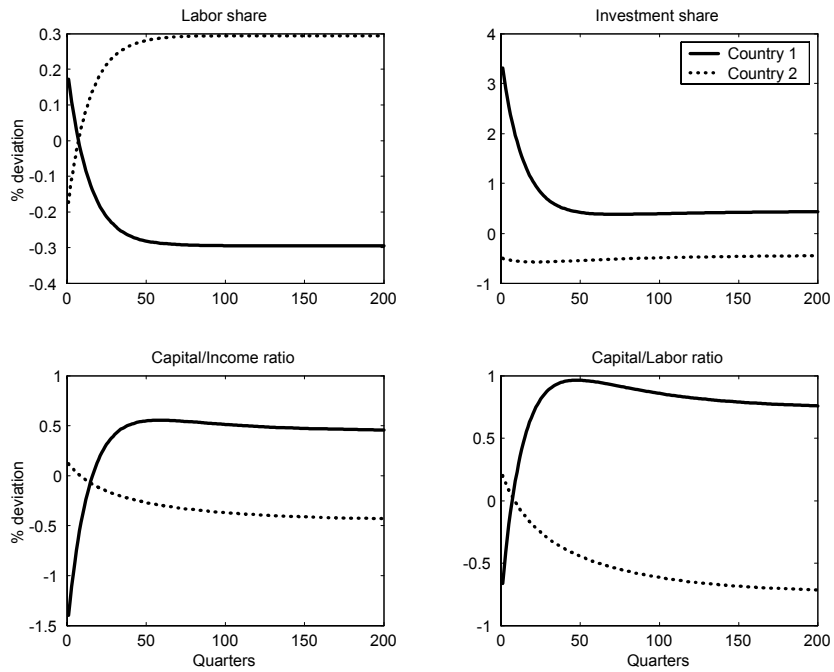


Figure 4: Ratios and shares.

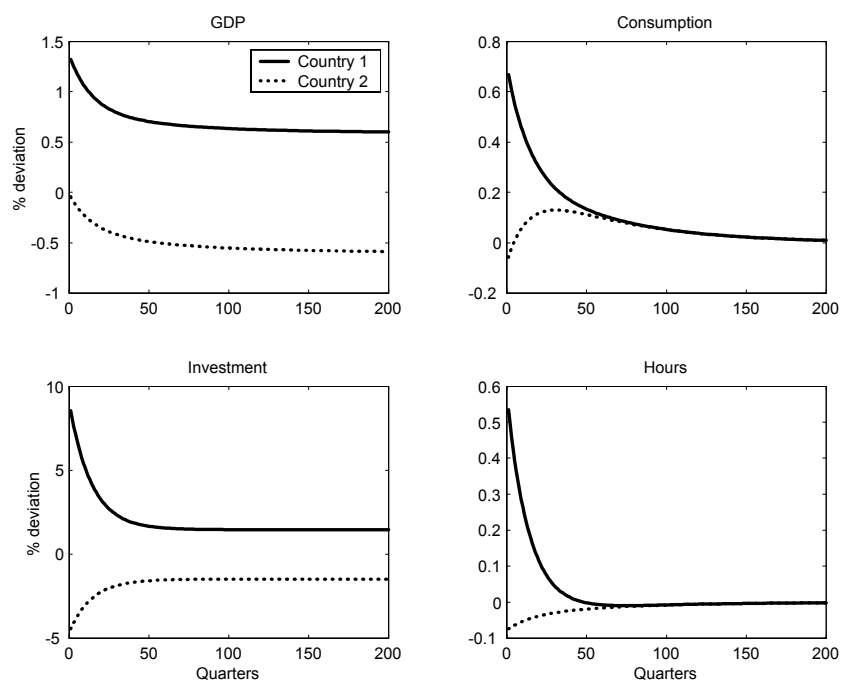


Figure 5: Income components and labor (complete markets).

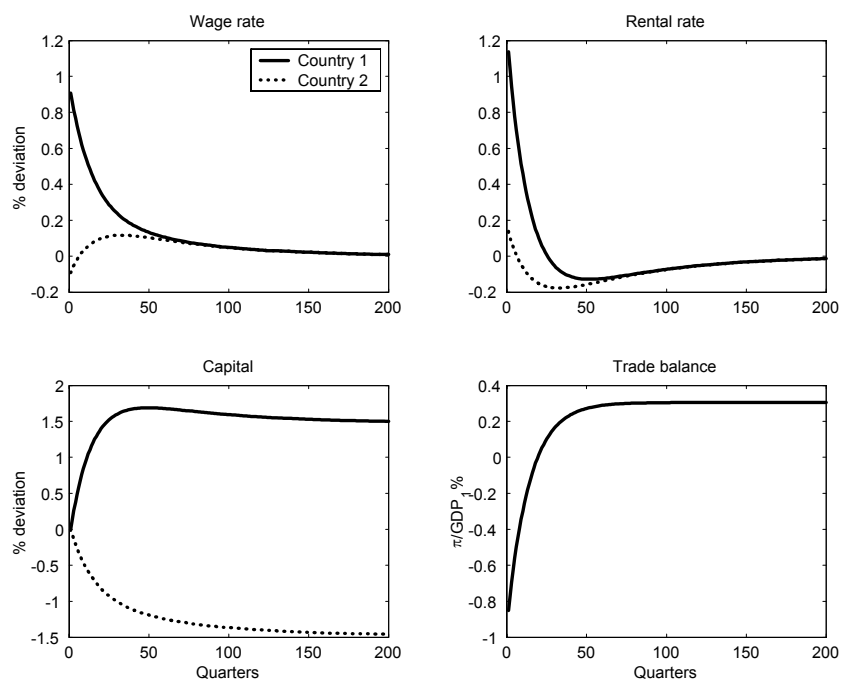


Figure 6: Factor prices, capital, and the current account (complete markets).

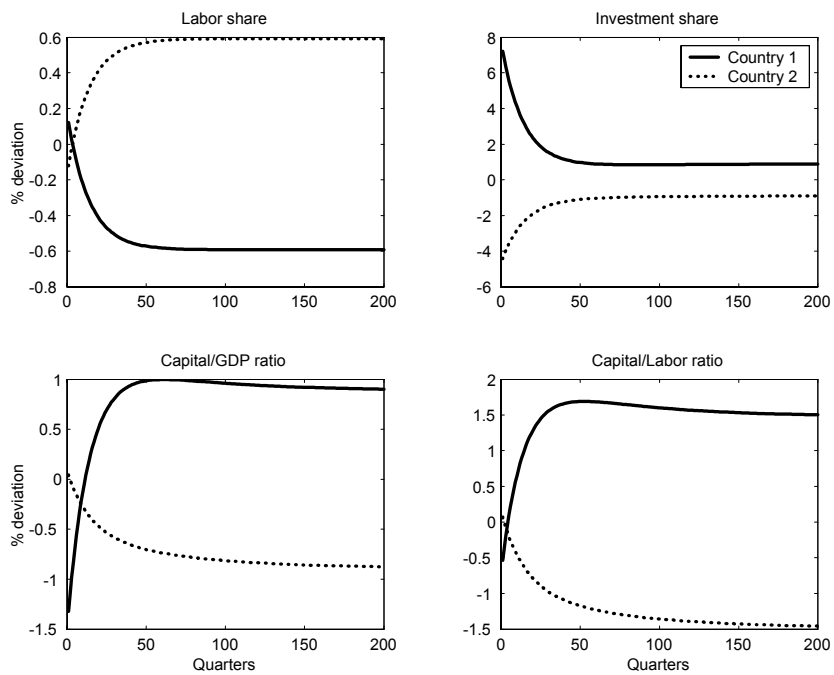


Figure 7: Ratios and shares (complete markets).

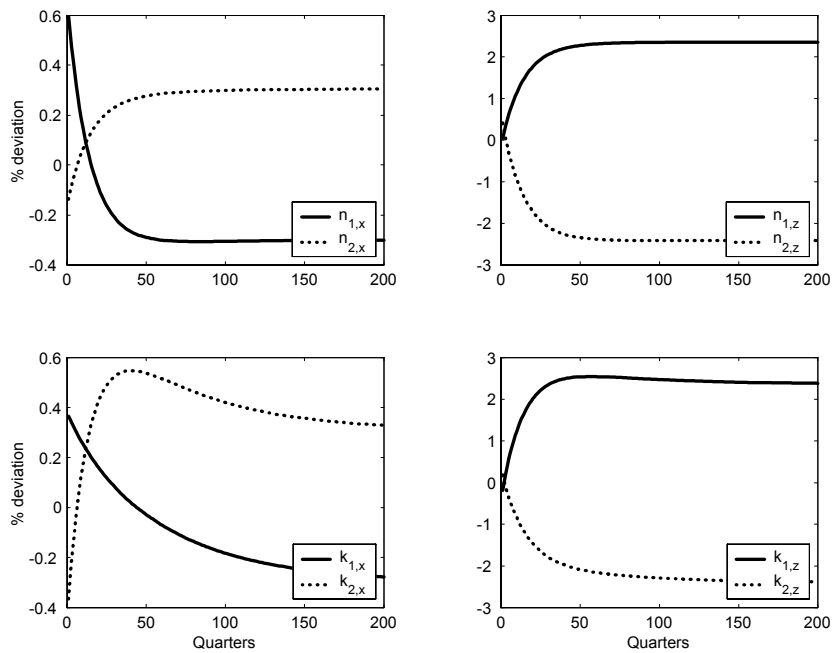


Figure 8: Sectoral allocation (complete markets).

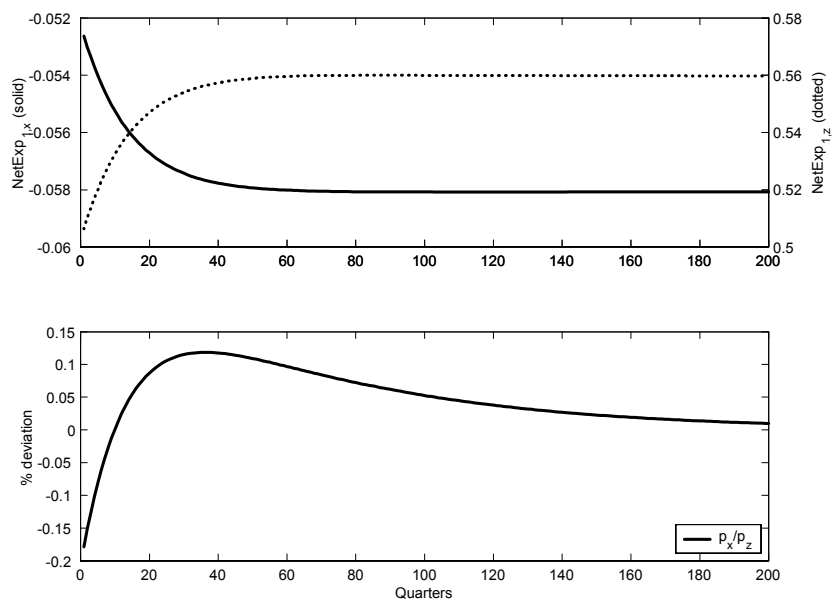


Figure 9: Net exports and the terms of trade (complete markets).