

BORROWING COSTS AND THE DEMAND FOR EQUITY OVER THE LIFE CYCLE *

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Borrowing costs and the demand for equity over the life cycle

by Steven J. Davis, Felix Kubler and Paul Willen

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Abstract

We analyze consumption and portfolio behavior in a life-cycle model with borrowing costs that exceed the risk-free investment return. We calibrate the model to U.S. data on life-cycle income profiles, the cost of borrowing and returns on risk-free and equity securities.

Even a modest wedge between borrowing costs and the risk-free return dramatically shrinks the demand for equity. When the cost of borrowing equals or exceeds the expected return on equity – the relevant case according to the data – households hold no equity during much of the life cycle. The correlation between consumption growth and equity returns is low at *all* ages. Risk aversion estimates based on the standard excess return formulation of the consumption Euler Equation are greatly upward biased. The bias diminishes, but remains large, for “samples” of households with positive equity holdings. The demand for equity is non-monotonic in borrowing costs, risk aversion and the magnitude of undiversifiable income risk.

1 Introduction

We analyze consumption and portfolio behavior over the life cycle using a model in which borrowing costs exceed the lending rate – the return on investment in the risk-free asset. We calibrate the model to U.S. data on life-cycle income profiles, the cost of borrowing and returns on risk-free and equity securities. Consumers in the model exhibit standard time-separable preferences with modest risk aversion and mild impatience.

We show that the wedge between the borrowing rate and the lending rate affects analysis of portfolio choice in at least four ways. First, even a modest wedge dramatically shrinks the demand for equity. Second, when the borrowing rate equals or exceeds the expected return on equity – the relevant case according to the data – households hold no equity during much of the life cycle. Third, the correlation between consumption growth and equity returns is low at *all* ages, although it rises with age. Fourth, risk aversion estimates based on the standard excess return formulation of the consumption Euler Equation are greatly upward biased. The bias diminishes, but remains large, for “samples” of households with positive equity holdings. In all four respects, the introduction of a wedge between borrowing costs and the risk-free return greatly improves the fit between theory and empirical evidence.

Table 1 provides evidence for the existence of a wedge between borrowing costs and the risk-free return. The bottom two rows show that household borrowing costs on unsecured loans exceed the risk-free return by about six to nine percentage points on an annual basis, after adjusting for tax considerations and charge-offs for uncollected loan obligations. Since 1987, roughly two percentage points of this wedge arise from the asymmetric income tax treatment of household interest receipts and payments. However, the bulk of the wedge between borrowing costs and the risk-free investment return arises from transactions costs in the loan market. Despite the evident size of these transaction costs, they have been largely ignored in theoretical analyses of life-cycle consumption and portfolio behavior. They have also been ignored in most empirical studies of asset-pricing behavior.

None of our results rely on preferences that exhibit strong risk aversion, a high degree of impatience, habit formation, other types of nonseparability or problems of self control. Nor do they rely on costs of participating in equity markets, informational barriers, serially correlated asset returns, enforcement problems in loan markets, or hard borrowing constraints.

Our comparative statics show that the shape of the expected lifetime income profile and the wedge between the borrowing rate and the lending rate affect life-cycle portfolio choice significantly. Undiversifiable labor income risk also plays a role as an

important source of cross-sectional heterogeneity in consumption and asset holdings conditional on age. However, the model more successfully matches some aspects of life-cycle behavior when the idiosyncratic component of risky labor income is smaller than suggested by the data.

Our analysis also uncovers other interesting results. For example, equity holdings and market participation rates are non-monotonic in the cost of borrowing, both reaching a minimum where the borrowing cost equals the expected return on equity. As this result suggests, the properties of our model are not “in between” those of the canonical model with equal borrowing and lending rates and alternative models with hard borrowing constraints. As another example, equity holdings and participation rates are not monotonic in the degree of risk aversion or the magnitude of undiversifiable income risks when borrowing costs exceed the risk-free return. These results, and the other results mentioned above, are based on numerical analysis of our life-cycle model.

On the asset-pricing front, our analysis helps resolve two closely related aspects of the equity premium puzzle: Why do people hold so little equity when faced with a substantial equity premium? And, why is the covariance between consumption growth and asset returns so low? Answers to both question follow from the low leverage premia implied by Table 1

The paper proceeds as follows. The remainder of the introduction discusses related research and reviews some basic facts about consumption and portfolio behavior over the life cycle. Section 2 describes the model and briefly explains our solution method. Section 3 calibrates the model. Section 4 discusses life-cycle portfolio choice and consumption in our model. Section 5 compares the predictions of our model with empirical evidence. Section 6 concludes by briefly considering some additional implications of our model and related empirical evidence.

1.1 Relationship to the Literature

In terms of its specification, our model departs quite modestly from the seminal work on life-cycle portfolio behavior by Merton (1969) and Samuelson (1969). Indeed, our model is identical to Samuelson’s discrete-time setup except for the wedge between borrowing and lending rates and the presence of undiversifiable income risks. These two features necessitate a computational approach to the analysis, which we pursue using methods developed by Judd, Kubler and Schmedders (2002).

Our analysis draws on insights from previous research in both macroeconomics and finance.

Brennan (1971) showed that a wedge between borrowing costs and the risk-free

return is easily handled in the standard mean-variance analysis of portfolio choice in a one-period setting. The wedge implies that households cannot attain points above and to the right of the tangency portfolio along the standard capital market line. Higher borrowing costs reduce the demand for equity in the one-period setting, given standard mean-variance preferences. We extend this analysis to a many-period setting that incorporates key life-cycle elements and endogenizes wealth accumulation. In our model, a higher cost of borrowing raises the demand for equity in reasonable circumstances. Heaton and Lucas (1997) showed that a borrowing rate that exceeds the lending rate reduces equity holdings for an infinitely lived agent with a flat life-cycle income profile. In our model, we show that the shape of the life-cycle income profile plays a crucial role in not only the shape of life-cycle equity demand but also in the level of equity demand and the decision to participate in equity markets.

Many researchers have explored the effects of explicit borrowing constraints on portfolio choice in a life-cycle model. Two prominent examples are Constantinides, Donaldson and Mehra (2002) and Campbell, Cocco, Gomes and Maenhout (2001). Constantinides, Donaldson and Mehra consider a three period model with no borrowing. Campbell et al. consider a calibrated many period life-cycle model. We show that allowing households to borrow at a rate above the lending rate leads to much more realistic levels of asset holding and market participation than a blanket prohibition on borrowing.

In Section 5, we show that our model generates a standard deviation of marginal utility growth that is an order of magnitude lower than the Sharpe ratio. Our work appears inconsistent with the work of Hansen and Jagannathan (1991), who argued that the Sharpe ratio represents a *lower* bound on the standard deviation of marginal utility growth. Our results should not come as a complete surprise, however. Other researchers (see He and Modest (1995) and Luttmer (1996)) have shown that the existence of trading frictions can *potentially* reduce the implied lower bound on the volatility of marginal utility growth. Thus, our results show that one particular friction does indeed reconcile a high Sharpe ratio with low volatility of marginal utility growth.

1.2 Some facts about life-cycle consumption and portfolio behavior

Our discussion of the quantitative implications of our model is motivated by three stylized facts from the literature. First, many households hold no equity. Second, those households that do hold equity hold very little. Third, the covariance of con-

sumption growth and equity returns is low.

Many researchers have noted low levels of equity market participation in the population – in fact it is often referred to as the “participation puzzle.” For example, according to Vissing-Jorgenson (2002), only 44 percent of households held positive amounts of stock in 1994, and the 1994 figure represented a dramatic increase over 1984, when only 28 percent of households held positive amounts of stock. Researchers have found participation generally increases with age (see Poterba and Samwick (2001)), education and income (see Mankiw and Zeldes (1991)). Further, self-employed workers are also more likely to hold stock (see Heaton and Lucas (2000)). In many cases, equity market participation reflects the fact that the household has no financial wealth at all (see Lusardi, et al. (2001)).

Second, equity holdings are relatively small. Vissing-Jorgensen (2002) reports that the median level of equity holdings for stockholding households is around 21 thousand dollars and that the mean level of stockholding is around 95 thousand dollars. Other researchers (see Poterba and Samwick (2001)) have found that the level of stockholding generally rises with education, income and over the life cycle.

Third, researchers going back as far as Grossman and Shiller (1982) have noted the low correlation of consumption growth and equity returns. In particular, with standard preferences the estimated covariance violates standard consumption based asset pricing equations. Several researchers have noted, however, that the covariance is higher for households who hold equity.¹

2 Model specifications

We consider a partial equilibrium model of household portfolio choice. Our household has a time-separable, isoelastic utility function. The household life cycle consists of two phases, work and retirement, which are distinguished only by the character of labor income. While a household works, we use a standard stochastic process for labor income which separates permanent and transitory shocks (like Gourinchas and Parker (2002), for example). First differences in log labor income (\tilde{y}_t) consist of a deterministic component (d_t), a permanent shock ($\tilde{\eta}_t$) and a transitory shock ($\tilde{\varepsilon}_t$),

$$\Delta\tilde{y}_t = d_t + \tilde{\eta}_t + \tilde{\varepsilon}_t.$$

During retirement, a household receives a fraction of their income in the last year of work. Ideally, we would like to provide retired households some fraction of, say, the

¹See, *inter alia* Mankiw and Zeldes (1991), Constantinides, Brav and Geczy (2002) and Attanasio, Banks and Tanner (2002).

highest n years of work income – consistent with most defined benefit pension plans and with social security. However, such a structure is computationally burdensome. Thus, we approximate the ideal approach in the following way. We first calculate the ratio of expected income in the highest n years to expected income in the final year of work. We then multiply the final year of realized work income by this ratio to get the retirement basis. To get retirement income, we multiply the retirement basis by a number between zero and one called the *replacement rate*. To see why this procedure is useful, look at Figure 1. Note that labor income in the final year of work is much lower relative to lifetime income for college educated workers than for more-than-college educated households. If we simply gave all households the same fraction of their last year of work income, college-educated workers would receive relatively low retirement income. Our adjustment approach takes into account the fact that college educated workers have relatively high lifetime income and gives them relatively high retirement income.

Households trade three financial assets. Households can buy equity with stochastic net return \tilde{r}_E , borrow at a risk-free net borrowing rate r_B and can save at a net risk-free rate r_L . Households cannot take short positions in equity and interest rates for borrowing and lending will generally be different.

We solve numerically for optimal portfolio choices using a backward induction algorithm. Details are given in the appendix.

3 Calibration

Table 2 summarizes our calibration of the model. We draw the reader’s attention to three things about our calibration.

First, for our life-cycle income process, we use parameters estimated by Gourinchas and Parker (2002) using the Consumer Expenditure Survey and the Panel Study of Income Dynamics. The Gourinchas and Parker income measure is “after-tax family income less social security tax payments, pension contributions, after-tax asset and interest income.” Gourinchas and Parker subtract “education, medical care and mortgage interest payments” from their measure of income, because “these categories of expenditure do not provide current utility but rather are either illiquid investments or negative income shocks.”² Relevant quantities are measured in 1987 dollars.

A fifth order polynomial characterizes the deterministic component of labor income. Gourinchas and Parker restrict their sample to male-headed households and attribute the characteristics of the male head to the entire household. In other words,

²Without these deductions, household income would be about 27% higher.

a household headed by a 35-year old college educated man is considered a college-educated household regardless of the educational attainment of other household members. We consider parameters estimated on the total sample (which we refer to as “total”) and on households defined by five different levels of educational attainment. Figure 1 shows the labor income profiles. The standard deviations of the transitory and permanent shocks are shown in Table 3.

Second, our baseline specification for asset returns is consistent with estimates reported by Campbell (1999).

We set the riskfree-rate equal to 2%, the return on equity equal to 8% and the standard deviation of returns equal to 15%. We set the correlation of equity returns and labor income shocks to zero.³ Finally, we set the baseline borrowing rate equal to 8%, which is consistent with the data reported in Table 1.

Third, we discretize the state-space using the Tauchen and Hussey (1991) algorithm. Our model has three independent sources of randomness: the permanent labor income shock, the transitory income shock and an asset return shock. We assume two discrete points for the permanent shock, two discrete points for the transitory shock and three discrete points for the asset return shock. Thus randomness in our model follows an 12-state Markov chain. The discretization procedure does not generate nor we do assume that households receive zero income in any state of nature. Thus, our specification differs from that of Gourinchas and Parker (2002), for example. And the difference is not innocuous: by assuming that agents have non-zero probability of zero income, Gourinchas and Parker make borrowing impossible.⁴ In our setup, households can and do borrow.

4 The demand for equity over the life cycle

In this section we explore portfolio choice in our model. First, we introduce some useful terminology and give basic intuition about what determines the demand for equity. Then we explore how four parameters of the household decision problem affect the demand for equity over the life cycle: (1) the borrowing rate, (2) risk aversion,

³Davis and Willen (2000) present evidence of non-zero correlations between labor income shocks and equity returns. They also consider the implications of a non-zero correlation for life-cycle portfolio choice in a model with equal borrowing and lending rates.

⁴If zero income is possible in the last period of life, households that borrow in the penultimate period of life run the risk of negative consumption in the last period of life. With isoelastic utility and relative risk aversion of at least 1, the possibility of negative consumption, no matter how remote, leads to infinitely negative utility. Thus no households borrow in the penultimate period. One can make a similar argument to show that no one will borrow in any other period either.

(3) undiversifiable labor income risk and (4) the life-cycle profile of household labor income. Lastly, we focus on the issue of non-participation in equity markets.

It will be helpful to define some terms. *Borrowing capacity* is the present value of future labor income discounted at the borrowing rate. The *equity premium* is the difference between the expected return on equity and the risk-free investment return. The *leverage premium* is the difference between the expected equity return and the borrowing rate. When the cost of borrowing exceeds the risk-free investment return, the equity premium exceeds the leverage premium. Hence, the net return on equity depends on the source of funds invested, as depicted in the following table.

Source of funds	Opportunity cost	Net return
Liquid wealth	Lending rate	Equity premium
Borrowing capacity	Borrowing rate	Leverage premium

Given a realistic equity premium (say 6%), a household invests all or much of its liquid wealth in equity. When the leverage premium is positive, a household may also borrow to finance equity holdings, but it holds less equity than an otherwise similar household with greater liquid wealth. When the leverage premium is negative, households do not draw on borrowing capacity to invest in equity. That is, with a zero or negative leverage premium, equity demand varies one-for-one with liquid wealth, which in turn depends principally on the shape of the lifetime income profile and the strength of consumption-smoothing motives. For example, a household with a sharply upward-sloping income profile saves little, and thus acquires little equity, early in life. The upshot is that the evolution of liquid wealth over the life cycle is the key determinant of life-cycle demand for equity.

4.1 Effect of the borrowing rate

How does the borrowing rate affect the demand for equity over the life cycle? First, the borrowing rate affects borrowing capacity – a higher interest rate reduces the present value of labor income when discounted at the borrowing rate. Second, the borrowing rate affects the leverage premium – all else equal, a higher borrowing rate means a lower leverage premium. And third, the borrowing rate affects the evolution of liquid wealth over the life cycle in several ways. A low borrowing rate reduces liquid wealth by encouraging greater borrowing for consumption smoothing purposes and by substituting for precautionary wealth holdings that households would otherwise acquire to smooth transitory labor income shocks. But a low borrowing rate can also increase liquid wealth: if the leverage premium is positive, borrowing to invest in equity enables the household to increase liquid wealth over time.

As these remarks suggest, there is a non-monotonic relationship between the cost of borrowing and the demand for equity. For example, consider our baseline specification with no labor income risk for a household with post-graduate education. Figure 2 shows its life-cycle equity holdings (averaged over many draws) for alternative borrowing rates. A higher borrowing rate raises or lowers equity demand, depending on whether the cost of borrowing is above or below the expected return on equity; i.e., whether the leverage premium is negative or positive.

When the borrowing rate equals the lending rate (2%), our baseline household invests large amounts in equity throughout the life cycle. Thus, the standard model with equal borrowing and lending rates implies equity holds that greatly exceed what we see in the data.

In comparison to the standard model, a borrowing rate of 5% implies much lower equity holdings throughout the life cycle. Why? An increase of the borrowing rate from 2% to 5% implies a reduction in the leverage premium from 6% to 3% and a consequent fall in investment of borrowing capacity in equity – visible at age 21 when liquid wealth is zero. This is not surprising for a household just starting out: since they have no liquid wealth, the leverage premium determines equity investment. Less obviously, the disparity between equity demand of households with 5% borrowing rates and 2% borrowing rates persists into retirement when liquid wealth is positive for all households. This persistence reflects two factors. First, households with a non-zero replacement rate still have borrowing capacity in retirement. Figure 3 shows that households with a positive leverage premium borrow positive amounts until the year they die. So even in retirement, the size of the leverage premium affects equity demand. Second, a higher leverage premium earlier in life leads, in expectation, to higher wealth accumulation at retirement. Figure 4 shows life-cycle liquid wealth for households with different borrowing rates. Note that the household with a borrowing rate of 2% has much higher liquid wealth at retirement than the household with a borrowing rate of 5%.

If the borrowing rate equals or exceeds the return on equity, then portfolio choice changes dramatically. With a borrowing rate equal to 8%, our baseline household with post-graduate education does not invest in equity until almost age 50 (Figure 2). Why? With a borrowing rate of 8%, the leverage premium equals zero and our household never invests its borrowing capacity in equity, meaning that equity demand depends entirely on liquid wealth. However, the household still borrows when young for consumption-smoothing purposes (Figure 3), implying that liquid wealth is negative. Although the household stops borrowing at age 48, asset accumulation and equity demand are much lower than that of investors who face lower borrowing

rates.

At borrowing rate above 8%, households hold more equity than for a borrowing rate equal to 8% (Figure 2). Higher borrowing rates lead to less borrowing early in life (see Figure 3) and more liquid wealth accumulation throughout the life cycle (Figure 4).

To conclude our discussion of the effect of borrowing rates on investment, we emphasize three points. First, a borrowing rate equal to the return on equity minimizes the demand for equity over the life cycle. This result is particularly notable since our estimates of the borrowing rate cluster around a value roughly equal to the return on equity. Second, when the borrowing rate is in the neighborhood of the return on equity, as suggested by the data in Table 1, the demand for equity is an order of magnitude lower than in the standard model with equal borrowing and lending rates. Third, even a modest wedge between borrowing and lending rates sharply reduces the demand for equity.

4.2 Effect of undiversifiable labor income risk

How does undiversifiable labor income risk affect the demand for equity over the life cycle? In two ways. First, higher undiversifiable labor income risk makes households with proper preferences effectively more risk averse – thus reducing demand for equity for a given level of liquid wealth and a given borrowing capacity. But, higher undiversifiable labor income risk also leads to more precautionary saving and thus higher liquid wealth accumulation. Figure 5 compares the asset holdings of households with our baseline levels of undiversifiable risk with households that face the same labor income profile but no undiversifiable risk. Figure 5 shows that the effects of undiversifiable risk are quite significant, especially early in life. If the borrowing rate equals the lending rate, then higher undiversifiable risk leads to much lower equity holdings early in life, reflecting the properness of preferences. But if the borrowing rate equals the return on equity, then higher undiversifiable risk leads to much higher equity holdings throughout the life cycle, reflecting the impact of the precautionary saving motive on wealth accumulation.

4.3 Effect of relative risk aversion

How does relative risk aversion affect the demand for equity over the life cycle in our model? In two distinct ways. On one hand, higher relative risk aversion reduces an household's appetite for risk. Thus for a given level of liquid wealth and a given borrowing capacity, higher relative risk aversion leads to lower equity holdings. On

the other hand, relative risk aversion has a powerful effect on the evolution of life-cycle liquid wealth. High risk aversion leads to higher caution, higher saving and higher liquid wealth. But higher risk aversion also implies lower elasticity of substitution for our preference specification, which, along with our baseline hump-shaped income profiles, leads to more borrowing and less wealth accumulation.

As a result, higher risk aversion can raise or lower the demand for equity for reasonable parameter choices. If the borrowing rate equals the lending rate, the level of equity holding behaves exactly as one would expect. Higher risk aversion leads to lower equity holding throughout the life cycle. If the borrowing rate equals the return on equity, the story is more complicated, as shown in Figure 6:

- A household with $RRA=0.5$ holds more equity than an household with $RRA=1$ throughout the life cycle.
- But a household with $RRA=4$ also holds more equity than an households with $RRA=1$ throughout the life cycle.
- Early in life a household with $RRA=8$ holds more equity than anyone else. At the end of its life, a household with $RRA=8$ holds less equity than anyone else.

What drives these results? Since the leverage premium is zero, the main issue is how liquid assets evolve over the life cycle. Why does a household with $RRA=0.5$ always invest more than a household with $RRA=1$? A household with $RRA=0.5$ high elasticity of substitution, which make it willing to reduce consumption early in life and thus invest in equity. And lower borrowing and more investment lead to higher liquid wealth accumulation throughout the life cycle. But a household with $RRA=4$ also holds liquid assets early in life, because it has a strong precautionary saving. Finally, what about the household with $RRA=8$? Early in life, it invests more than any of the other households, reflecting its strong precautionary saving motive – and consequent early asset accumulation. But all along, unlike the other households, it finds the riskless asset relatively attractive and as it ages moves more of its portfolio to the riskless asset and its equity holdings drop below those of the other households.

To sum up the effects of risk aversion on portfolio choice in our model, we point out that in the most reasonable region of the parameter space, one's natural intuition that higher risk aversion leads to lower equity holding need not be true – not only when investors are young but all through the life cycle.

4.4 Effect of life-cycle profile of labor income

When the borrowing rate equals the return on equity, demand for equity is highly sensitive to the shape of the life-cycle profile of equity. The intuition is quite straightforward: households that expect high income in the future borrow for consumption purposes today and thus have no liquid wealth to invest in equity. In particular, our simulations show that equity demand is highly sensitive to two aspects of the life-cycle profile on income: the shape of the working-age earnings profile and the replacement rate in retirement.

Figure 7 illustrates the effect of the shape of a household’s working-age earnings profile on equity demand. The line marked “Estimated” corresponds to a household with post-graduate education under our baseline specification with no labor income risk, as estimated by Gourinchas and Parker (2002). The line marked “Flat” considers the same household, with a life-cycle income profile that provides the same unconditional expected income but is flat. Note two things. First, the household with a flat labor income profile invests in equity throughout the life cycle whereas the household with the estimated labor income profile does not invest in equity until almost age fifty. Second, the household with a flat labor income profile accumulates considerable amounts of wealth through the equity premium. At age 48, the “estimated” household still has no equity and it never catches up with the “flat” household,

Figure 8 shows the effect on demand for equity over the life cycle for households with different retirement replacement rates. Simply put, the more income a household expects to receive in retirement, the less it saves and consequently, the less it invests in equity.

The amount and nature of retirement income also affect the household’s optimal portfolio mix. When replacement rates are low (or when pensions take the form of defined contribution plans invested heavily in equities), households are more likely to invest in the risk-free asset in addition to equity.

4.5 Non-participation in equity markets

Our model generates non-participation in equity markets without resorting to participation costs. When the leverage premium is zero or negative, a household invests in equity, if and only if its net liquid wealth position is positive. Since households can borrow in our model, liquid wealth is often negative or zero – and households do not invest in equity. Moreover, this pattern of borrowing and non-participation in equity markets is fully consistent with rational optimizing.

As with the level of equity holdings, the effects of various parameters on equity

market participation rates can be surprising. Table 4 shows participation rates for different age groups for our baseline households. When the cost of borrowing equals the return on equity, and labor income is not too risky, equity market participation rates are low or zero over much of the life cycle. A borrowing rate higher than the return on equity *increases* participation for precisely the same reason that higher borrowing rates lead to higher asset holdings. The effect of risk aversion on participation is ambiguous. For very low levels of risk aversion (< 1), participation rates are high, for intermediate levels ($1 \leq RRA \leq 4$) participation is low and for high levels of risk aversion ($RRA > 4$) participation is high again. The explanation for non-monotonicities in participation exactly parallels the explanation for non-monotonicities in asset holdings.

We note that undiversifiable income risk has a significant effect on participation. For example, at an 8% borrowing rate, participation rates are much higher for investors who face undiversifiable income risks than for those who don't.

5 Does our model match the data?

In Section 1, we took note of three well-established facts. First, many households hold no equity. Second, even among households that participate in equity markets, average holdings are modest (and much smaller than predicted by a standard model with equal borrowing and lending rates). Third, the covariance of consumption growth and equity returns is low.

5.1 Fact 1: Non-Participation

How does our model fare at replicating the facts about participation? In this regard, we focus on three predictions of the model: First, a significant share of the population holds no equity. Second, participation rates rise with age. Third, given our baseline income specifications, participation rates tend to decline with education (conditional on age). This last implication is contrary to the data, but it is unclear whether this constitutes a significant failure of the model. Small differences among education groups in borrowing costs, risk aversion and time discounting, or small changes in the baseline values for income variance, can alter the predicted relationship between participation rates and education.

Our model predicts that a significant fraction of the population holds no equity. Table 4 shows participation rates by age and overall for our the “total sample” household and households with post-graduate education. For the specifications without la-

bor income risk, we observe that participation rates are low or zero until middle age: “total” households do not participate until age 36, and households with post-graduate education wait until age 48. For our specifications with labor income risk, participation rates are higher. The median “total” household participates from the outset, but full participation does not occur until middle age. The median household with post-graduate education does not participate until age 35-39, and full participation does not occur until age 45-49.

The last column of Table 5 calculates average levels of participation weighted by age and education for different specifications of the model. Overall participation rates range from 27 percent for a model with high risk aversion and no labor income risk to 100 percent for a model with a borrowing rate below the return on equity and also for a specification with flat income profiles.

5.2 Fact 2: Life-cycle demand for equity

How does our model fare regarding the facts about life-cycle equity holdings? When the leverage premium is near zero, our model predicts equity demand that is roughly in line with the data and an order of magnitude lower than in the canonical model with equal borrowing and lending rates.

Table 6 shows equity demand over the life cycle for several different specifications of the model. Note, first, that equity demand is generally an order of magnitude lower when the borrowing rate equals 8 percent than when it equals 2 percent. This is true early in life, when many households don’t participate at all; and it remains true in retirement, when all households participate. Second, equity demand for the “total” household exceeds equity demand for the household with post-graduate education throughout the life cycle, in spite of the fact that the more educated household has higher income. This reflects the fact that the more educated household has a much steeper income profile early in life (see Figure 1).

The columns marked “Demand” in Table 5 compare the average demand for equity in our model (“DKW”) to the average demand for equity in the canonical model. We construct the averages by weighting each age-education group by its population share. For all specifications, the demand for equity in the standard model is at least 6 times greater than in our model. For our baseline specification with no labor income risk, our model generates overall demand for equity less than 1/20th of the canonical model. If we consider a relative risk aversion of 4, the overall demand for equity in our model is only 1/50th as large as in the canonical model.

5.3 Fact 3: The covariance of consumption and equity returns

How does our model fare in connection with facts about the covariance of consumption growth and equity returns? Much better, in three respects, than models that assume equal borrowing and lending rates. First, the covariance of minus marginal utility growth and equity returns almost always falls well short of the excess return on equity in our model, in line with the evidence. Second, if one applies standard formulas to calculate risk aversion using data generated by our model, one overestimates risk aversion, often by a factor of ten or more. This implication rationalizes the implausibly large estimates of risk aversion in consumption-based asset-pricing studies. Third, our model rationalizes the rejection of standard consumption-based asset-pricing relationships in samples that are restricted to households with positive equity holdings.

The covariance of minus marginal utility growth and equity returns almost always falls short of the excess return on equity in our model. Why is this a meaningful piece of information? According to the canonical model with equal borrowing and lending rates,

$$E\left(\tilde{R}_E\right) - R_L = \text{cov}\left(\tilde{R}_E, -\frac{\Delta\tilde{M}U}{E(\Delta MU)}\right) = \text{cov}_{MU} \quad (1)$$

That is, the excess return on equity equals minus the covariance between equity returns and the growth rate of marginal utility. Equation 1 is central to consumption-based asset pricing studies, and it fares poorly in confrontations with the data for standard preference specifications.

Equation (1) seldom holds in our model, as shown in Table 7. Column 3 in the table shows that cov_{MU} equals zero for young households and reaches a maximum of 1.9 shortly before retirement. In contrast, the canonical model implies that cov_{MU} equals 6.0 at all ages and for all education groups.

We can interpret cov_{MU} as the equity premium implied by consumption behavior. Suppose we know the utility function and observe the consumption process generated by our model. In addition, we maintain the assumption that households face equal borrowing and lending rates. Then, by fitting equation (1), we would conclude for our baseline case with no labor income risk that the equity premium equals 0.8%. In other words, we would mistakenly attribute the low covariance of consumption growth and asset returns to a low equity premium, not differential borrowing and lending rates. Table 5 shows that miscalculation of the equity premium from consumption data is particularly severe for our baseline case (top two rows). If we lower the borrowing rate below 8 percent (our baseline borrowing rate), the model implies a higher equity

premium. If we raise the borrowing rate above 8 percent, the model also implies a higher equity premium.

We can follow Hansen and Jagannathan (1991) and interpret the same information in yet another way. Equation 1 implies that the standard deviation of marginal utility growth must exceed the Sharpe ratio. We can use our model to calculate the actual standard deviation of marginal utility growth, reported in the column labeled “HJ” in Table 5. Note that only when the borrowing rate equals the lending rate does the HJ approach the Sharpe ratio (38%). When borrowing costs exceed the risk-free return, the actual standard deviation of marginal utility growth is much smaller than the Sharpe ratio.

Second, if we apply standard formulas to calculate risk aversion using data generated by our model, we greatly overestimate true risk aversion. Recall from Hansen and Singleton (1982) that, if asset returns and consumption are jointly log-normal and individual utility is of the time-separable isoelastic form, then equation (1) implies

$$\gamma = \frac{E(\tilde{R}) - R_f + .5 \text{var}(\tilde{R})}{\text{cov}(\tilde{R}, \tilde{c})} \quad (2)$$

where γ is the coefficient of relative risk aversion. The columns labelled “Implicit RRA” in Tables 5, 7, 8 and 9 calculate risk aversion using equation (2). Inspecting these tables reveals that equation (2) always overestimates risk aversion when borrowing rates exceed lending rates. For the baseline case with labor income risk, the bias is large: equation (2) implies that households with post-graduate education have a relative risk aversion coefficient of 14.6, more than seven times as large as the true coefficient; “total” households have an estimated coefficient of 12.5, more than six times as large as the true coefficient. If we assume that a household faces no labor income risk, equation (2) yields even more misleading estimates of risk aversion: 22.9 for households with post-graduate education and 14.1 for “total” households.

Third, focusing on market participants rather than the population as a whole only marginally narrows the gap between the implications of our model and the standard model. Researchers, starting with Mankiw and Zeldes (1991), have argued that for households who invest nothing in equity – non-participants – equations (1) and (2) are not a reasonable description of reality. But, they argue, perhaps equations (1) and (2) are a reasonable description of reality for participants – households who invest non-zero amounts in equity.

Table 9 reports the covariance of marginal utility growth with asset returns and the implied relative risk aversion for “samples” of equity market participants. Both tables show that focusing on participants reduces the gap between our model and the standard model. For example, using the baseline model for households with

post-graduate education and no labor income risk, the equity premium implied by cov_{MU} is 0.5% when fit to all households and 1.2 when fit to the subsample who hold equity – as compared to a true equity premium of 6%. Thus, restricting the sample to participants narrows the gap between the true equity premium and the equity premium implied by consumption behavior, but not by much.

The story with implied relative risk aversion coefficients is similar. For the baseline case with risky labor income, focusing on participants reduces the implied relative risk aversion from roughly twelve to seven for households with post-graduate education. For the case without risky labor income, focusing on participants reduces the implied relative risk aversion for more than college educated men from more than thirty five to “only” sixteen.

5.4 Facts we don’t match

Our model fails to match the facts on at least two dimensions. First, given our parametrizations, the model predicts lower participation rates for the most educated households. As we noted above, it is unclear whether this is a serious failure of the model. Second, our model predicts that households invest relatively small amounts in the riskless asset, and that they never hold positive amounts of the riskless asset without owning at least some stock. In fact, households invest relatively large amounts in the riskless asset and often invest in the riskless asset without owning any stock.

We now investigate the welfare significance of these model failures. Our basic message is simple: When borrowing rates are in the neighborhood of the return on equity, the welfare consequences of the wrong bond-equity mix are rather small. Even the right to hold equity has minor welfare implications in the model with a zero leverage premium as compared to the canonical model. These observations are not intended to downplay the shortcomings of our model. Rather, they suggest to us that there is ample scope for other factors – participation and transaction costs, alternative preferences, a desire for liquidity, information costs and so on – to strongly influence life-cycle equity holdings once we recognize that borrowing costs exceed the risk-free investment return.

Table 10 reports certain equivalent consumption levels for various specifications of the model. For example, the table shows that a less-than-high school educated household facing an eight percent borrowing rate, and that can trade equity, is indifferent between the optimal lifetime consumption plan and a fixed lifetime consumption level of \$15,550. If the same household is not allowed to trade equity, certain equivalent

consumption falls to \$15,410.⁵ In other words, the right to trade equity is worth about \$140 per year – or less than one percent of annual consumption. The row labeled 50/50 reports the certain equivalent consumption for a household that must buy a dollar of the riskless asset for every dollar of equity. In this case, the right to trade equity is worth around \$20 a year.

Table 10 shows that if households face a borrowing rate equal to the return on equity, the right to trade equity is rarely worth more than 1 percent of lifetime consumption. For example, a household that mistakenly believed equity investment to be actuarially unfair, and thus invested nothing in equity, would not suffer a great loss of lifetime welfare. This result stands in sharp contrast to the situation when the borrowing rate equals the lending rate. In this case, the gains to trading equity exceed 35 percent of lifetime consumption for all groups.

With respect to portfolio shares, our argument is similar to that of Vissing-Jorgensen (2002), who looks at actual portfolios in the Survey of Consumer Finance. She calculates that the welfare gains of increasing the share of household wealth in equity are small for most households.

6 Conclusions

We have shown that introducing realistic borrowing costs into the canonical model of Merton and Samuelson greatly reduces the demand for equity throughout the life cycle. According to our analysis, most households are constrained by their inability to borrow at the risk-free rate and, as a consequence, hold much less equity. Equity holding is an order of magnitude lower in our model than in a canonical model with cost of borrowing equal to the risk-free investment return. In this sense, equity demand is constrained for most households through the life cycle. Households hold no equity over much of the life cycle for sensible calibrations of the model.

We have also shown that the wedge between borrowing costs and the risk-free return improves the empirical performance of the model on several other dimensions. Our analysis also helps to understand how and why inferences based on models with equal borrowing and lending rates can be wildly misleading.

Our results contrast with those of earlier models that impose some type of fixed

⁵To calculate certain equivalent consumption, we first calculate lifetime expected utility of a given consumption profile – call it U . Then we choose the level of constant lifetime consumption \bar{c} that yields the same level of lifetime expected utility as the computed consumption profile. I.e. we solve $\sum_{t=0}^T \beta^t \frac{\bar{c}^{1-\gamma}}{1-\gamma} = U$ to get $\bar{c} = \left[\frac{1-\gamma}{\sum_{t=0}^T \beta^t} U \right]^{\gamma-1}$ where β is the subjective discount factor and γ is the coefficient of relative risk aversion.

borrowing limit at the risk-free rate. Allowing households to borrow, but at higher rates, significantly affects life-cycle wealth accumulation and thus changes the point in the life-cycle when household have liquid wealth to invest in equity. But because household can only borrow at a high rate, they do not leverage their investments in equity and thus invest relatively small amounts.

Our model provides at least three directions for future research. First, it provides a new perspective on life-cycle wealth accumulation. The predictions of our model that some household have no liquid wealth until relatively late in life are consistent with much empirical evidence on wealth accumulation. See Lusardi, Cossa and Krupka (2001), for example. Second, our model puts the spotlight on the role of borrowing costs in risky asset investment. In this paper, we did not consider in detail the possibility that investors can borrow against equity investments (i.e., take margin loans). One reason is that investors can only borrow up to 50% of an investment in equity. However, the limitations on borrowing are much lower for other risky assets, like corporate bonds, government securities, real estate and small business wealth. Thus an extended version of our model can shed light on portfolio choice in a broader context. See Kubler and Willen (2002) for a discussion. Third, our model has implications for the life-cycle profile of the covariance between consumption growth and equity returns. Specifically, it predicts that the covariance should rise with age.

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Appendix: Computational details

Using homotheticity of preferences we can write the ratio of optimal portfolio-choice and current income at z^t as a function of the current shock z_t , the household's age, t as well as on the ratio of cash at hand (i.e. pay-offs of last period's portfolio) to current income, ξ . We thus need to solve for functions $f_t^j(z, \xi)$ for $t = 1, \dots, N$, $z = 1, \dots, S$, $j = 1, 2, 3$ and $\xi \in A(t, z) \subset \mathfrak{R}$. Note that these are $3NS$ functions of a single continuous variable. The range of $f_t^j(z, \cdot)$ will generally depend on t and z . In practice we vary A only with t , assuming that each period agents save at most 30 percent of their average income and that agents never borrow more than they can pay back throughout their lifetime under the assumption that their income is deterministic and that they never consume less than $2/3$ of current income. In simulations we then verify that these bounds are never binding.

We approximate each unknown portfolio function using interpolating cubic splines (see Judd et al (2002) for details on spline interpolation). Since the true policy functions have non-differentiability we use 50 knots for each spline interpolation to obtain

sufficient accuracy. We solve for the unknown spline-coefficients by backward induction. Obviously f_N is identical equal to zero. Given the coefficients to approximate f_t , we solve for the coefficients of f_{t-1} by solving the household's Kuhn-Tucker conditions (which are necessary and sufficient for optimality) on a grid of 50 equi-spaced points in A_{t-1} and then interpolating the solution over A_{t-1} .

Through a simple change of variables (see Garcia and Zangwill (1981)) we can eliminate all inequalities in the Kuhn Tucker conditions and state the optimality conditions as a system consisting solely of equations. The resulting system has 3 unknowns.

Let $\eta_j \in \Re$ for $j = 1, 2, 3$ and define the Kuhn Tucker multiplier for asset j $\mu_j = (\max\{0, -\eta_j\})^3$ and the holding $\theta_j = (\max\{0, \eta_j\})^3$. Note that θ and μ are twice continuously differentiable and that the following relations hold:

$$\begin{aligned} (\max\{0, \eta_j\})^3 &= \begin{cases} (\eta_j)^3 & \text{if } \eta_j > 0 \\ 0 & \text{if } \eta_j \leq 0 \end{cases} \\ (\max\{0, -\eta_j\})^l &= \begin{cases} 0 & \text{if } \eta_j > 0 \\ |\eta_j|^3 & \text{if } \eta_j \leq 0 \end{cases} \end{aligned}$$

Moreover, the complementary slackness conditions hold:

$$(\max\{0, \eta_j\})^3 \geq 0, (\max\{0, -\eta_j\})^3 \geq 0, \text{ and } (\max\{0, \eta_j\})^3 \cdot (\max\{0, -\eta_j\})^3 = 0.$$

We implemented the algorithm using Fortran 90. As a non-linear equation solver a simple Newton method usually works well, in exceptional cases we had to rely on homotopy methods (as implemented in HOMPACT, see Watson et al (1987)). Maximial relative errors in Euler equations lie below 10^{-6} . Running times lie around 4 minutes per example.

Table 1: Household Borrowing Costs and Risk-Free Returns, Selected Years [tabx6]

	Year	1972	1980	1984	1987	1990	1995	2001
(1)	Average rate on two-year personal loans	12.5	15.5	16.5	14.2	15.5	13.9	13.2
(2)	Average marginal tax subsidy on borrowing	.181	.247	.249	0	0	0	0
(3)	After-tax borrowing cost (1 - row 2)*(row 1)	10.2	11.6	12.4	14.2	15.5	13.9	13.2
(4)	Rate on three-year U.S. Treasury Securities	5.7	11.5	11.9	7.7	8.3	6.3	4.1
(5)	Average marginal tax rate on interest income	.313	.428	.330	.279	.250	.282	.297
(6)	After-tax risk-free return (1 - row 5)*(row 4)	3.9	6.6	8.0	5.5	6.2	4.5	2.9
(7)	Pre-tax wedge between borrowing cost and risk-free return (row 1 - row 4)	6.7	4.0	4.6	6.5	7.2	7.7	9.1
(8)	After-tax wedge between borrowing cost and risk-free return (row 3 - row 6)	6.3	5.1	4.4	8.7	9.3	9.5	10.3
(9)	Charge-off rate on loans, net of recoveries				0.8	1.0	0.7	1.3
(10)	After-tax wedge net of charge offs (row 8 - row 9)				7.9	8.2	8.8	9.1
(11)	After-tax wedge net of charge offs, credit cards				9.0	8.5	7.9	6.5

Sources: Rows (1) through (8) for 1972 to 1987 are reproduced from Table 1 in Altig and Davis (1992). Data for later years as follows: Rows (1) and (4) are from various issues of the *Federal Reserve Bulletin* and the Federal Reserve's *Annual Statistical Digest*. Row (2) reflects the Tax Reform Act of 1986, which eliminated the tax deductibility of interest payments on non-mortgage loans. Row (5) is from Table 1 in Poterba (2001), which is calculated from the NBER TAXSIM model. Poterba's 1999 value is used for the 2001 entry in row (5). Row (9) is from www.federalreserve.gov/releases/chg-all_sa.txt (visited 3 April 2002). Other rows are calculated by the authors as indicated.

Notes: Borrowing costs, returns and charge-offs are expressed as annual percentage rates. Row (9) reports the value of loans removed from the books and discharged against loan loss reserves net of recoveries as a percentage of loans outstanding. Rows (10) and (11) show the difference between the household cost of borrowing and the rate of return on risk-free investments after adjusting for tax considerations and the charge-off rate. Row (11) is calculated in the same manner as row (10), except that it makes use of interest rate and charge-off data for credit cards instead of two-year personal loans.

Parameter	Baseline	Alternative values
RRA	2	0.5, 1, 4 8
Discount factor	0.95	
Age of labor force entry	21	
Age of retirement	65	
Age of death	80	
var ($\tilde{\eta}$)	see Table 3	
cov ($\tilde{\eta}, \tilde{r}$)	0	
var ($\tilde{\varepsilon}$)	see Table 3	
cov ($\tilde{\varepsilon}, \tilde{r}$)	0	
Replacement rate	80%	20%, 40%, 60%, 100%
r_B	8%	2%, 5%, 8%, 20%, 99%
r_L	2%	
r_E	8%	
std (\tilde{r})	15%	

Table 2: Calibration of our model.

Education group	$\sigma(\tilde{\varepsilon})$ (transitory)	$\sigma(\tilde{\eta})$ (permanent)
Some high school	26	15
High school diploma	21	17
Some college	18	15
College degree	20	12
Graduate school	22	11
Total	21	15

Table 3: Standard deviation of transitory and permanent shocks by education level measured in percentages. Estimates from Gourinchas and Parker (2002).

Table 4: Participation rates over the life-cycle. Specifications are baseline unless otherwise noted.[tabx1]

Age Group	<i>Total</i>						<i>More than College Educated</i>					
	Without inc. risk			With inc. risk			Without inc. risk			With inc. risk		
	$r_B =$			$r_B =$			$r_B =$			$r_B =$		
	8	20	99	8	20	99	8	20	99	8	20	99
22 - 24	0	0	0	55	53	70	0	0	0	0	31	52
25 - 29	0	0	0	78	79	96	0	0	0	3	40	73
30 - 34	0	100	80	95	97	100	0	20	40	31	79	90
35 - 39	20	100	100	100	100	100	0	100	100	76	97	99
40 - 44	100	100	100	100	100	100	0	100	100	97	100	100
45 - 49	100	100	100	100	100	100	40	100	100	100	100	100
50 - 79	100	100	100	100	100	100	100	100	100	100	100	100
Pop. avg	59	80	78	93	93	96	38	71	73	70	84	91

Table 5: Summary of results. '-' implies baseline specification. [tabx3]

r_B	RRA	RR	Labor income risk?	Profile	cov_{MU}	HJ	cov_C	Demand		Implied RRA	% Ptcp
								DKW	Std		
-	-	-	No	Est'd	0.8	5.3	0.4	24	467	12.4	74
-	-	-	Yes	Est'd	1.2	7.8	0.6	80	467	8.8	90
2	-	-	No	Est'd	5.7	36.6	2.8	467	467	1.8	100
99	-	-	No	Est'd	0.9	6.0	0.4	29	467	11.0	87
-	0.5	-	No	Est'd	1.2	7.3	2.2	107	467	2.3	98
-	1	-	No	Est'd	1.0	6.3	0.9	61	467	5.2	91
-	4	-	No	Est'd	0.7	4.7	0.2	9	467	28.0	37
-	8	-	No	Est'd	0.7	4.7	0.1	4	467	56.0	27
-	-	-	No	Flat	1.5	9.4	0.7	46	467	7.0	100
-	-	0.2	No	Est'd	1.7	11.0	0.8	50	467	6.0	82
-	-	1	No	Est'd	0.6	3.8	0.3	17	467	17.3	71

Notes: 1. Baseline specification: $r_B = 8\%$, $RRA = 2$, $RR =$ Replacement rate $= 0.8$.

2. $cov_{MU} = cov(\tilde{r}_E, -\frac{\Delta \tilde{M}U}{E(\Delta \tilde{M}U)})$. See Section ?? for details.

3. $HJ = std(\frac{\Delta \tilde{M}U}{E(\Delta \tilde{M}U)})$. See Section ?? for details.

4. $cov_C = cov(\Delta C, \tilde{r}_E)$.

5. DKW: our model. Std: standard model with equal borrowing and lending rates.

6. Implied RRA calculated using equation (2). See Section ??.

7. Relevant quantities in 1987 dollars.

Table 6: Demand for equity over the life-cycle in dollars. Specifications are baseline unless otherwise noted. Relevant quantities in 1987 dollars. [tabx2]

Age Group	<i>Total</i>				<i>More than College Educated</i>					
	w/o inc. risk		with inc. risk		income	w/o inc. risk		with inc. risk		income
	$r_B =$		$r_B =$			$r_B =$		$r_B =$		
	8	2	8	2		8	2	8	2	
22 - 29	0	486	4	235	18	0	591	0	221	19
30 - 39	0	506	21	493	22	0	629	10	507	29
40 - 49	14	504	59	859	25	0	620	62	924	34
50 - 59	47	477	111	1214	25	31	579	136	1288	33
60 - 69	69	388	144	1451	22	63	475	182	1500	29
70 - 79	43	202	90	978	20	42	248	116	988	27
Pop. avg	21	462	58	774	22	15	567	66	807	29

Table 7: Consumption and asset pricing over the life cycle. Baseline specification *with* labor income risk.[tabx7]

Age Group	cov_{MU}	<i>Total</i>			Implied RRA	<i>More than College Educated</i>			
		Demand		Implied RRA		cov_{MU}	Demand		Implied RRA
		DKW	Standard				DKW	Standard	
22 - 29	0.0	4	591	849.1	0.0	0	486	Inf	
30 - 39	0.4	21	629	27.5	0.2	10	506	75.9	
40 - 49	0.8	59	620	12.4	0.7	62	504	15.7	
50 - 59	1.4	111	579	7.5	1.3	136	477	7.9	
60 - 69	1.9	144	475	5.7	1.8	182	388	5.8	
70 - 79	1.9	90	248	5.4	1.9	116	202	5.6	
Pop. avg	0.8	59	567	12.5	0.7	66	462	14.6	

Notes: 1. See Table 5 for details.

2. Relevant quantities in 1987 dollars.

Table 8: Consumption and asset pricing over the life cycle. No labor income risk – otherwise baseline.[tabx8]

Age Group	<i>Total</i>				<i>More than College Educated</i>			
	cov_{MU}	Demand		Implied RRA	cov_{MU}	Demand		Implied RRA
		DKW	Standard			DKW	Standard	
22 - 29	0.0	0	591	Inf	0.0	0	486	Inf
30 - 39	0.0	0	629	Inf	0.0	0	506	Inf
40 - 49	0.4	14	620	26.2	0.0	0	504	Inf
50 - 59	1.4	47	579	7.4	0.7	31	477	14.0
60 - 69	2.3	69	475	4.5	1.8	63	388	5.8
70 - 79	2.5	43	248	4.1	2.1	42	202	5.0
Pop. avg	0.7	21	567	14.1	0.5	15	462	22.9

Notes: 1. See Table 5 for details.

2. Relevant quantities in 1987 dollars.

Table 9: Consumption and asset pricing over the life cycle for equity market participants. No labor income risk – otherwise baseline[tbx9]

Age Group	<i>Total</i>				<i>More than College Educated</i>			
	cov_{MU}	Demand		Implied RRA	cov_{MU}	Demand		Implied RRA
		DKW	Standard			DKW	Standard	
22 - 29	0.0	0	591	Inf	0.0	0	486	Inf
30 - 39	0.0	0	629	Inf	0.0	0	506	Inf
40 - 49	0.4	14	620	26.2	0.0	0	504	Inf
50 - 59	1.4	47	579	7.4	0.7	31	477	14.0
60 - 69	2.3	69	475	4.5	1.8	63	388	5.8
70 - 79	2.5	43	248	4.1	2.1	42	202	5.0
Pop. avg	1.2	35	567	8.4	1.2	38	462	8.8

Notes: 1. See Table 5 for details.

2. Relevant quantities in 1987 dollars.

Table 10: Welfare gains from trading equity. Table entries are certain equivalent life-cycle consumption measured in thousands of dollars. All parameters are baseline unless noted.[tabx11]

R_B	Income risk	Equity	Less than High School	High School Degree	Some College	College Degree	More than College
8	No	No	15.41	19.44	21.53	25.75	25.52
		50/50	15.43	19.45	21.56	25.79	25.53
		Yes	15.55	19.59	21.74	25.96	25.61
2	No	No	16.71	21.05	23.41	28.39	29.65
		Yes	22.39	28.21	31.37	38.06	39.75
8	Yes	No	12.59	15.01	17.10	21.97	21.96
		Yes	13.54	16.46	18.48	22.91	22.42

Notes: 1. See Section ?? for details.

2. Relevant quantities in 1987 dollars.

Figure 1: Baseline labor income in 1987 dollars over the life-cycle. Note that we exclude households with some college education from the picture. `figa7`

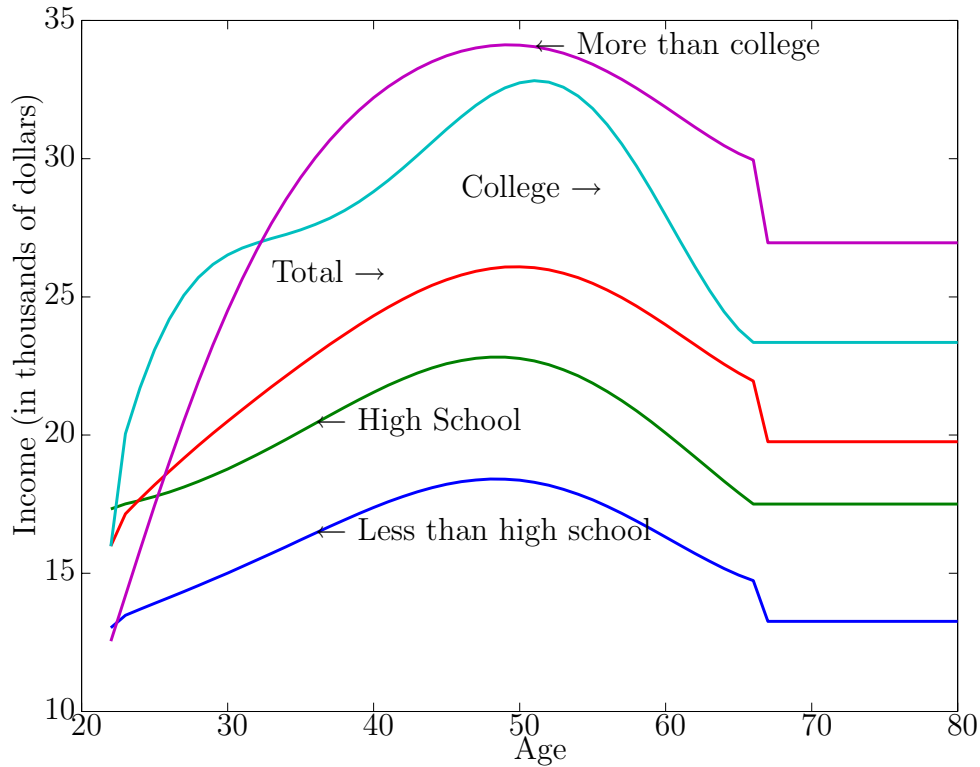


Figure 2: Life-cycle equity holdings in 1987 dollars for different borrowing rates. Baseline specification for more-than-college educated household with no labor income risk. `[figa1]`

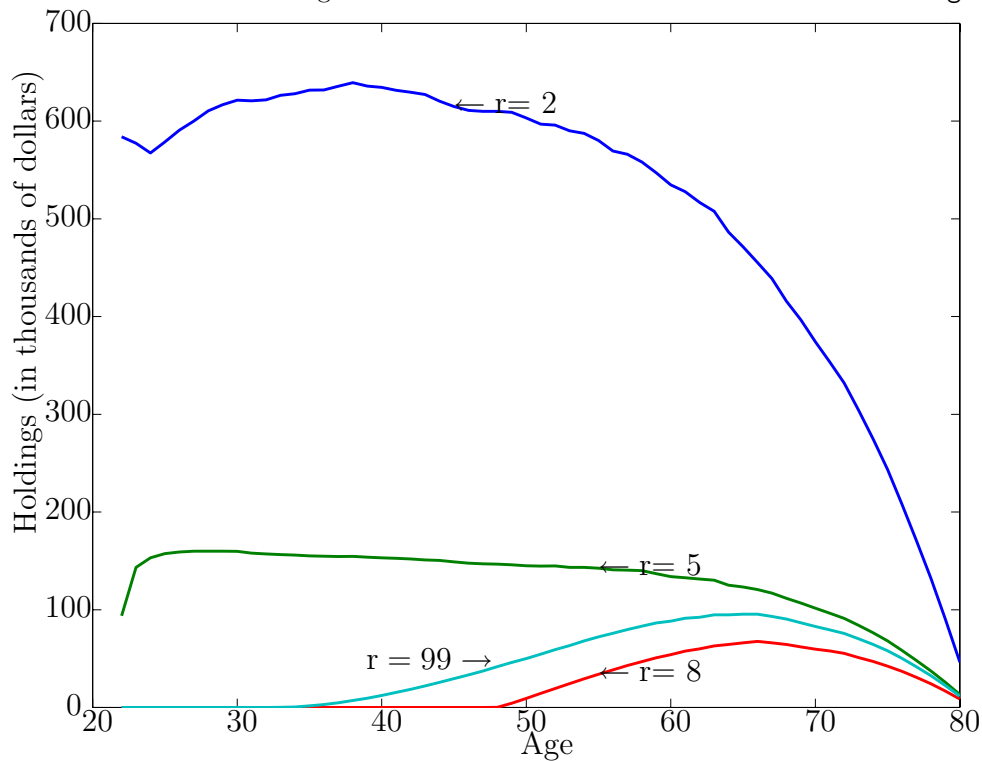


Figure 3: Life-cycle borrowing in 1987 dollars for different borrowing rates. Baseline specification for more-than-college educated household with no labor income risk. [figa2]

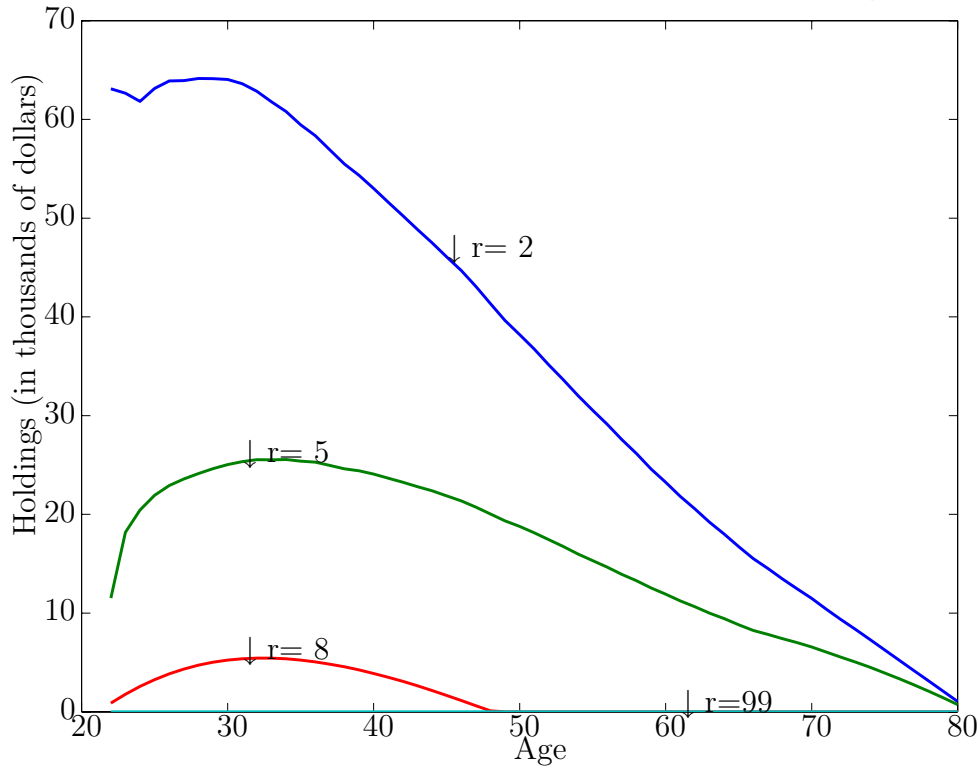


Figure 4: Life-cycle liquid wealth in 1987 dollars for different borrowing rates. Baseline specification for more-than-college educated household with no labor income risk. [figa3]

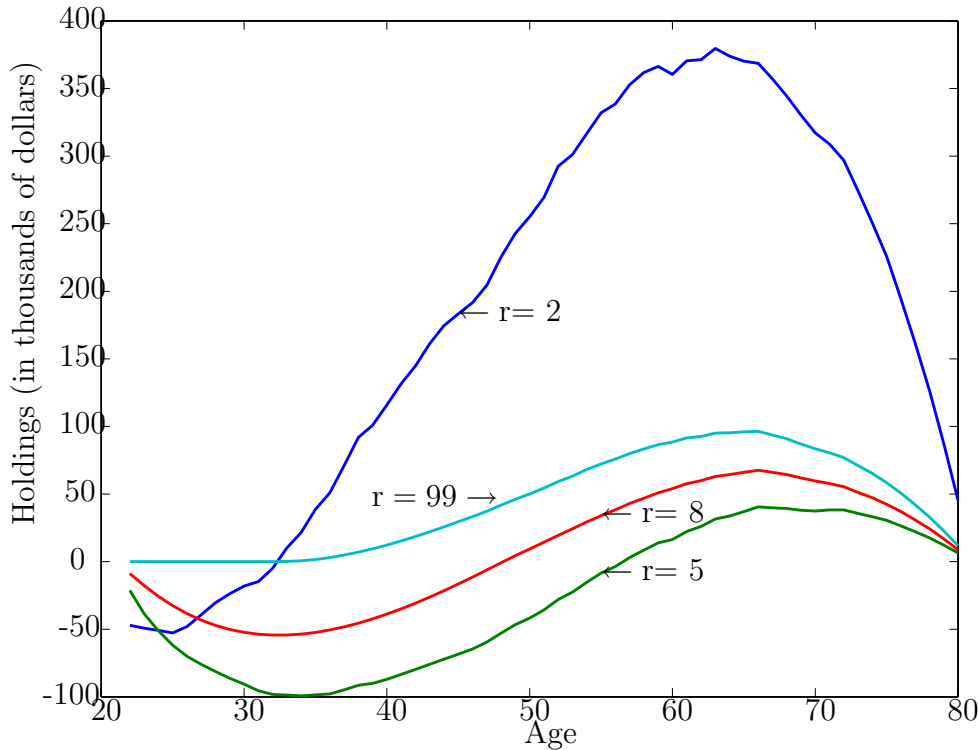


Figure 5: Life-cycle equity holding in 1987 dollars with and without labor income risk. Specification is for baseline more-than-college educated household otherwise.. [figa6]

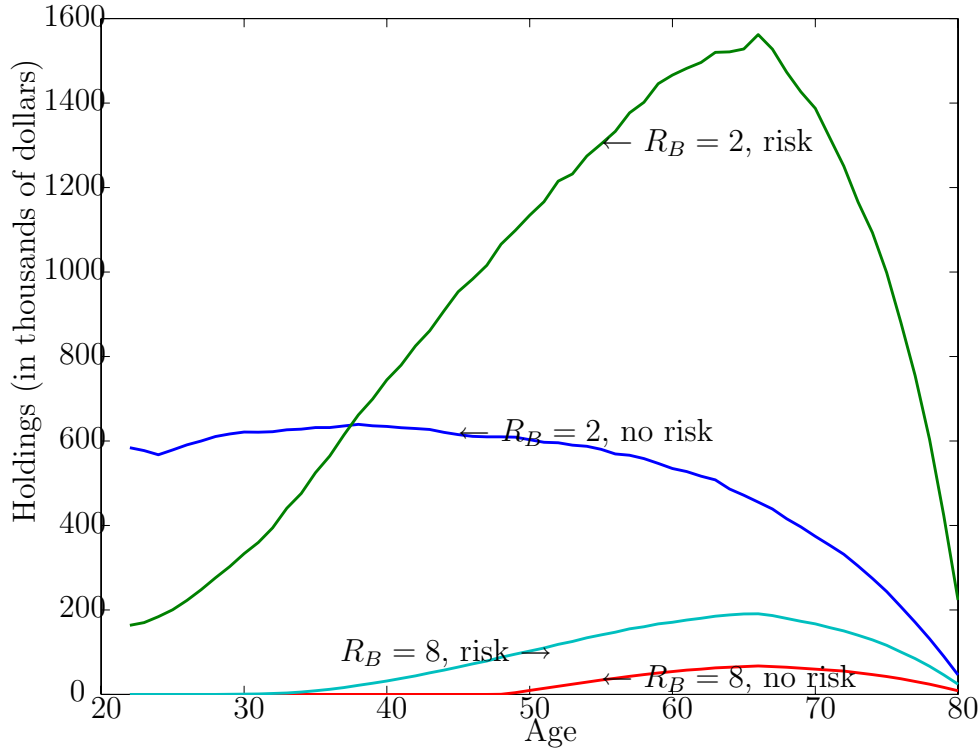


Figure 6: Life-cycle equity holding in 1987 dollars for different RRA. Specification is for baseline more-than-college educated household with labor income risk otherwise. [figa4]

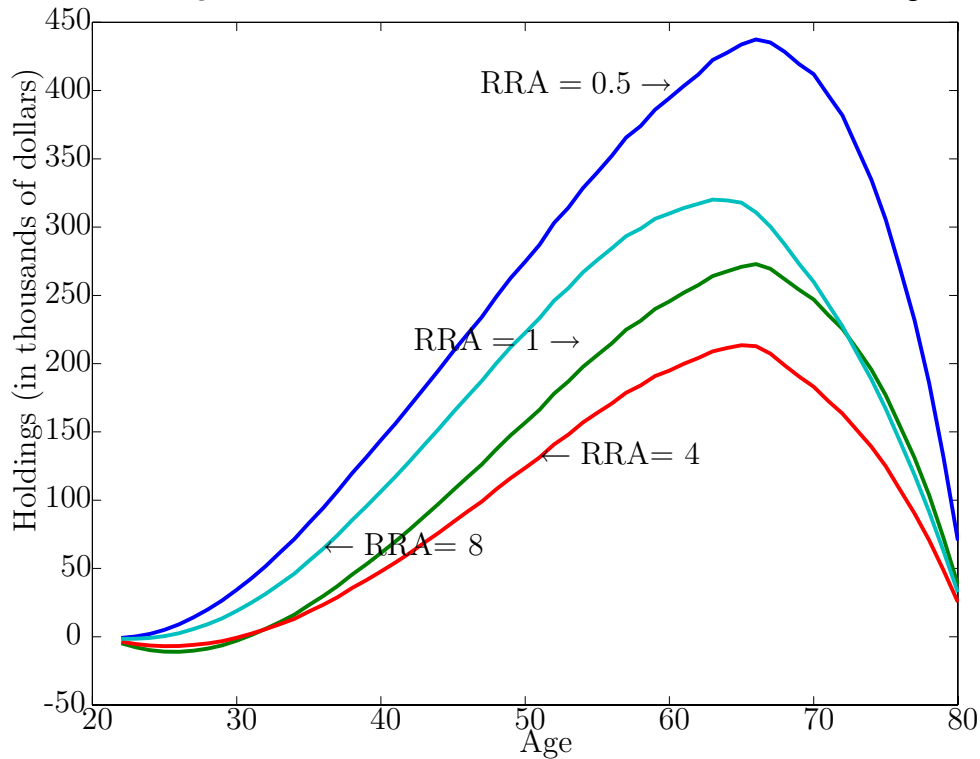


Figure 7: Life-cycle equity holding in 1987 dollars with flat and estimated life-cycle profiles. Baseline more-than-college educated household otherwise and *no* labor income risk. [figa9]

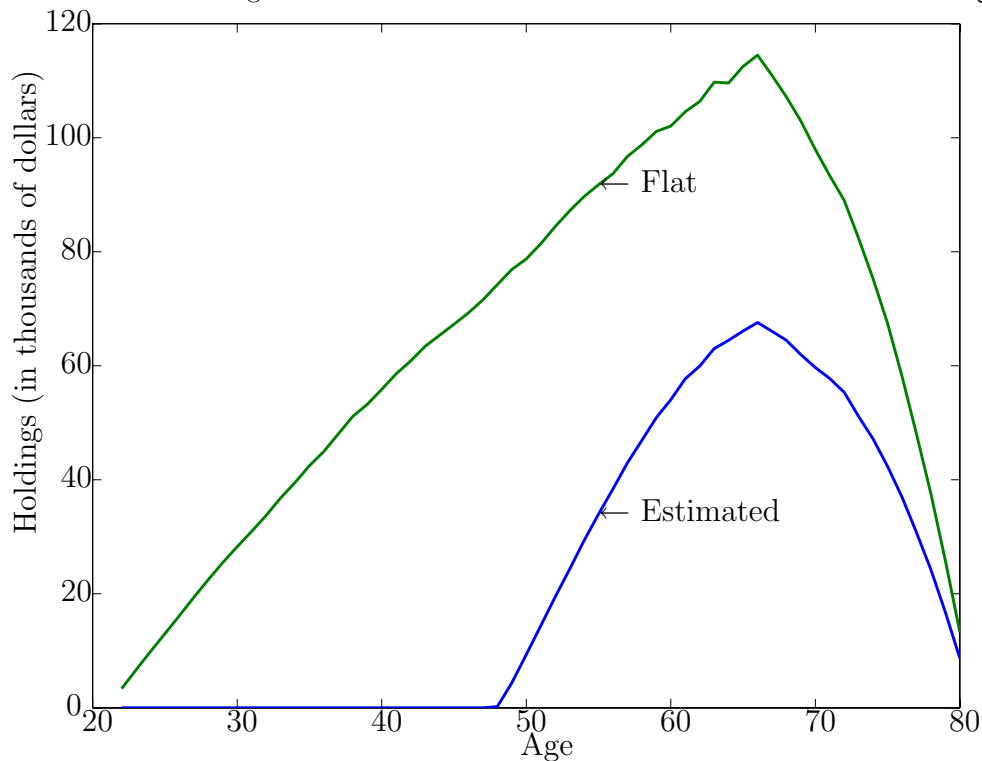


Figure 8: Life-cycle equity holding with different replacement rates. Baseline more-than-college educated household otherwise and *no* labor income risk. [figa8]

