Slow Boom, Sudden Crash

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Abstract

Many asset markets exhibit a pattern of slow booms and sudden crashes. This paper presents an explanation based on an endogenous speed of learning. In the model, more observable economic activity takes place in good times than in bad times. Since more activity generates more public information about the state of the economy, faster learning takes place in good times. If the economic state changes when times are good and learning is fast, asset prices adjust quickly and a sudden crash occurs. When times are bad and learning is slower, agents take longer to react as the economy improves; a more gradual boom ensues. Data from U.S. and developing-country credit markets support the theory.

Gradual booms and sudden crashes are a ubiquitous feature of financial markets. In the four months following the 1994 peso crisis, Mexican lending rates rose more than fourfold (21% to 91%). It took 33 months for rates to return to their pre-crisis level. During the financial crises in Thailand, Russia and Indonesia, asset price declines were shorter and sharper than any boom of equal magnitude. [1] Of the ten largest 1-day movements in the S&P 500, nine were declines. One measure of this asymmetry between booms and crashes is skewness of the distribution of price or interest rate changes. Using this criteria, bond and equity index changes are asymmetric for Thailand, Russia and Indonesia, as well as for emerging markets as a whole.[2]

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[1] A strain of finance literature systematically documents this asymmetry in U.S. equity returns. See Campbell and Hentschel, 1992; Engle and Ng, 1993 and Nelson, 1991.

[2] Emerging market equity indices from International Financial Statistics. Changes in regional composite indices for Asia, Latin America and Africa/Middle East exhibit negative skewness. Bond indices are from Lehman Brothers. Emerging market bond composite is constructed as a weighted average of country indices. At the country level, 21 of 25 emerging markets exhibit negative skewness.

A model where information flows increase as asset values rise, or as interest rates fall, can explain this asymmetry. Consider investors who chooses whether or not to invest in a risky asset. The probability of a positive return is the unobserved state of the economy. Investing generates information about this state. When investors believe the state is good (high probability of positive return), many investments generate a large sample of observations. If the state switches to bad, investors quickly deduce that the state has changed. Investment plummets and interest rates soar. When the state is bad and switches to good, the prevailing low investment levels create a small data set problem. Greater uncertainty slows reactions to the state change. The result is a gradual boom. Although the model is narrowly about a lending market, the idea of endogenous learning extends to other asset markets with asymmetry such as equity, venture capital, and foreign investment. The conclusion discusses some of these alternate interpretations.

The new contribution of the model is an endogenous rate of information flow that varies with the level of economic activity. The literature on herds and crashes (Welch, 1992; Banerjee, 1992; Bikhchandani, Hershleifer and Welch, 1992; Caplin and Leahy, 1994) and the portfolio insurance models of stock market crashes (Jacklin, Kleidon and Pfleiderer, 1992) use a constant speed of learning to generate sudden crashes. This paper proves that in such a constant-learning setting, large crashes are just as likely as large booms. These models explain sudden market movements, but not asymmetry. In this sense they are complements to this paper, which produces asymmetry but not overreaction. Coupling endogenous learning with an overreaction theory would yield a model that was capable of producing large asymmetric crashes, with only small changes in fundamentals.

To explain under-reaction to good news in bad times and overreaction to bad news in good times, Veronesi (1999) proposes a model where risk aversion makes asset price a convex function of beliefs. This means that asset price booms start gradually and build speed while crashes start suddenly and lose momentum. This is *conditional asymmetry* in price changes, a different feature of the data than the unconditional asymmetry this paper addresses. In such a model, beliefs are just as likely to move in one direction along a beliefs-price curve as in the other. Thus, any steep region of the curve that could produce large downward price movements must be just as likely to produce a same-size upward move. The results prove that no matter what the shape of the pricing curve, the price changes will still be *unconditionally symmetric*.

A variety of models produce unconditional asymmetry by exploiting exogenous asymmetries present in the markets they model. The source of the asymmetry in Hong and Stein (2002) is a short-sales constraint, which causes bad information to be revealed only when prices are already low. This mechanism is not active in a market like lending where short-sales constraints are not present. Chalkley and Lee (1998) produce unconditional asymmetry with noise traders whose presence does not diminish in bad times and who forget past information. Chamley and Gale's model (1994) shares the feature that investment enables learning. Irreversibility of investment delays the resumption of growth after a downturn.[3] Delay produces a distribution of changes with many observations near zero, but tails that are symmetric. This theory focuses on precisely the asymmetry in these tail events. Furthermore, interest rates on the smallest corporate loans, which we would not expect to be given to firms with the largest sunk costs, are the most asymmetric. (See section 5.) This finding is at odds with the irreversible investment theories but supportive of endogenous learning. Likewise, Boldrin and Levine's (2001) asymmetric spread of new technologies would not predict these cross-sectional differences in asymmetry.

Campbell and Hentchel (1992) explain asymmetry using GARCH return innovations and risk-aversion. Endogenous learning can be thought of as an economic foundation for the GARCH dividend process assumed by Campbell and Hentchel because it produces price volatility that is tied to the state, which is autocorrelated. The two papers are related in another sense: GARCH dividends create asymmetry by making good signals less informative than bad ones. Thus, the GARCH effect is a specific example of a learning asymmetry.

Analytical results show that endogenous learning generates unconditional asymmetry:

[3] Other papers that use a similar mechanism include Zeira, 1994; Zeira, 1999; Rob, 1991; Caplin and Leahy, 1993.

large, one-period interest rate increases that are more likely than equally large decreases. In contrast, a benchmark model with a constant information flow is shown to generate symmetric changes. The calibrated endogenous learning model generates asymmetry comparable to that in emerging market bonds. Since price changes are negatively related to interest rate (yield) changes, negative skewness in price data is consistent with the theory's positive skewness in interest rate changes. Finally, a cross-section examination of interest rate dynamics in three corporate lending markets supports an indirect prediction of the model that the riskiest markets should also be the most asymmetric.

1 The Model

There is a credit market with M perfectly competitive lenders and a finite number, $N < M$, of entrepreneurs who are potential borrowers. Both lenders and entrepreneurs are infinitely-lived, risk-neutral profit maximizers. Each period, lenders can either invest one indivisible unit of capital in a risk-free bond which pays a return of $(1 + r)$ next period, or they can loan capital to an entrepreneur borrower. The risk-free rate is exogenous and constant. If an entrepreneur borrows and the venture succeeds, the lender receives $(1 + \rho)$ next period. If the venture fails, the lender gets back nothing. The market lending rate is endogenous and depends on the expected rate of default.

Each period, an entrepreneur can borrow one unit of capital to start up a new venture or can work for a fixed wage. Entrepreneur borrowers know what their profit will be if the venture succeeds. If successful, borrower i receives an exogenously determined investment payoff $v_i \,\epsilon\, (\underline{v}, \bar{v})$ that is constant over time. These bounds are chosen so that an entrepreneur with an investment payoff in the interval could change a decision based on observed information. This assumption excludes uninteresting actors who would always invest or never invest. Section 5.2 relaxes this assumption and discusses its effects.

If unsuccessful, an entrepreneur who borrows receives zero profit. Borrowers also face an opportunity cost of undertaking a business venture: they forgo a riskless wage income w.

The probability of success for all new ventures is the same and depends on an unobserved state variable, $\omega \in \{\omega_g, \omega_b\}$. The state changes with a small probability λ each period. If the market is in the good state, ω_g, then the probability of a venture succeeding and a loan being repaid is θ_g. In a bad state, that probability is θ_b ($\theta_g > \theta_b$). By assumption, all agents begin with common beliefs. Furthermore, all successes and failures are publicly observable, so agents will always have common beliefs about the probability μ of being in the good state. Throughout the paper, I use the subscript t on μ and θ to denote beliefs about the probability of being in the good state and of a venture succeeding in period t, conditional on all information observed before the start of period t.[4]

The ordering of events in a period is as follows:

1. All agents enter the period with beliefs, μ_t.

2. Lenders set interest rates, and borrowers decide whether or not to accept a loan and invest in a venture.[5]

3. All lenders not matched with borrowers invest in the risk-free security. All borrowers who decide not to invest are matched with fixed-wage jobs.

4. The outcomes of all ventures are publicly observed, and all payoffs are received.

5. The state changes with probability λ, and beliefs are updated.

2 Equilibrium

Definition of Equilibrium

A rational-expectations subgame-perfect equilibrium, for a given initial belief μ_0 is a sequence of borrowing decisions by borrowers $\{b_{it}\}$, interest rates chosen by lenders $\{\rho_{jt}\}$, funded

[4]More precisely, if f_t is the set of all signals observed during period t, and $\mathcal{F}_t$ is the filtration generated by $\{\mu_1, f_1, \ldots, f_{t-1}\}$, then μ_t and θ_t are $\mathcal{F}_t$-adapted.

[5]In equilibrium, borrowers will be indifferent between loans from different lenders. However, off the equilibrium path, a market-clearing device may be needed. Auctioning loans to the bidder willing to pay the highest interest rate will suffice. In reality, capital is often allocated through an auction mechanism, such as the uniform price auction used for U.S. Treasury bills.

ventures $\{n_t\}$, updated beliefs about the probability of being in a good state $\{\mu_t\}$, and the probability of success of a venture $\{\theta_t\}$ for all agents, as shown below.

1. **Borrowers**. Given a set of available lending rates J_t, borrower i maximizes expected profit by choosing whether to borrow and which lender to borrow from.

$$\max_{b_{it}\epsilon\{0,1\}, j\epsilon J_{it}} b_{it}\theta_t(v_i - (1+\rho_{jt})) + (1-b_{it})w \tag{1}$$

2. **Lenders**. Given strategies of the other lenders, lender j chooses an interest rate ρ_{jt} to maximize his expected profit

$$\max_{\rho_j} l_{jt}\theta_t(1+\rho_{jt}) + (1-l_{jt})r \tag{2}$$

where $l_{jt} = 1$ if a borrower decides to borrow from lender j in period t and 0 otherwise.

3. **Number of Funded Ventures**. This number of ventures is equal to the number of borrowers who decide to take out a loan.

$$n_t = \sum_{i=1}^{N} b_{it} \tag{3}$$

4. **Beliefs**. All agents observe the number of successes and failures during period t and form posterior beliefs μ_t^P, using Bayes' Law. If s is the number of successes observed out of n funded ventures in period t, then the formula for posterior beliefs is

$$\mu_t^P = \frac{C_s^n \theta_g^s (1-\theta_g)^{n-s}\mu_t}{C_s^n \theta_g^s (1-\theta_g)^{n-s}\mu_t + C_{n-s}^n \theta_b^s (1-\theta_b)^{n-s}(1-\mu_t)}. \tag{4}$$

Since the combination terms C_s^n and C_{n-s}^n are equal, they drop out of the equation. These posterior beliefs are converted to next period's beliefs by adjusting for the probability of a state transition

$$\mu_{t+1} = (1-\lambda)\mu_t^P + \lambda(1-\mu_t^P). \tag{5}$$

The relationship between the belief about the current state and the expected probability of success of a venture is as follows: [6]

$$\theta_t = \mu_t * \theta_g + (1 - \mu_t) * \theta_b. \tag{6}$$

Equilibrium Outcomes

Since lenders are perfectly competitive, they make no profit in equilibrium.

Lemma 1 *In equilibrium, all loans accepted by borrowers have the common interest rate* ρ_t*, where* ρ_t *satisfies*

$$\theta_t(1 + \rho_t) = 1 + r.$$

Proof is in the appendix. Since all lenders charge identical interest rates, the choice of lending rates becomes irrelevant. The only choice left is to borrow or not. The solution is a cutoff rule in the payoff v_i. An entrepreneur borrows money to start a venture in period t if

$$\theta_t(v_i - (1 + \rho_t)) \geq w. \tag{7}$$

Another way to interpret this condition is that borrowers and lenders enter into a contract if and only if the expected joint surplus of that project is positive. The contract payoffs are the Nash bargaining outcome where borrowers have all the negotiating power.

A venture's expected probability of success enters into the borrowing rule in two ways. A higher θ increases the expected payoff of borrowing, and it decreases the market interest rate, ρ. Since both of these effects make borrowing more attractive, the number of loans that are extended is increasing in θ.

[6] One thing to note about this definition of equilibrium is that it considers agents to be adaptive learners and implicitly rules out active experimentation, in the sense of Bolton and Harris (1999). In other words, agents hold their current beliefs fixed when deciding future expected payoffs. This assumption is made for computational simplicity. However, since information is a public good in this setting, any experimentation would suffer from a free-rider problem, and its effect would likely be negligible.

Combining expressions for the expected probability of success for a venture and the market lending rate yields

$$1 + \rho_t = \frac{1 + r}{\mu_t \theta_g + (1 - \mu_t)\theta_b}. \tag{8}$$

Substituting the equilibrium values of θ and ρ in inequality 7, results in the following condition under which entrepreneur i will take out a loan:

$$v_i > \frac{1 + w + r}{\mu_t \theta_g + (1 - \mu_t)\theta_b}. \tag{9}$$

This inequality allows us to solve for the payoff bounds $\underline{v}$ and $\bar{v}$. An entrepreneur with payoff $\underline{v}$, who is certain he was in the good state in the previous period, would be indifferent between borrowing or working for a fixed wage. With payoff $\bar{v}$, an entrepreneur who is certain she was in the bad state would also be indifferent.

$$[\underline{v}, \bar{v}] = \left[\frac{1 + w + r}{\theta_g * (1 - \lambda) + \theta_b * \lambda}, \frac{1 + w + r}{\theta_g * \lambda + \theta_b * (1 - \lambda)}\right]. \tag{10}$$

A key variable is the number of ventures funded each period, n_t, because it is also the number of signals observed about the current state. Since n_t only depends on μ_t and fixed parameters, it can be expressed as a function $n(\mu)$.

$$n(\mu_t) = \text{count}_{i \epsilon \{1,2,\ldots,N\}} \left(v_i > \frac{1 + w + r}{\mu_t \theta_g + (1 - \mu_t)\theta_b} \right). \tag{11}$$

This function is non-decreasing; there are more signals per period when the expected probability of the good state is higher.[7]

To fully solve this model analytically, one would have to write out a function that took in the current state variables and produced the possible outcomes in the following period. Doing so for a given state is trivial. There are n_t signals, each of which can take on two

[7]To see this, let $\tilde{v}$ denote the right hand side of the inequality in (11). $\frac{\partial \tilde{v}}{\partial \mu} = -(\theta_g - \theta_b) \frac{1+w+r}{(\mu_t \theta_g + (1-\mu_t)\theta_b)^2} < 0$. By assumption, $\theta_g > \theta_b$ and $w, r > 0$, so the derivative is negative. For any v_i, and $\mu' > \mu$, if the inequality holds for μ, it must hold for μ'. Thus, the number of v_i that satisfy the inequality for μ' must be at least as large as $n(\mu)$.

possible values. So, there is a finite number of outcomes, and the probability of each outcome is determined by the unobserved state of the economy. However, the changing number of signals complicates the writing of a general function for every possible state. There would have to be a separate function for each $n \in \{0, 1, \ldots, N\}$. Since most markets involve a large number of actors, writing out an explicit solution is intractable.

3 Results

3.1 Analytical Results

This model can generate lending rate changes that have an asymmetric unconditional distribution where the probability of large interest rate increases is higher than the probability of equally-sized decreases. In contrast, when the information flow is constant, interest rate changes are symmetric, for any interest rate function of beliefs. Since the lending rate and the unobserved state form a joint Markov process whose transition probabilities are given by the model, the standard procedure for deriving the unconditional distribution of lending rates would be to derive the distribution that is a fixed point of the transition function. Because that function is non-algebraic, there is no analytic solution method for computing its fixed point. Therefore, the proof relies on a time-reversal argument to establish symmetry properties without deriving the distribution itself. One more assumption is needed to generate symmetry. Good and bad signals must be equally informative, meaning that the probability of success in good states must equal the probability of failure in bad states. ($\theta_g = 1 - \theta_b$) I impose this assumption in the theory to isolate the endogenous learning effect. When examining the model's quantitative predictions, the assumption will be relaxed.

Proposition 1 *In a constant learning economy with equally informative signals, an asset whose price is any function $p(\cdot)$ of beliefs μ, $p(\mu) : [0, 1] \to \Re$, has unconditionally symmetric price changes.*

The idea of the proof (in the appendix) is that the economy in the good state moving forward in time is identical to the economy in the bad state running backward in time.

Imagine two markets, G and B. The true state in market G will always be the good state, whereas the true state in market B will remain the bad state. All other parameters are identical. Between time zero and time $T \epsilon [1, \infty]$, economy G generates beliefs with an identical stochastic process as a B economy that starts at $\mu_{G,T}$ and runs backward from time T until time zero. Any function p of these two identical belief processes must generate two identical price processes p_G, p_B. If two price processes are identical for any $\mu_{G,T}$, the unconditional distributions of their changes, $f_G(\Delta p)$ and $f_B(\Delta p)$, must be identical. Running the B economy forward in time, rather than backward, generates the same distribution of price changes with the opposite sign: $f_G(\Delta p) = f_B(-\Delta p) \; \forall \; x$. If states are then allowed to change and the unconditional probability of being in each state is 1/2, then the unconditional distribution of the new process is $f(x) = 1/2 f_G(x) + 1/2 f_B(x) = 1/2 f_G(x) + 1/2 f_G(-x)$. This sum of two mirror-image distributions is symmetric. [8]

Corollary 1 *In a constant learning economy with equally informative signals ($\theta_b = 1 - \theta_g$), changes in lending rates have a symmetric unconditional distribution.*

The previous results established symmetry of constant learning models, as a benchmark against which to compare the asymmetry of the endogenous learning model. With endogenous learning, the largest increase in interest rates is bigger in magnitude than the largest decrease.

Proposition 2 *In the endogenous learning economy with equally informative signals, let $\hat{p}$ be the largest increase and $\underline{p}$ be the largest decrease in the price of an asset that occurs with strictly positive probability.*

1. *For any strictly decreasing function of beliefs $p(\mu) : [0, 1] \to \Re$, $|\hat{p}| > |\underline{p}|$.*

2. *For any strictly increasing function of beliefs $p(\mu) : [0, 1] \to \Re$, $|\hat{p}| < |\underline{p}|$.*

[8] Relaxing the assumption that signals are equally informative could produce asymmetry. For example, if bad signals contained more information than good ones ($\theta_g > 1 - \theta_b$), then beliefs would have larger but less frequent jumps down than up. However, the result that high-risk markets, with a higher frequency of bad signals, exhibit more asymmetry contradicts this explanation.

Corollary 2 *The unconditional distribution of changes in interest rates in the endogenous learning economy is asymmetric. The largest interest rate increase is larger in absolute value than the largest decrease.*

Proofs are in the appendix.

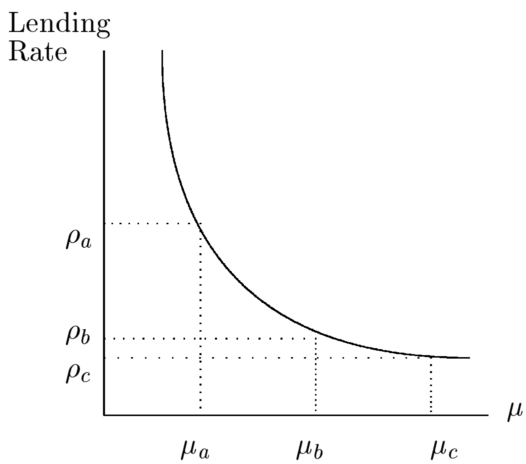

Figure 1: The relationship between changes in beliefs and changes in lending rates

To illustrate the difference between constant and endogenous learning and their relationship with conditional and unconditional asymmetry, consider the following example in Figure 1. The relationship between beliefs and the lending rate is given by the pricing rule in equation (8). In a constant learning model, the probability of beliefs moving from μ_c to μ_b is the same as beliefs moving from μ_a to μ_b. This produces conditional asymmetry in lending rate changes despite symmetry in beliefs because the nonlinear relationship between ρ and μ means that equal size upward and downward changes in beliefs do not produce equal size changes in lending rates. The increase in interest rate, $(\rho_b - \rho_c)$, will be much smaller than the equally likely decrease, $(\rho_b - \rho_a)$. This is the type of asymmetry that Veronesi (1999) discusses.

However, the fact that the transition probabilities for beliefs are reversible means that the unconditional probability of moving from ρ_a to ρ_b is the same as moving from ρ_b to ρ_a. So, the probability of this large interest rate decrease is the same as the equally large increase.

Veronesi's model and the herds models are unconditionally symmetric because upward belief movements in the good states are just as likely as the downward movements in the bad states.

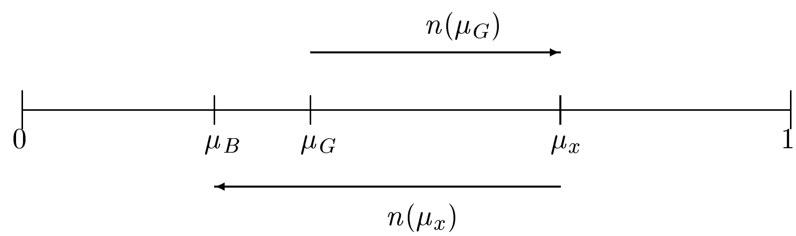

Figure 2: Asymmetric changes in beliefs in an endogenous learning economy

The reason for the asymmetry of the distribution of beliefs and prices in this model is that the change in beliefs in the good state is, on average, not as large as the reverse movement in the bad state. Figure 2 illustrates this point. If the economy starts with beliefs at μ_G and $\omega = G$, beliefs are likely to increase the following period (because μ is a submartingale in the good state) to a level such as μ_x. The size of that change will be affected by the number of signals observed during the period, $n(\mu_G)$.

Now, consider the economy starting with beliefs μ_x and $\omega = B$. Beliefs are likely to fall the next period and the size of the change will be affected by the number of signals observed during the period, $n(\mu_x)$. Recall that n is increasing in μ so that $n(\mu_x) > n(\mu_G)$. And since expected size of the movement in beliefs is increasing in n, the expected size of the downward movement in beliefs is larger than the expected upward movement. If large downward changes in beliefs are more probable than large upward movements in beliefs, then large, one-period lending rate increases will be more likely than large declines.

3.2 Simulation Results - Benchmark Model

One way to determine if this explanation for asymmetry is correct is to ask: with reasonable parameters, can the model produce the magnitude of the asymmetry we see in the data?

Choosing parameter values in such a stylized model is not straightforward. The benchmark parameters and the values deduced from various data sources are summarized in Table 1. The previous assumption that $\theta_g = 1 - \theta_b$ is dropped. To estimate a venture's conditional

Variable	Simulation Value	Value Implied by Data	Data Type
θ_g	0.97	0.97	Bond Default Rates
θ_b	0.95	0.95	Bond Default Rates
λ	0.027	0.027	Country Growth Data
N	25	25	Bond Price Crossing Times
r	0.42%	0.42%	U.S. Treasury Bills
w	1	unknown	
$\{v_i\}_{i=1}^N$	$\sim N(\frac{\underline{v}+\bar{v}}{2}, (\frac{\underline{v}+\bar{v}}{8})^2)$	unknown	

Table 1: Parameter values for benchmark simulation.

probability of success, yearly default rates on U.S. speculative grade bonds are split into two subsamples: recession years and boom years.[9] The reason for selecting U.S speculative grade bonds is that default rate data for the panel of emerging markets in the data is not available. Since emerging market bonds are likely to be riskier than U.S. corporate bonds, the riskiest grade of U.S. bonds provides the closest match. The default rate in recession years was 5%, whereas the default rate in non-recession years was 3%. Therefore, $\theta_b = 0.95$ and $\theta_g = 0.97$ for the benchmark model.

For the probability of a state transition, λ, world GDP data guides a reasonable estimate. For each of the country years listed in the Penn World Tables, boom (bust) labels are assigned to country years with positive (negative) growth of real GDP per capita. The number of years is converted to months: 36.5 months, on average, between state changes. The reciprocal of this number is the probability of a transition.[10]

The number of potential borrowers, N, is calibrated so that prices move at about the same speed in the model and the data. If N is very high, then agents learn almost immediately when the state has changed and beliefs converge a large fraction of the way to the truth in a short time. The N that matches average crossing times between upper and lower bounds of the output data to the simulation is $N = 25$.[11]

[9]Default rate data is from Moody's. (www.moodysqra.com/research/mdr.asp) Years of business cycle downturns are defined as years that have at least six months lying after the peak and before the next trough of a business cycle. Peak and trough dates are the NBER business cycle dates. Data spans 1970-1999.

[10]Using NBER recession dates to calculate the transition probability for the U.S. yields similar results.

[11]See section 4 for more on the construction of these bounds.

The entrepreneur's payoffs v_i have no data counterpart. They will be distributed normally with mean in the center of the interval $[\underline{v}, \bar{v}]$ where beliefs can affect investment choices. The variance is such that payoffs are in that interval with 95% probability. The remaining parameters only affect the scale of the lending rate; skewness is invariant in scale.

Table 2 lists two summary statistics for the simulated model. The average spread is the average percentage by which the lending rate exceeds the risk-free rate. Skewness is the sample skewness of the percentage changes in lending rates, $(\ln(\rho_t) - \ln(\rho_{t-1}))$. Ten thousand repeated runs of the benchmark model with 10,000 periods each produces an average skewness estimate of 2.35, with standard error of 0.05. With a skewness coefficient that is over 40 times as large as its standard error, there is little room to doubt that the benchmark model produces 1-period lending rate increases that are more extreme than the decreases.

Figure A.2 shows a histogram of the percentage changes in lending rates. The positive skewness is clearly visible. One strange feature of the histogram is the truncation of the distribution near -0.4. This is a result of having only two states and successes being very likely in both states. Having all ventures succeed is not such an unlikely event and causes lending rates to fall by a maximum of 0.4%. Observing many venture failures is a much less probable event and causes lending rates to react more. Having a richer state space alleviates this truncation problem. Figure A.2 is a histogram of percentage changes in interest rates in a simulation with 12 states.[12] Adding more states allows the shape of the distribution to match the data much more closely.

3.3 Comparative Statics

A sensitivity analysis was conducted by varying each baseline parameter, simulating the model for 50,000 periods, and measuring skewness. For a wide range of parameter values, the model consistently produces significant positive skewness in interest rate changes. (Table 2)

[12]The states used to generate this graphs had the following probabilities of success: $\theta_g \; \epsilon \; \{.8, .9, .95, .955., .96, .965, .97, .975, .98, .985, .99, .995\}$. Estimating conditional probabilities for so many states is not possible, given the small size of the data set on default rates (30 observations and 12 parameters). Therefore, parameters were chosen to be close to the observed success rates and on the basis of their ability to replicate the data.

model variation	average spread	skewness
Benchmark model	4.20%	2.35 (0.06)
Thick market $N = 4,000$	4.20%	4.43 (0.15)
Thin market $N = 2$	4.20%	4.97 (0.09)
Volatile state $\lambda = 0.2$	4.20%	1.57 (0.03)
Stable state $\lambda = 0.001$	4.20%	4.86 (0.39)
Noisy signals $\theta_g = .51$, $\theta_b = .49$	100.6%	0.29 (0.04)
Clear signals $\theta_g = .99$, $\theta_b = .5$	42.0%	3.67 (0.13)
Uniformly distributed investment payoffs (v)	4.20%	1.60 (0.04)

Table 2: Simulation results. Monte Carlo standard errors in parentheses. Spread is lending rate minus risk-free rate.

In every parameterization, out of 10,000 sample skewness coefficients generated, not one was negative.

High and low values of the parameters are as extreme as possible without causing learning to be either trivial or impossible. For the number of borrowers, when $N > 4,000$ posterior beliefs becomes numerically indistinguishable from zero or one. [13] Likewise, as the probability of a state change λ approaches 1/2, learning becomes impossible because states each period become independent of previous periods' states. However, as λ approaches zero, very few booms and crashes are observed. The θ's make learning either very difficult (noisy signals, θ_g close to θ_b) or very easy (clear signals, $\theta_g >> \theta_b$). Lastly, the v_i's that were previously normally distributed are now uniform, the uniform distribution being the most natural alternative. The parameters that have no effect on skewness except through other parameters: r, μ_0, and w, remain at benchmark values throughout all simulations.

[13] The model can accommodate larger N if signals become more noisy. For any N , there is a pair of conditional probabilities θ_g, θ_b such that learning is not too "too fast." To see the basis for this argument, examine equation 4, multiply the number of observed successes s and the number of funded ventures n by a factor γ. Than take the limit $lim_{\gamma \to \infty} \mu_t^P$. Realize that this converges to either 0 or 1, unless $\theta_g \to \theta_g$. If the θ's converge at just the right speed, it is possible for μ_t^P to be stable.

Table 2 also illustrates some comparative statics. When the state changes frequently (λ high), beliefs revert faster to their unconditional mean. Because mean reversion is symmetric, stronger mean reversion equalizes learning between high and low investment markets, reducing asymmetry. Likewise, when signals are very noisy, the symmetric effect of state changes has a larger relative effect on changes in beliefs, reducing asymmetry.

The relationship between the number of potential borrowers N and the skewness of interest rate changes is non-monotonic. There are two competing effects to consider: First, more agents means more signals and faster learning in all states. That dampens skewness because it becomes easier for beliefs to rise quickly from a low level. Second, doubling the number of signals in each state results in a greater difference between the number of signals observed in good and bad times. This effect makes learning speeds more asymmetric and boosts the skewness of the model outcomes.

Changing the distribution of investment payoffs, v, from normal to uniform increases the density of payoffs close to $\bar{v}$. As beliefs begin to rise from their minimum level, entrepreneurs start to borrow at a faster rate. Because the mapping from μ to n is steeper when μ is low, escaping from a slow learning state is faster; booms are more sudden, and asymmetry is less.

Examining conditional variance in the benchmark model reveals another property: The variance of lending rate is smaller in good states than in bad. This finding is consistent with observations from the stock market showing that volatility is higher when prices are lower (see e.g. Cox and Ross, 1976; Bekaert and Wu, 2000). The reason for the lower variance is an idea similar to the central limit theorem. More information makes beliefs more accurate and less variable.

4 Asymmetry in Emerging Market Bonds

Theory predicts that lending rates of corporate loans should have larger positive outliers than negative outliers. To verify this prediction, bond price data is mapped into yields, in order to measure the skewness of yield changes. Loans in the model are 1-period bonds with yields

	Skewness of Changes in Yield	Probability of skewness ≤ 0	# observations
Monthly Data	2.90 (1.59)	4.1%	1,053
Daily Data	0.74 (0.41)	3.6%	22,971

Table 3: Descriptive statistics for 1-month changes in emerging market bond yields, inferred from prices. Probability and standard errors (in parentheses) from bootstrap.

equal to ρ. [14]

The data is monthly and daily dollar-denominated bond price indices calculated by Salomon Brothers for a panel of 13 emerging markets, between January 1994 and October 2000. Table 3 displays the skewness statistics for the yields, their standard errors, and the probability that the null hypothesis of non-negative skewness is true. Both monthly and daily yields exhibit significant positive skewness. Furthermore, the magnitude of the skewness in implied yield is very close to the value predicted by the model, 2.90 and 2.35 respectively. The model's predicted skewness lies well within the 95% confidence interval of the data.

Figure A.2 shows a histogram of percentage changes in the monthly implied yields.[15] The fat tail on the right side, which indicates positive skewness in the yields, is clearly discernible.

Another feature of the data is the presence of excess kurtosis, a well-known property of financial asset returns.[16] The kurtosis of the daily and monthly data sets is quite close. (K=36 and 41) The kurtosis of the model with two states is much lower than that in the data. (K=10) This is because of the two-state truncation problem discussed at the end of section 3.2. However, when more states are added, the model can produce kurtosis similar to that of the data. (K=30) The kurtosis of the model with 12 states is within the 95% confidence interval of the data.

[14] A first-order Taylor approximation to the bond pricing equation relates bond prices to yields: $\frac{\Delta P_t}{P_t} = -M\,\frac{\Delta(1+\rho_t)}{(1+\rho_t)}$. where P_t is the bond price, ρ_t is the yield, and M is a constant equal to 100 times the bond's Macaulay duration. Since skewness is invariant in a constant multiplier, positive skewness in the model's lending rates corresponds to negative skewness in bond prices, of equal magnitude.

[15] To convert the bond price data into implied yields, an estimate of the constant M is needed. It is inferred from the data by comparing the variances of the percentage yield changes in the data and the model and taking the square root of their ratio.

[16] See Campbell et. al. (1997) or Pagan (1996) for a discussion of the excess kurtosis literature.

There are a couple of potential data problems that might be a concern. The first potential problem is the presence of fixed country effects when data is pooled across countries. However, country fixed effects in bond prices should be eliminated when prices are differenced. Remaining heteroscedasticity across countries is accounted for by using Monte Carlo distributions of the skewness statistics. The second potential problem is the presence of one extreme outlier in the monthly data, corresponding to the 1998 financial crisis in Russia. While the Russia crisis is important since it captures the type of sudden state change the model describes, it is worrisome if the results depend on a single event. Two observations mitigate these fears. First, when the Russia observation is removed, the skewness remains positive at 1.41. Second, daily data, which has twenty times as many observations and no obvious outliers, has a skewness of 0.74, which is significant at the 5% level.[17]

Asymmetry in Crossing Times

The idea of long boom times and short crash times can be captured with an alternative measure of the asymmetry using crossing times. If prices take longer to move from lower to upper bound than from upper to lower, then booms are more gradual than crashes. The procedure used to determine upper and lower bounds was to calculate the maximum and minimum price or lending rate p_{max}, p_{min} in each country or simulation run and set the upper and lower bounds equal to $(1-\alpha)p_{max} + \alpha p_{min}$ and $\alpha p_{max} + (1-\alpha)p_{min}$ for some $\alpha < 1/2$. Then, each time the price started above the upper bound and crossed the lower bound, or vice-versa, the crossing time was recorded. The ratio of average upward crossing times to downward crossing times provides an alternate measure of asymmetry.

Table 4 shows the results of this crossing time analysis. The average speed of market movements in the model and the data are similar. This is not surprising since N was chosen to match this feature of the data. The striking feature of this table is that the ratio of boom times to crash times is similar for the different bounds and data sources, and that it

[17]The conversion constant for the daily changes in implied yield was calculated by comparing the data variance to the variance of a simulation where state transitions are adjusted to happen 1/30th as frequently.

α	Data Source	Average Boom Time	Average Crash Time	Boom/Crash Ratio
1/5	Monthly Bonds	17.25	8.69	1.98
1/5	Daily Bonds	12.16	5.80	2.09
1/5	Simulation	15.16	8.65	1.75
1/10	Monthly Bonds	19.00	12.07	1.57
1/10	Daily Bonds	21.01	11.16	1.88
1/10	Simulation	22.30	14.63	1.52

Table 4: Booms refer to price increases, or lending rate declines. Boom times for monthly and daily data are measured in months. For simulations, the unit of time is one period.

is consistently above one. That means that booms are slower than crashes and the relative speed of booms and crashes produced by the model matches the data well.

5 High and Low Risk Markets

Developed-country lending markets tend not to exhibit as much asymmetry in their interest rate movements as developing countries. This section uses the model to explain why asymmetry varies by market and then looks at a panel of U.S. lending data to support the explanation.

Predictions of the Model

The theory presented so far is primarily a story about high-risk markets, meaning markets where a large fraction firms will fail in bad times. If most firms in a market are not at risk and will continue to operate in good and bad times, then only a small amount of extra information will be observed on good times. To model a low-risk market, the model will be altered to include a number of actors who receive publicly observable signals every period, no matter what their beliefs are. This constant flow of information reduces asymmetry by making the number of signals and the ability to learn more similar in good and bad states.

Another way of interpreting this setup is that there are a number of entrepreneurs in this economy who have venture payoffs, v_i's, that lie above the interval where actions depend on

Number of External Signals	Average Spread	Skewness of Change
High Risk $I = 0$	4.20%	2.35 (0.06)
Medium Risk $I = 25$	4.20%	1.13 (0.05)
Low Risk $I = 250$	4.20%	0.58 (0.11)

Table 5: Simulation results for 50,000 periods. Benchmark parameters. Monte Carlo standard errors in parentheses.

beliefs. Thus, they always borrow, generating more signals every period. [18] The setup of the model is the same with one exception: If n_t is the number of entrepreneurs who borrow at t, the number of signals observed is $n_t + I$. I is the number of riskless firms.

The results in Table 5 suggest that lower risk markets are less likely to exhibit the positive skewness in interest rates that this model predicts. The benchmark model ($I = 0$) is a high-risk market. 100% of firms are at risk. When $I = 25$, the market is medium-risk: 50% of firms are at risk. Finally, $I = 250$ is a low-risk market where less than 10% of firms are at risk. This is the type of market in which large, successful U.S. companies operate. Skewness in this type of market is very low. In a setting with more noise in lending rates and fewer observations, skewness would likely be insignificant.

What is important about adding more signals each period is not the increased amount of information, but the convergence in the quantity of information available across states. If there are more signals observable each period, no matter what beliefs are, the ability to learn in the bad state catches up to learning in the good state. As learning becomes more symmetric, skewness in interest rates falls.

Empirical Results From High and Low Risk Markets

This section uses data on U.S. lending rates of different sized loans to test if high risk markets exhibit more interest rates asymmetry than low risk markets. The reason for being interested

[18]The effect of allowing v to fall below the interval would only be to reduce the number of active borrowers, N. Since agents with v_i's below $\underline{v}$ never invest and never generate information, they are irrelevant.

Loan Size	Average Risk Premium	Skewness of Change
Less than $ 100,000	4.21%	1.12 (0.28)
Between $100,000 and $1 million	3.36%	0.48 (0.24)
Greater than $1 million	1.50%	-0.23 (0.22)

Table 6: Descriptive statistics for quarterly U.S. lending rate spreads. Bootstrap standard errors for skewness in parentheses.

in loan size is that it is related to firm risk. This relationship is evidenced by the differences in risk premia charged for each class of loans. Low-risk companies are more likely to qualify for large loans of over $1 million. Riskier firms are more likely to take out smaller loans of less than $100,000.

The data are the Federal Reserve's commercial and industrial loan rates, measured by the spread (or risk premium) over the federal funds rate.[19]

The fact that the skewness coefficients are decreasing as the size of the loan increases supports the prediction of the model. Of course, this data is still far from perfect. Ideally, one would have data that contained a time series of these interest rate spreads for markets with different amounts of risk or public information. Given that such data is not available, market risk and size of loans appear to be reasonable proxies because of their correlation with the risk premium. Thus, U.S. interest rate data also lend modest support to the endogenous learning explanation for credit market asymmetries.

6 Conclusion

What is crucial for endogenous learning asymmetry is that rate of information flow is correlated with the asset price. The model presented here creates that tie by relating the asset value to real economic activity which produces information. Interpreted literally, this model seems to ignore important sources of information about the state of a market or company

[19] The data can be found at http://www.federalreserve.gov/releases/E2/e2chart.txt

that can be learned without investing, such as information about costs or demand. However, perhaps an even more important channel for endogenous learning is information transmission. If more information is reported to investors about markets that are performing well, this learning asymmetry could also generate asset price asymmetry.

One avenue for future research is to examine market incentives for information provision in order to explicitly model a process of faster learning about more profitable firms. The following example illustrates the idea. An investor in a new firm has difficulty obtaining firm information. Changes in corporate strategy are not covered by the media. If a project succeeds, confidence in the firm rises as the information slowly disseminates (slow boom). Five years later, the firm is a market leader. Any new product release or management change is broadcast in real time on CNN. The cost of borrowing is much lower because it is less risky. If the firm's profits soared, that would also be learned quickly. However, because the lending rate was already close to the risk-free rate, it wouldn't fall by much. If something bad happened to this firm, its cost of future borrowing would rise within minutes (sudden crash).

One of the drawbacks of the model presented here is that its simplicity makes it difficult to directly match the model to data. Van Nieuwerburgh and Veldkamp (2002) show that learning asymmetry emerges in a dynamic general equilibrium model, with unobserved production shocks, where agents learn about the technology level. In this richer economic environment, a close comparison of a calibrated model and the data is possible, at the expense of theoretical results.

The ideas in this paper can also be applied to demand for consumption goods. For example, consumers learn about the risk of side effects of a new drug (such as Fen Phen) by hearing about the experiences of numerous other consumers who have tried the drug. The more popular the drug, the faster consumers will learn if there are problems. A variation of this model with random unobserved changes in product quality could predict slow growth and big crashes in demand for new goods. The same logic could explain asymmetric venture capital in a market of uncertain profit potential, or the asymmetric firm entry and exit in an industry.

Finally, the assumption of public signals can be relaxed to apply the idea to settings where signals are more likely to be private. If actions partially reveal private signals, as in herding models, then more private signals would generate more informative actions and faster learning. As long as the flow of information is connected to beliefs about the state, and signals can be transmitted, however imperfectly, to market participants, then learning and asset price movements should be unconditionally asymmetric.

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A Appendix

Throughout the proofs, let $\bar{\mu}$ denote beliefs in an environment where the number of signals observed each period, n, is constant. Furthermore, let $\theta \equiv \theta_g = 1 - \theta_b$.

A.1 Lemma 1: In equilibrium, all loans accepted by borrowers have the common interest rate $\rho_t = (1+r)/\theta_t - 1$.

Suppose a lender charges $\rho_{jt} < \rho_t$. Then, this lender can do strictly better by charging $\rho_{jt} > \rho_t$, having no borrower accept his loan, and getting the risk-free return. Suppose a lender charges $\rho_{jt} > \rho_t$, and this rate is accepted by a borrower in equilibrium. If a borrower accepts this loan, then it must have been one of the lowest posted interest rates. Since there are more lenders than borrowers, some lender is charging $\rho_{kt} \geq \rho_{jt}$, not matching with a borrower and earning only the risk free return. Lender k can gain from lowering his rate to $\rho_{jt} - \epsilon > \rho_t$. The higher rate ρ_{kt} earns $1+r$. The deviation earns $\theta_t(1 + \rho_{jt} - \epsilon) > 1 + r$, in expectation. Thus, neither ρ_{jt} can be supported as an equilibrium.[20]

One thing to note about this proof is that it assumes contracts to be one period. However, the same argument as in the lemma 1 proof also proves that any multi-period contract must have an expected net present value of $(1+r)$. Therefore, borrowers and lenders maximizing expected net present value would be indifferent between the 1-period contract and any equilibrium multi-period contract. See appendix B for more on multi-period contracts.

A.2 Lemma: Time-Reversibility

Prove: In a constant learning economy with equally informative signals, the stochastic process for beliefs in the good state is the time-reverse of the process for beliefs in the bad state: $P[\bar{\mu}_{G,t+1} = x | \bar{\mu}_{G,t} = y] = P[\bar{\mu}_{B,t} = x | \bar{\mu}_{B,t+1} = y] \qquad \forall x, y \in [0,1]$.

Consider belief changes from y to x that are realized with positive probability in the G economy. Each must have an associated number of observed success signals z that produced this posterior belief. The relationship between x, y, and z is given by Bayes' Law:

$$y = \frac{\theta^z (1-\theta)^{n-z} x}{\theta^z (1-\theta)^{n-z} x + (1-\theta)^z \theta^{n-z} (1-x)} \tag{12}$$

Likewise, for any change from x to y in the B economy, there must be a number of observed venture successes, v, that produces the posterior belief x.

$$x = \frac{\theta^v (1-\theta)^{n-v} y}{\theta^v (1-\theta)^{n-v} y + (1-\theta)^v \theta^{n-v} (1-y)} \tag{13}$$

Suppose that z solves equation (12) for a given x, y, then $v = n - z$ solves equation (13). To verify this, substitute $n - z$ for v in equation (13) and substitute equation (13) in to (12) and show that they reduce to an identity.

$$x = \frac{\theta^{n-z} (1-\theta)^z y}{\theta^{n-z} (1-\theta)^z y + (1-\theta)^{n-z} \theta^z (1-y)}.$$

$$y = \frac{\frac{(1-\theta)^n \theta^n y}{\theta^{n-z}(1-\theta)^z y + (1-\theta)^{n-z}\theta^z(1-y)}}{\frac{(1-\theta)^n \theta^n y}{\theta^{n-z}(1-\theta)^z y + (1-\theta)^{n-z}\theta^z(1-y)} + \frac{(1-\theta)^n \theta^n y}{\theta^{n-z}(1-\theta)^z y + (1-\theta)^{n-z}\theta^z(1-y)}}$$

Canceling terms results in

$$y = \frac{y}{y + (1-y)} = y$$

[20] Lenders can charge higher rates in equilibrium as long as none of the higher rates are accepted by borrowers. Since lenders are indifferent between charging ρ_t and charging a higher rate which gets them the risk-free return, equilibria where some lenders charge higher rates can be sustained. Since no lending occurs at these rates, they do not influence outcomes. This paper ignores this class of equilibria and focuses on the symmetric equilibrium.

If C_z^n is the number of combinations of z elements from a set of size n, the formula for the transition probabilities is then

$$P[\bar{\mu}_{G,t+1} = y | \bar{\mu}_{G,t} = x] = \begin{cases} C_z^n \theta^z (1-\theta)^{n-z} & \text{if } \exists\, z \,\epsilon\, \{0,1,\ldots,n\} \text{ s.t. equation 12 holds} \\ 0 & \text{otherwise} \end{cases}$$

$$P[\bar{\mu}_{B,t+1} = x | \bar{\mu}_{B,t} = y] = \begin{cases} C_v^{n-v} \theta^v (1-\theta)^{v} & \text{if } \exists\, v \,\epsilon\, \{0,1,\ldots,n\} \text{ s.t. equation 13 holds.} \\ 0 & \text{otherwise} \end{cases}$$

When, $z = n - v$, these two probabilities will be equal. This proves the proposition for the case where $P[\bar{\mu}_{U,t+1} = y | \bar{\mu}_{U,t} = x] > 0$.

If $P[\bar{\mu}_{U,t+1} = y | \bar{\mu}_{U,t} = x] = 0$, then $P[\bar{\mu}_{B,t} = y | \bar{\mu}_{B,t+1} = x] = 0$. If not, then some $z \neq n - v$ would solve (12) and (13), which contradicts the previous result.

A.3 Proposition 1: In a constant learning economy with equally informative signals, an asset whose price is any function of beliefs $p(\cdot) : [0,1] \to \Re$, has unconditional probability of price changes $g(\cdot)$, such that $g(\Delta p) = g(-\Delta p)$, for all p.

Step 1: Suppose a state transition from ω_b to ω_g occurs at time t, and the beliefs at that date are $\mu_t = x$. Show that the following two conditional probabilities are equal for all s:

$$P[\mu_{t+s} = y, \mu_{t+s+1} = z | \omega_t = \omega_g, \mu_t = x] = P[\mu_{t-1-s} = y, \mu_{t-2-s} = z | \omega_{t-1} = \omega_b, \mu_t = x] \quad (14)$$

From lemma 2, we know that

$$P[\mu_{t+1} = y | \omega_t = \omega_g, \mu_t = x] = P[\mu_{t-1} = y | \omega_{t-1} = \omega_b, \mu_t = x]$$

holds for all n in the n-signal world. This means that it also holds for all s in any n-signal world without state changes ($\lambda = 0$, except at the one point where we assume that the state changes) in the following sense

$$P[\mu_{t+s} = y | \omega_t = \omega_g, \mu_t = x] = P[\mu_{t-1-s} = y | \omega_{t-1} = \omega_b, \mu_t = x]. \quad (15)$$

The reason this is true is that an s-period transition in an n-signal world identical to a 1-period transition in an (ns)-signal world when states don't change.

Now, consider allowing for state changes. Let Λ be the Markov transition matrix for the state $\Lambda = \begin{bmatrix} 1-\lambda & \lambda \\ \lambda & 1-\lambda \end{bmatrix}$. The probability $P[\omega_{t+s} = \omega_g] = \Lambda^s[1,1]$ and $P[\omega_{t-1-s} = \omega_b = \Lambda^s[2,2]$. Since Λ is symmetric, Λ^s is symmetric and $\Lambda^s[1,1] = \Lambda^s[2,2]$. Therefore, the probability of being in state ω_g at $t+s$ is the same as being in ω_b at $t-1-s$. Since the state probabilities are symmetric at each date, the probability of observing $\tilde{n}$ bad signals at date $t-1-s$ is still the same as the probability of observing $\tilde{n}$ good signals at date $t+s$. Therefore, the original proof of proposition 2 follows with relabeled probabilities to adjust for state changes, and equation 15 holds when $\lambda > 0$.

The fact that the probability of observing $\tilde{n}$ good signals at $t+s$ is the same as $\tilde{n}$ bad signals at $t-1-s$ also means that

$$P[\mu_{t+s+1} = z | \mu_{t+s} = y, \omega_t = \omega_g] = P[\mu_{t-2-s} = z | \mu_{t-1-s} = y, \omega_{t-1} = \omega_b].$$

Multiplying (15) and (A.3) yields (14).

Step 2: Let ψ^s be the s-period transition kernel for beliefs. Show that ψ satisfies the conditions for weak convergence, meaning that

$$\lim_{s\to\infty} \psi^s(y|x) = \psi^\star(y) \;\; \forall\, x, y \,\epsilon\, [0,1].$$

By Stokey and Lucas' (1989) Theorem 12.12, ψ has this weak convergence property if

1. ψ has the Feller Property
2. ψ is monotone
3. There $\exists\, c \,\epsilon\, [0,1]$ and an $s \geq 1$ s.t. $P[\mu_s \epsilon [c,1] | \mu_0 = 0] \geq \varepsilon$ and $P[\mu_s \epsilon [0,c] | \mu_0 = 1] \geq \varepsilon$.

Step 2.1: Show that ψ has the Feller Property.
Let $M(\mu, s)$ be the updated beliefs, given prior beliefs μ and s success signals out of n funded ventures. Define an operator T on any function f as

$$(Tf)(\mu) \equiv \int_0^1 f(\mu')\psi(\mu'|\mu)d\mu' \;\; \forall \mu$$

Then, ψ has the Feller Property if for any bounded, continuous function f, Tf is continuous in μ.

Since $\psi(\mu'|\mu)$ only takes a non-zero value where there is an $s \epsilon \{0, 1, \ldots, n\}$ such that $M(\mu, s) = \mu'$,

$$(Tf)(\mu) = \sum_{s=0}^{n} f(M(\mu, s)) * P(s)$$

By inspection of equation 4, we know that $M(\mu, s)$ is continuous for all $\mu \epsilon [0, 1]$ and all z. Since a composite function of two continuous functions is continuous, as is a linear combination of continuous functions, Tf is continuous in μ.

Step 2.2: Show that ψ is monotone.
ψ is monotone if for any bounded, continuous function f, Tf is also increasing. Realize that if $M(\mu, s)$ is increasing, then since $P(s) \geq 0$ for all z, then Tf will also be increasing.

To see that $M(\mu, s)$ is increasing in μ, examine equation 4, which gives the formula for the posterior belief, before adjusting for state changes. Realize that the posterior μ_t^P is increasing in μ_t iff $1/\mu_t^P$ is decreasing in μ_t.

$$\frac{1}{\mu_t^P} = 1 + \frac{C_{n-s}^n (1-\theta)^s \theta^{n-s} (1-\mu)}{C_s^n (1-\theta)^{n-s} \theta^s \mu}$$

Since all these terms are positive, the numerator is decreasing in μ, while the denominator is increasing in μ, which means that μ_t^P is increasing in μ_t. To get μ_{t+1}, use the formula

$$M(\mu, s) = \mu_{t+1} = \mu_t^P (1-\lambda) + (1-\mu_t^P)\lambda.$$

Because $\lambda < 1 - \lambda$, μ_{t+1} is increasing in μ_t^P and μ_t. Therefore, the monotonicity condition holds.

Step 2.3: Show that condition 3 holds.

Let $c = 1/2$ and $\varepsilon = (1-\theta)^{ns}$.
Start by showing that $\exists$ an s such that

$$P[\mu_s \epsilon [1/2, 1] | \mu_0 = 0] \geq (1-\theta)^{ns}$$

No matter what the unobserved state ω is, the probability of observing a good signal is always $\geq (1-\theta)$. So, the probability of observing ns good signals is always $\geq (1-\theta)^{ns}$. Now, it just remains to be shown that there is an s such that after ns consecutive good signals, $\mu_s \geq 1/2$.

If only good signals are observed, it is easy to verify that μ_s is a submartingale and that as $s \to \infty$, it converges to a limit $\mu^\star > 1/2$. By definition of a convergent sequence, for every η, there must be an s such that

$$|\mu^\star - \mu_S| < \eta \;\; \forall \; S \geq S$$

If $\eta = \mu^\star - 1/2$, then $\mu_S > 1/2 \; \forall \; S \geq s$. Therefore $\mu_s \geq 1/2$ and the condition holds.

Since all three conditions hold, the transition kernel for beliefs ψ converges to a unique unconditional distribution of beliefs as $s \to \infty$.

Step 3: Show that $P[\mu_{T+1} = z, \mu_T = y] = P[\mu_{S+1} = y, \mu_S = z]$ for all T, S.

The unobserved state process ω converges to a unique unconditional distribution $\lim_{s\to\infty} \Lambda^s = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$.

So, the joint Markov process $[\mu, \omega]$ converges to a unique unconditional distribution.

Taking the limit of both sides of equation 14 as $s \to \infty$ and applying weak convergence, produces the equality in unconditional probabilities.

Step 4: Show that $P[p_{T+1} = y, p_T = x] = P[p_{T+1} = x, p_T = y]$ for all T, and any function p.

First consider the case where p is invertible. Then, symmetric transitions in μ imply symmetric transitions in p.

$$P[p_{T+1} = y, p_T = x] = P[\mu_{T+1} = \rho^{-1}(y), \mu_T = \rho^{-1}(x)]$$
$$= P[\mu_{T+1} = p^{-1}(x), \mu_T = p^{-1}(y)] = P[p_{T+1} = x, p_T = y]$$

Now consider a function p that is not one-to-one. For any p_{T+1}, p_T, there will be sets of beliefs $\mathcal{M}_{T+1}, \mathcal{M}_T$ of beliefs that correspond to the prices. Then,

$$P[p_{T+1} = y, p_T = x] = \sum_{m_T \epsilon \mathcal{M}_T} \sum_{m_{T+1} \epsilon \mathcal{M}_{T+1}} P[\mu_{T+1} = m_{T+1}, \mu_T = m_T]$$

$$= \sum_{m_T \epsilon \mathcal{M}_T} \sum_{m_{T+1} \epsilon \mathcal{M}_{T+1}} P[\mu_{T+1} = m_T, \mu_T = m_{T+1}] = P[p_{T+1} = x, p_T = y]$$

Step 5: Show that the distribution function $g(p_{T+1} - p_T)$ is symmetric.

The conditional of interest rate changes is related to the distribution of interest rate levels in the following manner:

$$g(\Delta p) = \int_0^1 P[p_{T+1} = x + \Delta p, p_T = x]dx.$$

Using the equality from step 4,

$$= \int_0^1 P[p_{T+1} = x, p_T = x + \Delta p]dx = g(-\Delta p).$$

A.4 Corollary 1: In a constant learning economy with equally informative signals, if the unconditional probability of a change in the interest rate is $g(\cdot)$, then $g(\Delta\rho_t) = g(-\Delta\rho_t)$, for all $\Delta\rho_t$.

Equation 1 shows that the interest rate ρ is a function of beliefs. By proposition 1, it follows that lending rate changes are symmetric.

A.5 Lemma: $|E[\mu(j,t+1) - \mu(j,t)|\mu_{j,t}]|$ is increasing in $n(t)$, for $j \epsilon \{U, D\}$.

An n-signal one-period jump is the same as an n-period change in the 1-signal world without state changes. So, this amounts to proving that in the 1-signal world, $|E[\mu(j,t+s) - \mu(j,t)|\mu_{j,t}]|$ is increasing in s. This follows directly from μ_U and μ_D converging to the truth in expectation, and therefore being sub- and super-martingales.[21]

A.6 Proposition 2

Let $\hat{p} = \max\{\Delta p : g(\Delta p) > 0\}$, $\underline{p} = \min\{\Delta p : g(\Delta p) > 0\}$. Prove: $\hat{p} > -\underline{p}$ when $p(\mu)$ is decreasing.

If $\underline{p}$ occurs with positive probability, then there must be a set of $n(\mu_t)$ signals that generates $p(\mu_{t+1}) = p(\mu_t) + \underline{p}$. Since this is the largest possible downward movement in price, and price is decreasing in μ, the set of signals must be $n(\mu_t)$ venture successes.

From proposition 1, we know that if $\mu_t = x$ and a set of n_t observed successes generates $\mu_{t+1} = x + \delta$, then the opposite set of signals (n_t observed failures) and $\mu_t = x + \delta$ will generate $\mu_{t+1} = x$. Recall that n is monotone increasing in beliefs: $n(x+\delta) > n(x)$.[22] Since μ_{t+1} is strictly decreasing in the number of failures observed at time t, $n(x+\delta)$ failure signals and $\mu_t = x + \delta$ will produce $\mu_{t+1} = x - \eta < x$ with positive probability.

For the largest price decrease, $p(x+\delta) - p(x) < 0$, there is a set of signals observed with positive probability that generate a price increase $p(x-\eta) - p(x+\delta) > p(x) - p(x+\delta)$, of larger absolute value.

[21] A formal proof of martingale properties is available upon request.

[22] This statement requires a weak assumption on the distribution of the v_i's that they have a rich enough distribution so that some borrower changes her investment decision in the event of the largest possible change in lending rates.

When p is increasing in μ, the largest increase in p will be generates by n_t observed successes. μ_t and $n(\mu_t)$ will produce a $\mu_{t+1} > \mu_t$. There will then be $n(\mu_{t+1}) > n(\mu_t)$ possible failure signals observed with positive probability that will produce a larger price decline than the increase.

A.7 Corollary 2

Since the interest rate is a strictly decreasing function of beliefs, by Proposition 2, the largest increase occurring with positive probability will be larger in absolute value, than the largest decrease. Since the the model is stationary, the unconditional mean of price changes is zero. Therefore, $g(\Delta\rho) \neq g(-\Delta\rho)$ implies asymmetry of the distribution function g.

B Appendix: Multi-Period Projects

This section relaxes the assumption that all ventures are single period-lived projects. It shows that interest rates are still positively skewed, holding all other parameters fixed.

The reason this is an important robustness check is that the relative size of booms and crashes in the data can be sensitive to the time interval between observations. Since the data the model is being compared to is monthly data, ventures in the model would have to succeed or fail in one month. Since this is obviously unrealistic, showing that frequent observations on long-lived projects produces similar results is important.

Suppose that all ventures undertaken at time t either succeed or fail at time $t + s$. The true but unknown probability of success of the venture depends on the state variable ω_{t+s}. Based on information known at the beginning of time t, the expected probability of success s periods later is:

$$\begin{bmatrix} \mu_{t,t+s} \\ 1-\mu_{t,t+s} \end{bmatrix} = \begin{bmatrix} 1-\lambda & \lambda \\ \lambda & 1-\lambda \end{bmatrix}^s \cdot \begin{bmatrix} \mu_t & 0 \\ 0 & 1-\mu_t \end{bmatrix}$$

This is a simple linear transformation that moves all the belief measures closer to 1/2. Since the upper and lower bounds for $\mu_{t,t+s}$ are closer to 1/2, the relevant interval for investment payoffs is a strict subset of $(\underline{v}, \bar{v})$. So, holding the v_i's fixed, lengthening the time to project completion effectively reduces the number of borrowers, N, and results in more borrowers outside the relevant interval. Although the effect of v_i's outside the relevant interval is to reduce skewness, the effect of reducing N is ambiguous; thus the overall effect is unknown. However, as the simulation results show, the positive skewness in interest rates is still present.

The skewness of interest rate spread changes in the multi-period setting with these parameters is slightly less than in the original model but is still very significant. Another difference is the lower variance of the interest rate spreads in the time to build model. This difference appears because adjusting the beliefs to predict the state 24 periods in the future dampens their movement and makes the lending rate less volatile.

The same strategy would allow pricing of assets with streams of payoffs, where each period's payoff depends on that period's state. As the number of periods in the future became large, the effect of the next period's payoff on skewness would converge to zero, and the overall skewness measure should converge. In this way, the result of this model that price changes are asymmetric could be extended to cover assets with infinite dividend streams.

Model	Average Spread	Variance of Spread	Skewness of Change
Benchmark	6.22%	0.06	0.72 (1.00)
Time To Build	6.22%	0.05	0.65 (1.00)

Table 7: Simulation results with benchmark parameters. Time to build is 24 periods (two years). 50,000 periods simulated.

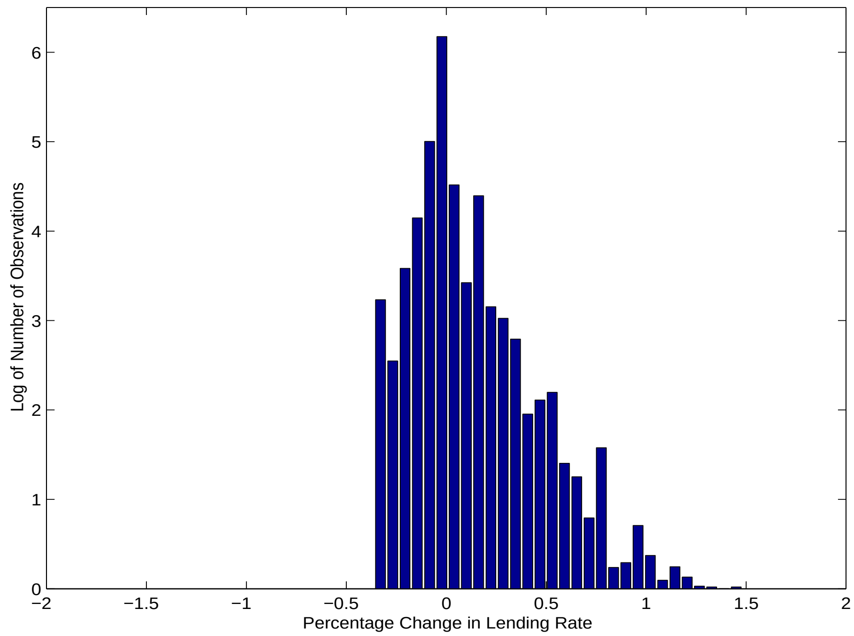

Figure 3: **Benchmark Model** - Histogram of Changes in Interest Rates
Skewness = 2.35 Kurtosis = 9.66

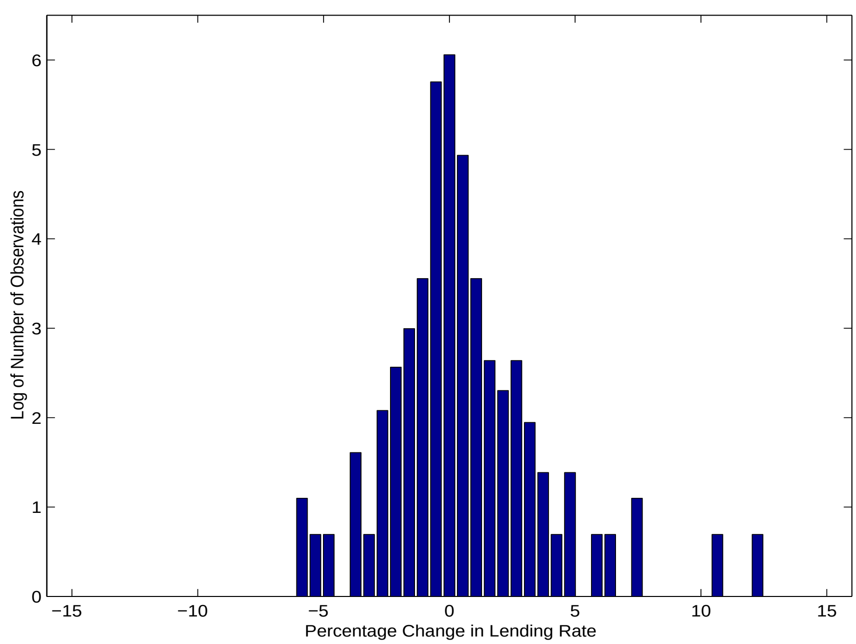

Figure 4: **Model with 12 States** - Histogram of Changes in Interest Rates.
Skewness = 2.35 Kurtosis = 29.8

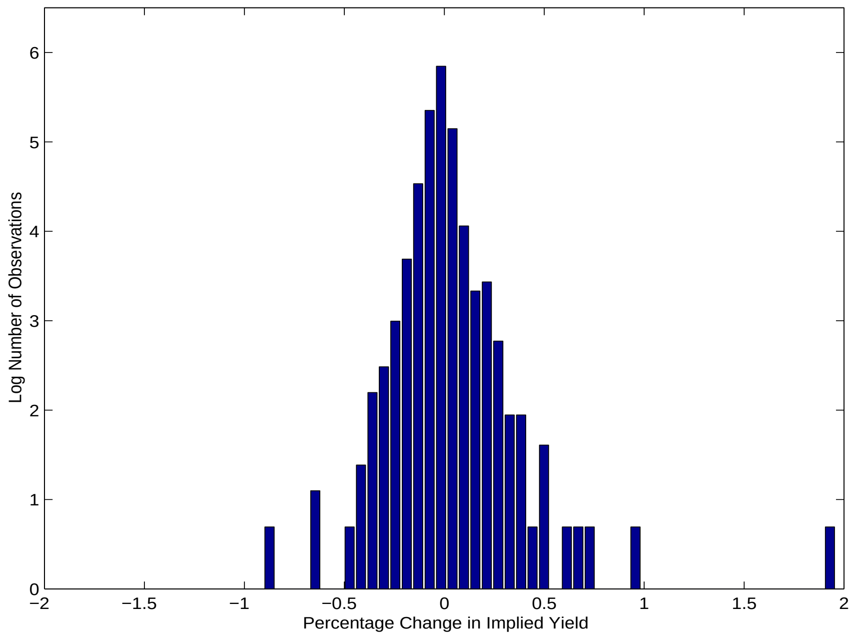

Figure 5: **Monthly Data** - Histogram of Changes in Implied Bond Yields. 1053 observations.

Skewness = 2.90 Kurtosis = 41.4