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Advertising, Entry Deterrence, and Industry Innovation

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Abstract

This paper studies how advertising influences firms' incentives to invest in R&D. The link between advertising and industry innovation is important, not only because advertising can spur R&D by spreading product knowledge, but also because advertising can discourage new innovative firms from entering the industry.

This paper finds that a worse advertising technology can result in local improvements in industry innovation rates. Globally, however, a complete ban on advertising always reduce industry growth. This result is significant because industry advertising spending is quantitatively significant and there are potential connections between public policy towards advertising and R&D.

This paper presents a variant of the Grossman and Helpman (1991) quality ladder model. The key difference is that the model in this paper allows advertising to gradually spread product awareness among consumers. This model differs from the entry deterrence literature by assuming perfect price discrimination. Technically, this assumption allows a fully tractable model and analytical characterization of a stationary equilibrium in a dynamic setting, which is not previously available. In terms of economic analysis, this assumption eliminates the extra profit incentives for new firms to enter early, and makes incumbent firms more inclined to use advertising as a deterrent.

JEL Classification: L15, L25, M37

Keywords: Advertising, Entry Deterrence, Innovation

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1 Introduction

Economic growth is widely recognized as being driven in large part by innovation (i.e. by research and development). Consequently, the abundant theoretical literature on industry growth (e.g. Grossman and Helpman (1991))[1] focuses on firms' incentives to invest in R&D. One factor that may specifically influence R&D, which has largely been ignored by the literature, is advertising. Advertising allows firms to make consumers aware that it has developed an innovation over the existing state-of-the-art product. Therefore, improvements in advertising spur R&D.

There is another force in industries in which strategic interaction plays an important role. If current state-of-the-art products are heavily advertised, they may discourage innovations by new entrants, as they will face intense competition from well-entrenched incumbents. In fact, there is ample literature dating back to Bain (1949), arguing that advertising can deter entry by new firms. Then in industries where strategic interaction is important, there will be two offsetting forces regarding how advertising influences R&D.

The goal of this paper is to study the impact of advertising on the innovation and growth of an industry in which strategic interaction is a key element. In particular, if there is an improvement in advertising technology, which of the two offsetting forces will prevail? This research finds that as advertising technology improves (i.e. per unit advertising cost decreases), it is possible that the entry deterrence effect of advertising dominates. Therefore, a worse advertising technology can result in local improvements of industry innovation rates. However, this paper finds that a complete ban on advertising always reduces industry growth.

This paper presents a variant of the Grossman and Helpman (1991) quality ladder model. The key difference is that a new product is not known by all consumers immediately after entry. Instead, consumers become aware of a product only gradually. In this gradual process of product knowledge diffusion, the more a product is advertised, the more consumers become

[1] The literature on the relationship between competition and growth is much too large to cite comprehensively. Besides Grossman and Helpman (1991), a few recent examples include Aghion and Howitt (1992), Jones (1995), Segerstrom (1998), and Horner (2004)

aware of the product. In this model, each firm has a particular quality level product and invests in advertising to inform consumers about the existence of the product. Each product is protected by a patent. Eventually, the product becomes obsolete upon patent expiration, and the firm exits the market. Possibly before or after an incumbent's patent expiration, a new firm enters the market. Each entrant firm decides upon its time of entry. Upon entering, a firm chooses how much R&D to invest in making its product better than the existing variety, and how much advertising to make consumers aware of its new product.

This model provides a comparative static analysis of innovation, advertising and entry time with respect to changes in advertising technology. Different advertising technology results in three different firm-entry modes: blockaded entry, accommodated entry and entry deterrence. Blockaded entry occurs when large structural entry barriers, such as an expensive advertising technology, exist in an industry, thereby preventing future entry. Accommodated entry occurs when advertising technology is good enough so that any entry-deterring strategies become ineffective. In both blockaded and accommodated cases, advertising complements innovation. Locally, as advertising technology improves, innovation improves, new firms enter sooner, and industry grows at a faster rate. If advertising technology is in between the above two settings, firms can invest heavily in advertising to strategically deter future entry. Locally, when entry is deterred, an incumbent's intensive advertising reduces entrant firms' entry values, thereby dulling incentives for innovation. Both innovation and industry growth suffer as a result.

This paper extends the analysis to include global comparative statics by considering the extreme case of advertising ban. Interestingly, no-advertising-at-all proves to be much more detrimental to innovation and growth than even heavy entry-deterring advertising.

The results of this paper are significant due to a couple of considerations. First, this paper extends our understanding of industry dynamics by incorporating a quantitatively important variable: advertising. In most industries, advertising spending equals or even exceeds R&D spending[2]. In particular, the pharmaceutical industry is known to be an extremely innovative

[2] Information comes from the Compustat company database from Standard and Poors, which contains information for 18,000 companies.

industry in which R&D spending is 19 percent of sales. Yet spending on advertising in this industry is virtually the same as it is for R&D (18 percent of sales)[3]. Second, this paper investigates the potential connections between public policy towards advertising and R&D. In most countries, direct-to-consumer advertising of pharmaceutical products is banned, but such advertising is legal in the United States. This model of advertising and innovation gives an clear answer to the impact of public policy on advertising on long-run growth.

A long strand of IO literature has studied entry deterrence via advertising. Although mostly in a static setting and failing to address the issues of R&D and industry growth, this literature converges upon an interesting, but ambiguous result. As suggested by Schmalensee (1983)[4] and Fudenberg and Tirole (1984), although advertising can lower the demand of a new entrant firm by strengthening an incumbent's hold on consumers, the resulting expansion in consumer awareness creates an incentive for an incumbent to price its product higher. This increase in profit margin extends to the entrant, and may make a new firm more likely to enter.

Aside from incorporating industry R&D and industry growth, the key way that my paper differs from this literature is that I assume firms can perfectly price discriminate. That is, firms in my paper are allowed to charge different prices to consumers who differ in product awareness. This assumption simplifies the theoretical analysis by making the effect of advertising on entry deterrence unambiguous: advertising aids entry deterrence. Indeed, this assumption allows an incumbent to maintain a high price for those consumers who are solely aware of its product, while it lowers its price aggressively for those consumers who are aware of both its product and its rival's product. This assumption eliminates the extra profit margin for a potential entrant, and thus dulls its incentive for early entry.

Pure uniform pricing, as assumed by the literature, and perfect price discrimination are two polar case assumptions. Neither is likely to be exactly true in the real world. For example, due to the immense complexity of the health care system, pharmaceutical companies certainly do

[3]Sources of the figures: *2006 Pharmaceutical Industry Profile*.

[4]Ishigaki (2000) studies Bertrand pricing competition under the Schmalensee setting. Ishigaki finds that only mixed strategy pricing equilibria are possible.

not charge uniform pricing to all consumers. Although pharmaceutical firms cannot perfectly price discriminate either, the existence of such industries is enough to make an analysis of allowing price discrimination to be of interest.

This paper is closely related to that of Doraszelski and Markovich (2005)[5]. These researchers incorporate the advertising and industry dynamics in Pakes and Ericson's (1995) framework. They assume uniform pricing and numerically simulate results for a symmetric Markov-perfect equilibrium. However, my paper differs from that of Doraszelski and Markovich (2005) in both its technical aspects and economic intuition. Technically, the assumption of perfect price discrimination allows a much more tractable model. My model has no issues concerning the existence of a pure strategy price equilibrium, and can obtain full analytical comparative static results. As for economics, since perfect price discrimination allows the elimination of extra profit incentives for new firms to enter early, incumbent firms are more inclined to use advertising as a deterrent.

The remainder of the paper is organized in the following fashion: the model is formally presented in the next section; Section 3 introduces the single entry case as a benchmark; Section 4 extends to a general setting with sequential entry, and defines the Stationary Markov-perfect Equilibrium; Section 5 characterizes a stationary equilibrium with entry deterrence; Section 6 presents numerical analysis; and Section 7 concludes.

2 Model

The model is in continuous time and has an infinite horizon. Firms are risk neutral and discount the future at a rate of $\rho > 0$. Each firm produces a unique product, distinct in two dimensions: consumer awareness and quality level. Each firm is, in turn, able to expand its product's awareness through advertising, and raise its quality through innovation. A firm operates for a

[5]The results of Doraszelski and Markovich (2005) indicate that strategic under-investment in advertising can deter entry. The intuition is similar to Boyer and Moreaux (1999), although Boyer and Moreaux (1999)'s model is static.

finite T units of time (T is exogenous); then, its product becomes obsolete. There are, at most, two firms in the market at any instant in time[6]. In addition, there is a numeraire good of which all consumers are aware, so that the prices of the two products are bounded away from infinity.

Timeline. Firms enter market sequentially, and only one firm can enter at any given instant in time. A large number of potential entrants wait to enter at any given moment. Each potential entry firm must decide how long to wait to enter the market after the last entry. This waiting time is denoted by l. For example, if firm 1 enters the market at t_1, and firm 2 decides the entry time to be t_2, then $l_2 = t_2 - t_1$. The innovation stepsize denoted by "z" and the time length of advertising denoted by "a" are decided at the time of a firm's entry.

Firms pay an entry cost to enter the market. This entry cost is a function of innovation stepsize z and waiting time l. It has the additive separative form: $I(z) + ke^{-\eta l}$. The cost associated with innovation $I(\cdot)$ is time invariant. For simplicity, $I(z)$ is assumed to be $I \cdot z^{\gamma}$ ($\gamma \geq 2$) for the remainder of the paper. However, all of the results presented in this paper are robust, as long as $I(\cdot)$ is increasing and convex in stepsize z, with $I(0) = 0$ and the inverse of its derivative $I'^{-1}(\cdot)$ exists and it is weakly concave. Notice that $I(0) = 0$ implies that the new entrant would have at least the same quality level as its predecessor. In addition, the convexity in innovation cost bounds the optimal solutions away from infinity. The cost associated with entry waiting time: $ke^{-\eta l}$ implies that the more time a potential entrant waits, the better the innovation environment, and the cheaper it is to enter the market. This provides an incentive for firms to delay entry.

Once in the market, a firm can produce the product at zero marginal cost. And there is no other variable costs or fixed costs associated with the production process.

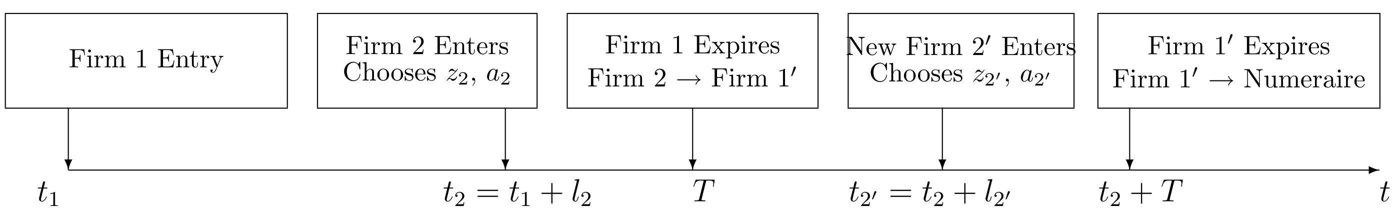
All firms exit after T units of time. This fixed length of time can be seen as the patent length of a new innovation. When a firm enters the market, its good is patented regardless of quality and the patent expires after T periods. Immediately after the patent expiration, a firm's

[6]Generalization to more than 2 firms is conceptually obvious. For notational convenience, this paper only focuses on the two-product case.

product becomes known to all consumers in the market. This patent-expired good becomes the new numeraire good until it is replaced by the next patent-expired good.

I assume at most two patented goods can be in the market simultaneously, thereby I use subscript $i \in \{1, 2\}$ to denote the order of entry. Firm 2 enters the market after firm 1, and innovate over firm 1's product. In addition, I use subscript $i = 0$ to denote the numeraire good. At the time of firm 1's product patent expiration, the firm/product subscript changes. Firm 1's product becomes the new numeraire good indexed by "0," firm 2 becomes the new firm "1," and a new entrant will be indexed as the new firm "2."

The timeline is depicted in the following chart (To avoid confusion, at the time of product 1's patent expiration, I denote the new firm 1 by $1'$ and the new firm 2 by $2'$. Here, firm 2 becomes firm $1'$.):



From above, firm 2 enters at t_2 and expires at t_2+T. Firm 2 overlaps with firm 1 for $t \in [t_2, T]$. After firm 2 becomes firm $1'$, it overlaps with firm $2'$ for $t \in [t_{2'}, t_2 + T]$.

Innovation and product quality. Consumers value a product by its quality level x. Firms innovate to improve product quality. Innovation is modeled in a similar fashion as in Grossman & Helpman (1991), and the quality index is continuous. Upon entry, each firm can innovate once and only once. Firm i chooses an innovation stepsize denoted by z_i. The innovation results in an improvement in quality over the latest industry innovator; hence, innovation is cumulative. For example, if firm 1 is the latest innovator with quality level x_1, and new entrant firm 2's innovation stepsize is z_2, then firm 2's quality level is $x_2 = x_1 + z_2$. Moreover, the arrival of innovation is instantaneous.

Advertising and consumer awareness. There is a unit measure of consumers. The share of consumers who are aware of a product i is $s_i \in [0, 1]$. When a new product is introduced, the initial level of awareness[7] is $\sigma \in (0, 1]$. Additional consumers can only be made aware of a new product through advertising. Once a consumer is made aware of a product, she never forgets, hence a product's consumer awareness level never shrinks.

For each instant in time, a firm can pay a constant flow cost $\theta > 0$ to advertise. For each time it is advertised, a product's awareness expands at a rate of $\phi > 0$. A firm chooses to advertise in $a_i \geq 0$ units of time after entry, and stops advertising thereafter[8]. Product awareness accumulates according to the following process[9]:

$$s_i(t) = \begin{cases} 1 - (1-\sigma)e^{-\phi t}; & \text{if } t \in [0, a_i] \\ s_i(a_i); & \text{if } t \in (a_i, T] \end{cases} \quad \text{for } i \in \{1, 2\} \tag{1}$$

Notice that once a firm stops advertising at time a_i, its awareness remains constant thereafter at $s_i(a_i)$. The key feature of this awareness accumulation process is the decreasing return to advertising. This feature allows a new product to expand in market awareness quickly at the beginning. The increasing difficulty in getting new consumers to know about a new product would eventually make a firm stop advertising. Full awareness is not possible to obtain within finite T units of time.

Consumer preference and market segmentation. Given product awareness and quality levels, firms compete in the product market by setting prices. Consumers are homogenous in their tastes. Consumers are indifferent between x units of quality 1 products and 1 unit of quality x product. Each consumer consumes exactly one good at each instant in time, and she can only consume a product of which she is aware. Given the prices, the utility of a consumer

[7]This positive level of initial awareness can be justified as word-of-mouth advertising, see Fishman and Rob (2003).

[8]Given a discount rate greater than zero, a firm would always prefer to advertise early in its life cycle. This result is formally proven in the Technical Appendix.

[9]This process can be generated using a Poisson distribution, assuming that the advertising message is hitting consumers according to this distribution (see Technical Appendix).

from purchasing a product with quality x and price p is:

$$x - p$$

Recall that there are three products in the market: the product of the incumbent firm denoted by "1," product of the new entrant denoted by "2," and the patent-expired numeraire good denoted by "0." Notice that consumers are different only in the number of products of which they are aware. Since all consumers are aware of product "0," the market is partitioned into four segments: the portion of consumers who are aware of both product 1 and 2, denoted by m^{12}; the portion who are aware of product 1, but not product 2, denoted by m^1; the portion who are aware of product 2, but not product 1, denoted m^2; and the portion who are aware of neither product 1 nor product 2, denoted by m^0. In terms of consumer awareness, market segments are defined as the following:

$$\begin{aligned}
&m^{12}(t) = s_1(t) \cdot s_2(t) \\
&m^{i}(t) = s_i(t) \cdot (1 - s_{-i}(t)) \quad \text{for } i = 1, 2 \\
&m^{0}(t) = (1 - s_1(t))(1 - s_2(t))
\end{aligned}$$

Price discrimination and demand. The key assumption in this paper is that firms can perfectly price discriminate across different market segments. This assumption not only helps to avoid dealing with the price effect in entry deterrence, but also ensures that the model has a unique pure strategy Nash equilibrium. For any given market segment, firms engage in a static Bertrand competition. The pricing decisions for each segment are independent of the firm's other decisions. Since there is no production cost, it is easy to derive the pricing policy as the following: denote firm i's price in market segment m^j, $j \in \{12, 1, 2\}$ as p_i^j, then

$$p_1^1 = z_1; \quad p_1^{12} = 0; \quad p_2^2 = z_1 + z_2; \quad p_2^{12} = z_2$$

In firm i's exclusive market segment "m^i," it charges the highest possible price in order to break even with the numeraire good. In the share market segment m^{12}, firm 2 bids the price down until firm 1 cannot charge a positive price, at which point firm 2 gains the whole segment. Hence, the resulting flow revenue is:

$$\begin{aligned} R_1 &= z_1 \cdot s_1(1 - s_2) \\ R_2 &= (z_1 + z_2) \cdot s_2(1 - s_1) + z_2 \cdot s_2 s_1 = z_2 \cdot s_2 + z_1 \cdot s_2(1 - s_1) \end{aligned}$$

At the time of product 1's patent expiration, it becomes the new numeraire good in the market. At this time of expiration, it becomes instantly known by all consumers. Meanwhile, the market segment which is aware of product 2 but not product 1, namely m^2, disappears, insofar everyone can acquire product 1 at zero price. Because of the high profitability of m^2, a firm has an incentive to enter early and secure this exclusive market for a longer time.

3 Equilibrium with A Single Entrant

In this section, I restrict the model to allow only a single entrant. Once one firm enters, no others are allowed to enter the market. The analysis of this restricted model serves two purposes. First, it is helpful for expositional purposes to work through the mechanics of this simple case. Second, it can serve as a benchmark for later analysis. An equilibrium is defined in this section, and comparative statics are investigated. The key comparative statics regard how a firm's decisions[10] on innovation stepsize "z," advertising length "a," and entry waiting time "l" change with respect to lowering the advertising cost "θ".

In this particular setting, since there is only one firm, consumers are grouped into two market segments: those who know the product of the single entrant, and those who do not. Since the firm competes only with the numeraire good, it charges price $p = z$, and all consumers aware of its product would buy from the firm, so $R(t) = z \cdot s(t)$.

[10]Subscript i is dropped since only one single patented good exits in the market.

Given the consumers' product choices, an *Equilibrium with Single Entry* consists of optimal decisions $\{z^s, a^s, l^s\}$ (superscript "s" denotes the single entrant), and a value function $V^s(z, a, l)$ for which the following conditions are satisfied:

1. Given l, a firm chooses z^s and a^s to maximize the after-entry value[11]:

$$V^s(z^s, a^s, l) = \max_{z,a} \int_{t=0}^{T} e^{-\rho t} z \cdot s(t)\, dt - \int_{t=0}^{a} e^{-\rho t}\theta\, dt - I z^{\gamma} - k e^{-\eta l} \tag{2}$$

2. The zero profit condition:

$$V^s(z^s, a^s, l^s) = 0$$

   If there is more than one l satisfying the above condition, entry time l^s is the earliest l possible.

Here, I assume that the entry cost is high enough so that no firm can enter at time zero with a non-negative profit. The zero profit condition must be satisfied because potential entrants compete to enter the market. One entrant firm will choose to enter the market at the first possible opportunity when it can make a non-negative profit.

In analyzing the equilibrium, the first order necessary conditions for interior optimal z and a are:

$$\frac{\partial V}{\partial z} = \int_{t=0}^{T} e^{-\rho t} s(t)\, dt - I\lambda z^{\lambda-1} = 0 \tag{3}$$

$$\frac{\partial V}{\partial a} = (1-\sigma)\phi e^{-\phi a} \int_{t=a}^{T} z \cdot e^{-\rho t}\, dt - e^{-\rho a}\theta = 0 \tag{4}$$

Notice that z^s and a^s are independent of entry time l. Regardless of when an entrant enters, the firm always chooses the same optimal level of innovation stepsize and advertising.

In Equations (3) and (4), z is increasing in a. This result shows that innovation stepsize and advertising are complements. Given more advertising, consumer awareness expands, and

[11] Because the entry fixed cost $ke^{-\eta l}$ enters the value function additive separately, the optimal decisions on z and a do not depend on the entry time l.

it becomes more profitable to increase innovation stepsize. Meanwhile, innovation becomes increasingly more expensive, and the marginal benefit of innovation shrinks. On the other hand, if a higher innovation stepsize raises the prices charged in the advertised market segment, a firm would have a higher incentive to invest in advertising. However, advertising becomes increasingly more difficult in reaching a higher proportion of the population, so the marginal benefit of advertising also decreases.

Proposition 1. *An equilibrium exists in the single entry case, and it is always unique.*

Proof. First, I show that for any parameters, a firm always chooses to enter and innovate at a positive stepsize z. Notice that a firm always enters given enough waiting time, since the fixed entry cost $ke^{-\eta t}$ goes to zero as $t \to \infty$. As the cost approaches zero, a firm can innovate $z = \varepsilon > 0$ arbitrarily small at which point $\int e^{-\rho t}\, dt > I \cdot \varepsilon^{\gamma-1}$. At this point, even when a firm does not advertise at all, it can make a strictly positive profit by entering the market. This also shows that given a (z, a) pair, there is always a $l > 0$ satisfying the zero profit condition. Next, if a firm enters the market, then it innovates at $z > 0$. This is obvious, since if $z = 0$, a firm competes with the numeraire good and can only charge price $p = 0$. However, the entry cost is strictly positive, so a firm cannot enter the market without a positive innovation.

The above statement states that a market can be in either three cases: a corner case in which $a = 0$, a corner case in which $a = T$, or an interior solution. Next I look into each of the three cases. If a firm does not advertise ($a = 0$), its market awareness stays at the initial exposure level σ throughout. Thus, the firm solves the maximization problem:

$$\max_{z} z\sigma \int_{t=0}^{T} e^{-\rho t}\, dt - Iz^{\gamma}$$

Therefore, $z = [\sigma \int_{t=0}^{T} e^{-\rho t}\ dt/(I\gamma)]^{1/(\gamma-1)}$; consequently, an equilibrium exists and is unique. The result of the corner solution case in which $a = T$ follows a similar rationale.

I next look into the interior case in which Equations (3) and (4) are both satisfied. I can rewrite these two equations as the following functions:

$$\text{Equation (3)} \rightarrow \widetilde{z}(a) = \left[\frac{\int_{t=0}^{T} e^{-\rho t} s(t)\, dt}{I\gamma}\right]^{\frac{1}{\gamma-1}}$$

$$\text{Equation (4)} \rightarrow \widetilde{\widetilde{z}}(a) = \frac{\theta\rho}{\phi(1-\sigma)e^{-\phi a}(1-e^{-\rho(T-a)})}$$

It can be easily verified that $\widetilde{z}(a)$ is increasing and concave in a, and that $\widetilde{\widetilde{z}}(a)$ is increasing and convex in a.

In a space where where "a" is on the x-axis and "z" is on the y-axis, a typical example of $\widetilde{z}(a)$ and $\widetilde{\widetilde{z}}(a)$ is illustrated in Figure 1. In this figure, the $\widetilde{z}(a)$ is represented by the blue solid line and $\widetilde{\widetilde{z}}(a)$ is represented by the red dotted line.

The potential equilibrium (z, a) pair satisfies both equations. Therefore, it must be an intersection point of $\widetilde{z}(a)$ and $\widetilde{\widetilde{z}}(a)$. As shown in Figure 1, it is possible that these two functions equal at 2 points. However, there is a local maximum only when the slope of $\widetilde{\widetilde{z}}(a)$ is greater than the slope $\widetilde{z}(a)$ (negative semi-definite Hessian matrix), and the second order sufficient condition is satisfied. Thus, a unique equilibrium exists. □

Next, I turn my attention to the comparative statics of optimal decisions with respect to a change in the advertising cost θ. From first order conditions, it is clear that a change in advertising cost θ would not affect Equation (3), nor would it affect the decision on innovation stepsize z directly. However, a change in θ would change Equation (4), and a higher cost in advertising would reduce a firm's incentive to advertise, thereby indirectly reducing the incentive to innovate.

Proposition 2. *Under equilibrium with a single entry, z^s and a^s decrease, and l^s increases with respect to the advertising cost θ.*

Proof. This statement is obvious in the corner solutions for $a = 0$ and $a = T$. I then focus on the interior case. As shown in Figure 1, an increase in θ shifts $\widetilde{\widetilde{z}}(a)$ upward, while $\widetilde{z}(a)$ is unchanged. Due to the properties of both $\widetilde{\widetilde{z}}(a)$ and $\widetilde{z}(a)$, it is clear that z^s and a^s both decrease.

I next turn my attention to the optimal entry time l^s. Consider $\theta_1 < \theta_2$, and call the payoff maximizing (z, a) pair (z_1, a_1) and (z_2, a_2) respectively. Also define the "zero profit" waiting time l_1 and l_2. Holding all else equal, $V^s(z_2, a_2, l_1|\theta_2) < V^s(z_2, a_2, l_1|\theta_1)$, simply because $\theta_1 < \theta_2$. Then since (z_1, a_1) are payoff maximizing after entry, then $V^s(z_2, a_2, l_1|\theta_1) \leq V^s(z_1, a_1, l_1|\theta_1)$. By the zero profit condition, for all $i = 1, 2$, $V^s(z_i, a_i, l_i|\theta_i) = 0$. Then $V^s(z_2, a_2, l_1|\theta_2) < V^s(z_1, a_1, l_1|\theta_1) = V^s(z_2, a_2, l_2|\theta_2)$, and this implies that $l_1 < l_2$, hence proving the claim. □

This result is intuitive because a lowering in advertising cost θ raises the firm's incentive to advertise. Since advertising and innovation are complementary, innovation stepsize correspondingly increases upon entry. The raised profitability upon entry caused by increases in both advertising and innovation efforts prompts firms to enter at an earlier date. The immediate implication of Proposition 2 is that the innovation growth rate z/l increases as θ is lowered.

Next, this paper looks into the general case in which sequential entries are allowed.

4 Equilibrium with Sequential Entry

In contrast to the restricted model with single entry, the general model allows multiple firms to enter the market sequentially. Each one brings a new innovation into the market and generates industry growth.

A notion of the *Markov-perfect equilibrium* (MPE) for the model with sequential entry is defined in this section. Given the consumers' product choices, an MPE with at most two firms overlapping consists of:

1. The *Payoff Relevant* states are summarized by the vector $\{\bar{z}, \bar{a}\}$. At the time of a firm's entry, $\bar{z}$ and $\bar{a}$ are its predecessor firm's innovation stepsize and advertising respectively. Notice that the entry time of the predecessor firm is irrelevant to the new entry firm's decision-making; thus, I normalize the predecessor entry time to be $\bar{l} = 0$.

2. At the time of entry, a firm chooses innovation stepsize z and advertising length a to

maximize the entry value given $(\bar{z},\bar{a})$ and given a new firm enters the market following the entry time policy function $\widehat{l}(z,a)$[12].

$$\begin{aligned}
&\max_{z,a} \widehat{V}(z,a,l|\bar{z},\bar{a}) \\
&= \max_{z,a} \int_{t=0}^{\max\{T-l,0\}} e^{-\rho t}\bar{z}(1-\sigma)e^{-\phi\bar{a}} \cdot s(t)\ dt + \int_{t=0}^{\hat{l}(z,a)} e^{-\rho t} z \cdot s(t)\ dt \\
&- \int_{t=0}^{a} e^{-\rho t}\theta\ dt - Iz^{\gamma} - ke^{-\eta l} + VC(z,a)
\end{aligned}$$

The resulting optimal policy functions are denoted $\widehat{z}(\bar{z},\bar{a})$ and $\widehat{a}(\bar{z},\bar{a})$.

3. VC is the incumbent firm's value after a new firm's entry, given that the new firm follows the optimal policies $\widehat{z}(\cdot)$ and $\widehat{a}(\cdot)$:

$$\begin{aligned}
&VC(z,a) = \int_{t=\hat{l}(z,a)}^{T} e^{-\rho t} z s(t)(1-s'(t))\ dt \\
&\text{where:} \quad s'(t) = \begin{cases} 1-(1-\sigma)e^{-\phi t}; & \text{if } t \in [0,\widehat{a}(z,a)] \\ 1-(1-\sigma)e^{-\phi\widehat{a}(z,a)}; & \text{if } t \in (\widehat{a}(z,a),\widehat{l}(z,a)] \end{cases}
\end{aligned}$$

4. The Zero Profit condition holds:

$$\widehat{V}(\widehat{z},\widehat{a},l|\bar{z},\bar{a}) = 0$$

If there is more than one l satisfying the above condition, the entry time is the earliest l possible. The resulting policy on the entry time l is $\widehat{l}(\cdot)$.

The Markov-perfect equilibrium consists of policy rules $\{\widehat{z},\widehat{a},\widehat{l}\}$, and the value function $\widehat{V}$. For the analysis of this paper, I restrict my attention to a *Stationary* MPE. The *Stationary* MPE consists of $\{z^*,a^*,l^*\}$ and the value function V^*, where $z^* = \widehat{z}(z^*,a^*)$, $a^* = \widehat{a}(z^*,a^*)$,

[12]Here, I focus only on the case in which firms do not overlap in advertising. Parameters are such that a firm enters after the incumbent firm has stopped advertising.

$l^* = \widehat{l}(z^*, a^*)$ and $V^*(z^*, a^*, l^*) = \widehat{V}(z^*, a^*, l^*|z^*, a^*)$[13].

Under this notion of equilibrium, I first look into the entry blockaded case.

4.1 Blockaded Entry

For some parameters, the equilibrium solution in the single entry case $\{z^s, a^s, l^s\}$ also constitutes a Stationary Markov-perfect Equilibrium in the sequential entry case. In particular, this is the case when advertising cost θ is high enough to block all possible entry within the duration of an incumbent's patent length: this is the case of blockaded entry.

I first look at how the value function behaves under the blockaded entry case. Given that the successor firm can optimally enter the market only after its patent expiration, $\widehat{l}(z, a) \geq T$, a firm's value function V^b is defined as the following:

$$\begin{aligned} V^b(z, a, l|\bar{z}, \bar{a}) &= \int_{t=0}^{\max\{T-l,0\}} e^{-\rho t}\bar{z}(1-\sigma)e^{-\phi\bar{a}}s(t)\, dt + \int_{t=0}^{T} e^{-\rho t} z s(t)\, dt - \int_{t=0}^{a} e^{-\rho t}\theta\, dt - Iz^{\gamma} - ke^{-\eta l} \\ &= \int_{t=0}^{\max\{T-l,0\}} e^{-\rho t}\bar{z}(1-\sigma)e^{-\phi\bar{a}}s(t)\, dt + V^s \\ &= W(l)\bar{z}(1-\sigma)e^{-\phi\bar{a}} + V^s \end{aligned}$$

where $W(l) = \int_{t=0}^{\max\{T-l,0\}} e^{-\rho t}s(t)\, dt$.

I denote the optimal policies (z^b, a^b), which maximize V^b given predecessor's decisions $(\bar{z}, \bar{a})$. In addition, I denote l^b to be the entry time satisfying the zero profit condition. Then $\{z^b, a^b, l^b\}$ and V^b constitute a stationary Markov-perfect equilibrium if $\bar{z} = z^b$, $\bar{a} = a^b$, and an entry firm chooses $l^b > T$. This equilibrium only exists if the advertising cost is high enough. To characterize this equilibrium and make comparisons to other cases of a stationary MPE, I first introduce two definitions:

Definition 1. [14] Given that all firms choose z^b and a^b, $\overline{\theta}$ is the advertising cost at which for

[13] In general, a stationary Markov-perfect equilibrium exists following similar intuitions as those in Dorazelski and Satterthwaite (2003). However, the following analysis does not rely on their results.

[14] Notice that at $\theta = \overline{\theta}$, the zero profit waiting time l may not be equal to T, since the value function may not be monotone in the waiting time l. Thus, $\overline{\theta}$ is not the θ in which $l^s(\theta) = 0$.

all $\theta > \overline{\theta}$, the entry time $l^b > T$; and for all $\theta < \overline{\theta}$, the entry time $l^b < T$.

Implicit in the above definition is that l^b is monotone increasing in θ, or a firm's entry time gets delayed with rising advertising costs. This is true following the same intuition as in the single entry case, which is proven formally in Proposition 2.

Notice that V^b may be different from V^*, because in V^b, $\widehat{l}(z,a) \geq T$ is always the case, but in V^*, it is possible that $\widehat{l}(z,a) < T$. Therefore, similar to the definition of $\overline{\theta}$, I can define $\underline{\theta}$ for V^*:

Definition 2. Given that all firms choose z^* and a^*, $\underline{\theta}$ is the advertising cost at which for all $\theta > \overline{\theta}$, the entry time $l^* > T$; and for all $\theta < \overline{\theta}$, the entry time $l^* < T$.

Notice that $\overline{\theta} \geq \underline{\theta}$. This is because $V^* \leq V^b$ for any $\{z, a, l\}$. Next, I characterize the stationary MPE under the blockaded entry case:

Proposition 3. *For all $\theta > \overline{\theta}$, the policies $\{z^b, a^b, l^b\}$ and the value function V^b constitute a Stationary MPE, if and only if $z^b = z^s$, $a^b = a^s$, $l^b = l^s$ and $V^b(z^b, a^b, l^b) = V^s(z^s, a^s, l^s)$.*

Proof. Since $\theta > \overline{\theta}$, $l^b > T$, thus, there is no firm overlapping in the market at any time, so $V^* = V^s$. I have already shown in the single entry case, (z^s, a^s) maximizes the value function V^s given the entry time. Additionally, the entry time l^s satisfies the zero profit condition under the value function V^s. Hence, if $z^b = z^s$, $a^b = a^s$, $l^b = l^s$ and $V^b(z^b, a^b, l^b) = V^s(z^s, a^s, l^s)$, the policies $\{z^b, a^b, l^b\}$ and the value function V^b constitutes an MPE. The proof of stationarity is obvious.

I next show that if the policies $\{z^b, a^b, l^b\}$ and the value function V^b constitute a Stationary MPE, then $z^b = z^s$, $a^b = a^s$, $l^b = l^s$ and $V^b(z^b, a^b, l^b) = V^s(z^s, a^s, l^s)$. If $l > T$ and $V^* = V^s$, I have already shown that the equilibrium with (z^s, a^s, l^s) is unique. The only thing remaining to be shown is that for any $l < T$, a firm can only make a negative profit. This is clear, since by the definition of $\overline{\theta}$, for all $\theta > \overline{\theta}$, even if a firm optimally chooses z^b and a^b, it will still enter at $l^b > T$. □

Since $z^b = z^s$, $a^b = a^s$, $l^b = l^s$ and $V^b(z^b, a^b, l^b) = V^s(z^s, a^s, l^s)$, I derive the following corollary.

Corollary 1. *Under a Stationary Markov-perfect equilibrium with blockaded entry, z^b and a^b decrease, and l^b increases with respect to the advertising cost θ.*

If $\theta < \overline{\theta}$, a firm would have an incentive to enter the market before its predecessor has exited. Then a couple of questions arise: is it possible to strategically innovate and advertise in order to deter a successor's entry time? If this is possible, what is the implication for industry growth? Next, I introduce a notion of entry deterrence and answer the above questions.

5 Entry Deterrence

In some equilibrium settings, each entrant firm enters the market after the previous incumbent's patent expiration. However, dissimilar to the entry blockaded setting, each incumbent firm chooses to advertise and innovate differently from the levels of the single entry case (z^s, a^s). This deviation from the equilibrium optimality under no entry threat must be due to incumbents' strategic concerns of entry deterrence. Intuitively, an incumbent manipulate its own advertising in order to delay a new entry. In doing so, an incumbent gains a higher profitability.

In this section, I define a stationary MPE under entry deterrence, shows its existence, explores comparative statics with respect to the advertising cost θ, and compare the optimal decisions with the blockaded entry setting.

An *Entry Deterrence Equilibrium* is a stationary Markov-perfect equilibrium in which $l^* > T$, but $z^* \neq z^s$ and $a^* \neq a^s$.

In an entry deterrence setting, a firm chooses z and a not only to maximize its after entry profit, but also to force potential entrants to wait until after its patent expiration, or $\widehat{l}(z, a) > T$. In other words, given (z, a), no firm can make a non-negative profit by entering before T. To make sure of this, I first need to find out at what entry time $l < T$ a potential entrant attains a maximum value.

Lemma 1. *For any given pair of $(\bar{z}, \bar{a})$, V^b attains a unique local maximum at an entry time $l \in (0, T)$.*

Proof. See Technical Appendix. □

The above lemma shows that for $l < T$, V^b takes an "inverted U" shape, as depicted in Figure 2. This shape is the result of two opposing forces. When t is small, V^b increases in t because fixed entry costs are reduced sharply. Then V^b starts to decrease because the benefit of profiting from a lower quality numeraire good declines with the predecessor's patent expiration quickly approaching. I then can define the unique local maximum point $l^{\max}$ as the following:

Definition 3. For any such $(\bar{z}, \bar{a})$, V^b attains a local maximum at $l^{\max}(\bar{z}, \bar{a}) < T$.

Since this interior maximum point must be attained where the slope of V^b with respect to l is zero, l^{max} satisfies the following:

$$W'(l^{\max}(\bar{z}, \bar{a}))\bar{z}(1-\sigma)e^{-\phi\bar{a}} + k\eta e^{-\eta l^{\max}(\bar{z}, \bar{a})} = 0 \tag{5}$$

To deter entry, a firm must choose (z, a) so that:

$$V^b(z, a, l^{max}(z, a)) < 0 \tag{6}$$

Then an entering firm solves the following problem:

$$\max_{z,a} V^b(z, a, l) \qquad \text{subject to: Inequality (6)}$$

Denote (z^d, a^d) to be the optimal decisions solving the above problem, and $l^b > T$ satisfies the zero profit condition, in which:

$$V^b(z^d, a^d, l^d) = 0$$

Notice that Equations (5) and (6) define a functional relationship between z^d and a^d. We can call this function $z^d(a^d)$. Then the first order necessary condition for an interior maximum is:

$$\frac{\theta\rho}{\phi(1-\sigma e^{-\phi a})(1-e^{-\rho(T-a)})} z^d(a) - e^{-\rho a}\theta + \phi z^d(a)(\int_0^T e^{-\rho t}s(t)\, dt - I\gamma(z^d(a))^{\gamma-1}) = 0 \qquad (7)$$

Given that$\theta \in (\underline{\theta}, \overline{\theta}]$, z^d and a^d have to satisfy both Equations (5) and (6). Hence, it is easy to verify that $z^d \neq z^s$ and $a^d \neq a^s$. Moreover, $l^d > T$; therefore, $\{z^d, a^d, l^d\}$ and V^b constitute an Entry Deterrence Equilibrium. Next, I show that this equilibrium exists.

Assumption 1. $\left[\frac{\int_{t=0}^T e^{-\rho t}s(t)\, dt}{I\gamma}\right]^{\frac{1}{\gamma-1}} (1-\sigma)e^{-\phi a}$ is increasing in a.

Proposition 4. *Given that Assumption 1 holds, there exists an $\varepsilon > 0$ that is arbitrarily close to zero, and for a $\theta = \overline{\theta} - \epsilon$, an Entry Deterrence Equilibrium exists.*

Proof. Essentially, the above statement is equivalent to proving that $\overline{\theta} > \underline{\theta}$, or $\exists\theta \in (\underline{\theta}, \overline{\theta})$, such that $V^b(z^b, a^b, l^{\max}(z^b, a^b)|\theta) > 0 > V^*(z^*, a^*, l^{\max}(z^*, a^*)|\theta)$.

For any $\theta < \overline{\theta}$, $V^b(z^b, a^b, l^{\max}(z^b, a^b)) \geq V^b(z^b, a^b, l^{\max}(z^*, a^*))$, this is true by the definition of $l^{\max}(z^b, a^b)$. Also $V^b(z^b, a^b, l^{\max}(z^*, a^*)) \geq V^b(z^*, a^*, l^{\max}(z^*, a^*))$ because (z^b, a^b) are the optimal decisions given V^b. In addition, $V^b(z^*, a^*, l^{\max}(z^*, a^*)) > V^*(z^*, a^*, l^{\max})(z^*, a^*))$ because $l^{\max}(z^*, a^*) < T$. Then define $\chi(\theta) = V^b(z^*, a^*, l^{\max}(z^b, a^b)|\theta) - V^*(z^*, a^*, l^{\max}(z^*, a^*)|\theta)$. Then $\chi(\theta) > 0$ for any $\theta < \overline{\theta}$.

Furthermore, $V^b(z^b, a^b, l^{\max}) = 0$ for $\theta = \overline{\theta}$. Given Assumption 1, I know that $z^b(1-\sigma)e^{-\phi a^b}$ is decreasing in θ. Thus, for any $l < T$, $W(l)z^b(1-\sigma)e^{-\phi a^b}$ is decreasing in θ. In other words, V^b shifts up as the advertising cost θ is lowered, then $\exists\varepsilon > 0$ is small enough, so that $V^b(z^b, a^b, l^{\max}(z^b, a^b)|\overline{\theta} - \varepsilon) - V^b(z^b, a^b, l^{\max}(z^b, a^b)|\overline{\theta}) < \epsilon < \chi(\overline{\theta} - \varepsilon)$. This implies that $V^*(z^*, a^*, l^{\max}(z^*, a^*)|\overline{\theta} - \varepsilon) < 0$. Also, by definition of $\underline{\theta}$, $\overline{\theta} - \varepsilon > \underline{\theta}$. Hence, an Entry Deterrence Equilibrium exists at $\overline{\theta} - \varepsilon$. □

In fact, the above proposition has shown that the Entry Deterrence equilibrium exists for all $\theta \in (\underline{\theta}, \overline{\theta}]$. How value functions vary with respect to θ is illustrated in Figure 3. Next, I investigate the comparative statics within the Entry Deterrence Equilibrium with respect to changes in the advertising cost θ.

Proposition 5. *Given that $\theta \in (\underline{\theta}, \overline{\theta}]$, if θ decreases, and the model has an interior solution, the innovation stepsize z^d decreases, the advertising a^d increases, and the entry time l^d decreases, but is always greater than T, as well as the innovation growth rate z^d/l^d decreases.*

Proof. See Technical Appendix. □

As noted in the single entry case, advertising and innovation are complementary. An increase in one raises the marginal benefit of the other. Thus, as advertising cost is lowered, both advertising length a and innovation stepsize z go up. However, in the entry deterrence case, lowering advertising cost puts increasing pressure on the incumbent to delay entry. As an incumbent firm raises its advertising investment, this would effectively shrink the size of the new entrant exclusive market segment m_2. In doing so, the marginal profitability of innovation decreases, and each subsequent entrant innovates less. In addition, the value function for $l > T$ continues to rise as θ falls, causing l to decrease. Due to the significant loss in z, the overall effect on industry growth z/l is negative.

Next I compare $\{z^d, a^d, l^d\}$ to $\{z^b, a^b, l^b\}$, and show the effect of entry deterrence on firm decisions.

Assumption 2. $\int_{t=0}^{T} e^{-\rho t} s(t)\, dt - I\gamma(z^d(a))^{\gamma-1}$ is decreasing in a.

Proposition 6. *Under Assumptions 1 and 2, given that $\theta \in (\underline{\theta}, \overline{\theta}]$, $z^d < z^b$ and $a^d > a^b$. Also $l^d > l^b$. Thus, $z^d/l^d < z^b/l^b$.*

Proof. See Technical Appendix. □

Compared to the blockaded entry setting, a firm tends to over-invest in advertising in order to deter entry. As a result, the industry growth rate is lower in the entry deterrence case.

6 Numerical Analysis

The analysis so far has focused on the comparative statics of innovation stepsize and growth with respect to local changes in advertising cost. This section extends these local results by considering numerical examples that illustrate the effect of a global policy change.

The main focus of this section centers on how a comprehensive ban on advertising ($\theta = +\infty$) influences the market. The effects of an increase in advertising costs on innovation can be subtle. The analytical results have shown that innovation stepsize and growth can be reduced as advertising costs rise, as in the case of blockaded entry; however, innovation and growth can also be higher, as in the case of entry deterrence. Despite the ambiguity in local comparative statics, results from this section show that a complete ban on advertising always proves to be detrimental to growth, even compared to the case of entry deterrence.

In addition, the analysis in this section extends the model beyond the two outcomes on which this paper has focused so far. This section considers numerical examples that illustrate both of these possibilities, blockaded entry and entry deterrence, as well as a third case. In this third case, entry occurs sufficiently frequent that firms overlap [15]. The equilibrium comparative statics in this case are similar to those of blockaded entry. An improvement in advertising (i.e., a reduction in "θ") leads to higher advertising spending, higher innovation stepsize, shorter entry time, and higher industry growth. Finally, I can show that in the case of entry deterrence, firms over-invest in advertising compared to the case of blockaded entry.

I have verified that the qualitative results above on the following grid of parameters: patent length[16] $T \in [1, 20]$ with an increment size of 1; initial awareness level $\sigma \in [0.01, 0.5]$ with an increment size of 0.01; the advertising diffusion rate $\phi \in [0.01, 1]$ with an increment size of 0.01; the entry cost parameters $\gamma \in [2, 10]$ with an increment size of 0.5, $I \in [0.1, 2]$ with an increment size of 0.1, $k \in [0.01, 1]$ with an increment size of 0.01 and $\eta \in [1, 3]$ with an increment size of 0.01.

Moreover, in the model presented above, I use a specific functional form of innovation cost $I(z) = I \cdot z^{\gamma}$. In this section, I allow for more flexibility and have tested the model for the following functional form (the functional forms must satisfy the assumptions that $I(\cdot)$ is

[15] I do not present the analytical results of the overlapping case in this paper. Analytical results are possible with additional assumptions. Please contact the author if interested.

[16] I assume that the patent length is 20 years and firm discount future at 98.5% annually. Hence, for each patent length T, there is a corresponding discount rate. For example, if $T = 1$, a year is $\frac{1}{20}T$, so $\rho = 0.3$.

increasing and convex in stepsize z, with $I(0) = 0$ and the inverse of its derivative $I'^{-1}(\cdot)$ exists and it is weakly concave): $I(z) = I \cdot (e^{\gamma \cdot z} - 1)$ for the range of parameters $I \in [0.1, 2]$ with an increment size of 0.01 and $\gamma \in [1, 10]$ with an increment size of 0.5.

The qualitative implications are robust if the parameters and functional forms satisfy the following three conditions. First, Assumptions 1 and 2 are satisfied, these assumptions are needed to ensure that there is a region of entry deterrence. Second, the parameters must be such that no three firms overlap in their entry. Third, the parameters must be such that no two firms overlap in their advertising. All of the results are qualitatively the same to the following representative example.

For the example I illustrate here, the parameters are the following: the patent length is set to be $T = 1$; the discount rate $\rho = 0.3$; the entry cost functional form is $0.6 \cdot z^2 + 0.06 \cdot e^{1 \cdot l}$; the advertising diffusion rate $\phi = 0.3$, and the initial awareness level $\sigma = 0.2$. I vary the advertising cost θ in the interval $[0.005, 0.015]$, with an increment size of 0.0005.

Figure 3 illustrates the evolution of the value function shifts as the advertising cost is lowered. Specifically, I compare the value functions in the blockaded case V^b and in the stationary Markov-perfect equilibrium V^*, when innovation stepsize and advertising are optimally chosen. Here V^b and V^* are both functions of the entry time l. In case (a), $\theta_1 > \overline{\theta}$, $V^* = V^b$, and all firms enter at $l > T$. In cases (b) and (c), $\theta \in (\underline{\theta}, \overline{\theta})$, firms deter entry, as a result, $V^* = V^d < V^b$, and firms' entry times are delayed to $l > T$. Comparing cases (b) and (c), $\theta_2 > \theta_3$, V^d is distorted further and further away from V^b, and the entry time l gets closer and closer to T. In case (d), $\theta_4 < \underline{\theta}$, two firms overlap in the market, $V^* < V^d$, $l < T$.

For each equilibrium indicated in Figure 3, I can determine the optimal innovation stepsize z, advertising a, entry time l, and innovation growth rate z/l. Figure 4 shows the comparative statics of these quantities with respect to advertising cost θ. In the figures, advertising cost is on the x-axis and is in reverse order; hence, the advertising costs decreases from left to right. Clearly, Figure 4 shows the optimal quantities under a complete advertising ban. It is clear that firms innovate at a much lower stepsize, and the entry is infrequent. The resulting industry growth z/l is much less even than in the entry deterrence case.

In addition, for this particular example, $\overline{\theta}$ is approximately 0.0115 and $\underline{\theta}$ is approximately 0.0075. As shown, if $\theta > \overline{\theta}$, or in the blockaded case, as the advertising cost θ drops, z increases, a increases, l decreases and z/l increases as shown in the analytical results. When $\theta < \underline{\theta}$, in the overlapped entry case, as θ decreases, the comparative statics are exactly the same as they are in the blockaded case. The exception is when $\theta \in (\underline{\theta}, \overline{\theta}]$, in which entry deterrence occurs. As a result, as θ decreases, z decreases, and a increases, l decreases but kept above T. Industry growth z/l decreases.

Finally, Table 1 compares $\{z^b, a^b, l^b\}$ with $\{z^d, a^d, l^d\}$ for those advertising costs $\theta \in (\underline{\theta}, \overline{\theta}]$. The important conclusion to keep in mind is that firms under entry threat, over-invest in advertising so as to keep rival entry time $l > T$. As a result, the industry growth slows down for all of these θ's.

7 Conclusion

This paper studies the effect of firms' strategic behavior in advertising on industry innovation. A novel feature of this model is the assumption of perfect price discrimination. By allowing firms to price discriminate across different groups of consumers, this paper presents a fully tractable model and analytically characterizes equilibrium behavior under entry deterrence in a dynamic setting.

I found in this paper that advertising can act as tool for entry deterrence. As a result, as advertising technology improves, it is possible that the industry innovation rate will slow down. In addition, this paper shows that although it is locally possible that industry growth improves as advertising becomes worse, a complete ban on advertising always reduces industry growth.

For future research considerations, this paper can consider the different effects that advertising may have on consumer perceptions (other than the awareness effect), such as the prestige effect, the brand name effect, etc. In this paper, consumers are assumed to be passive at all times. Ackerberg (2003) provides a good example as to how one may incorporate different effects of advertising into a consumer-learning environment. In addition, it would be of interest to extend this model to include differentiated products (i.e. Grossman and Shapiro (1984)) and directions of technological innovation (i.e. Mitchell and Skrzypacz (2006)). Fleshing out these issues would surely require more detailed and complex modeling than has been attempted in this paper.

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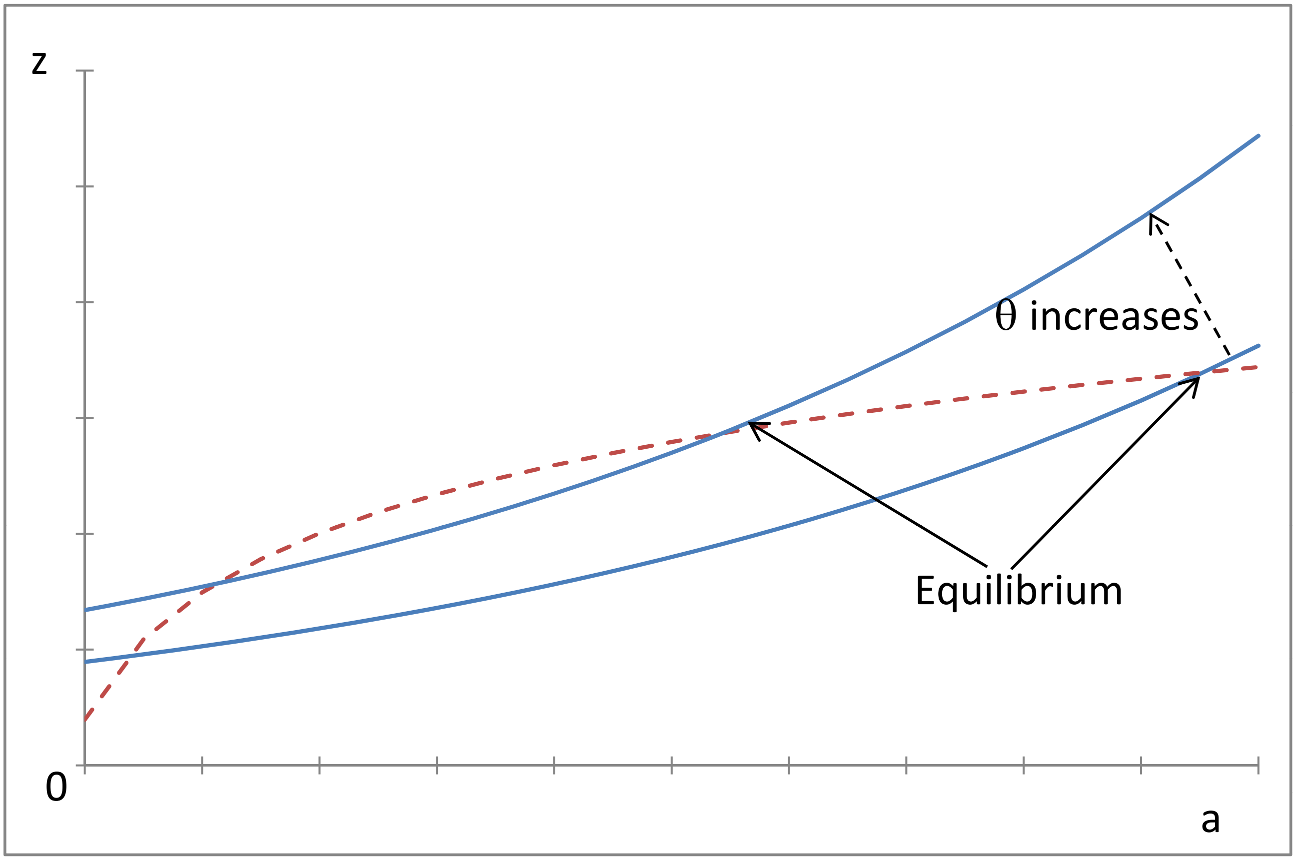


Figure 1: Functions $\widetilde{z}(a)$ (Solid) and $\widetilde{\widetilde{z}}(a)$ (Dotted)

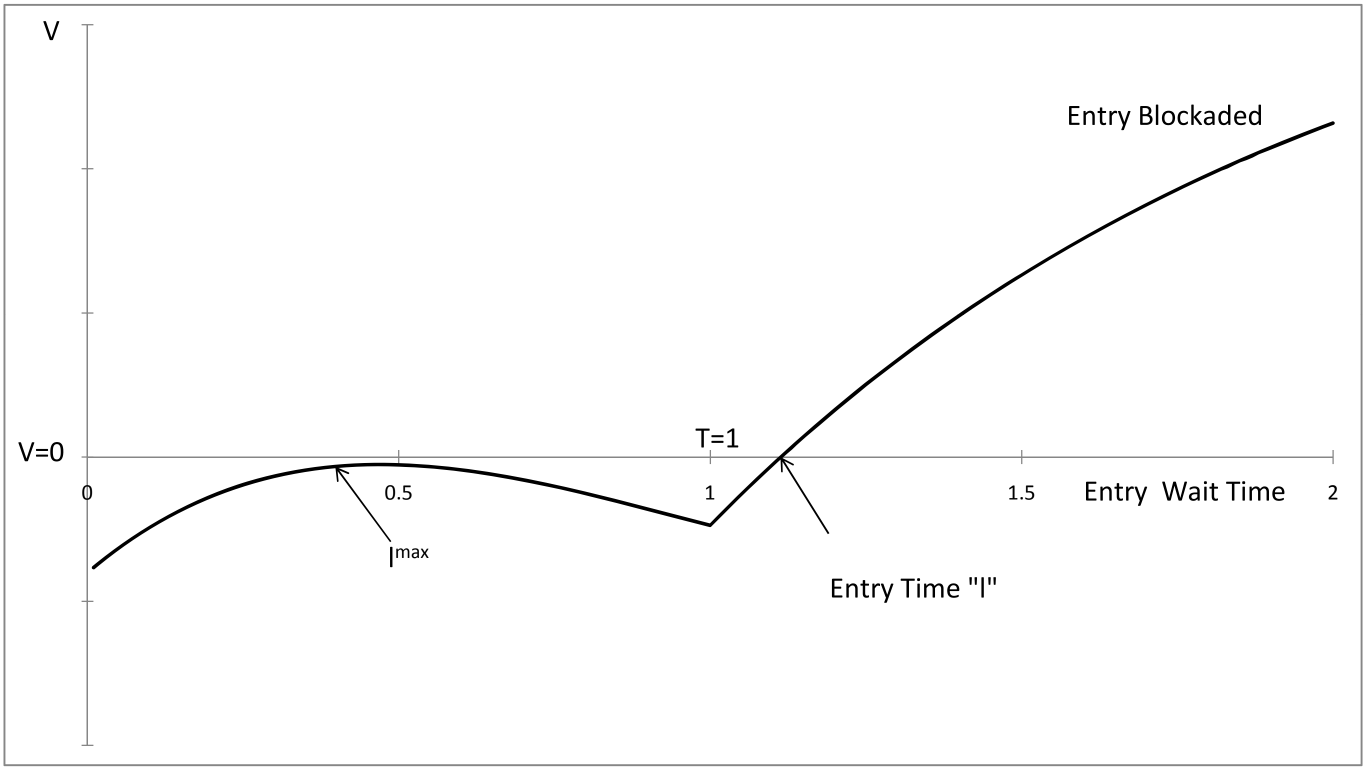
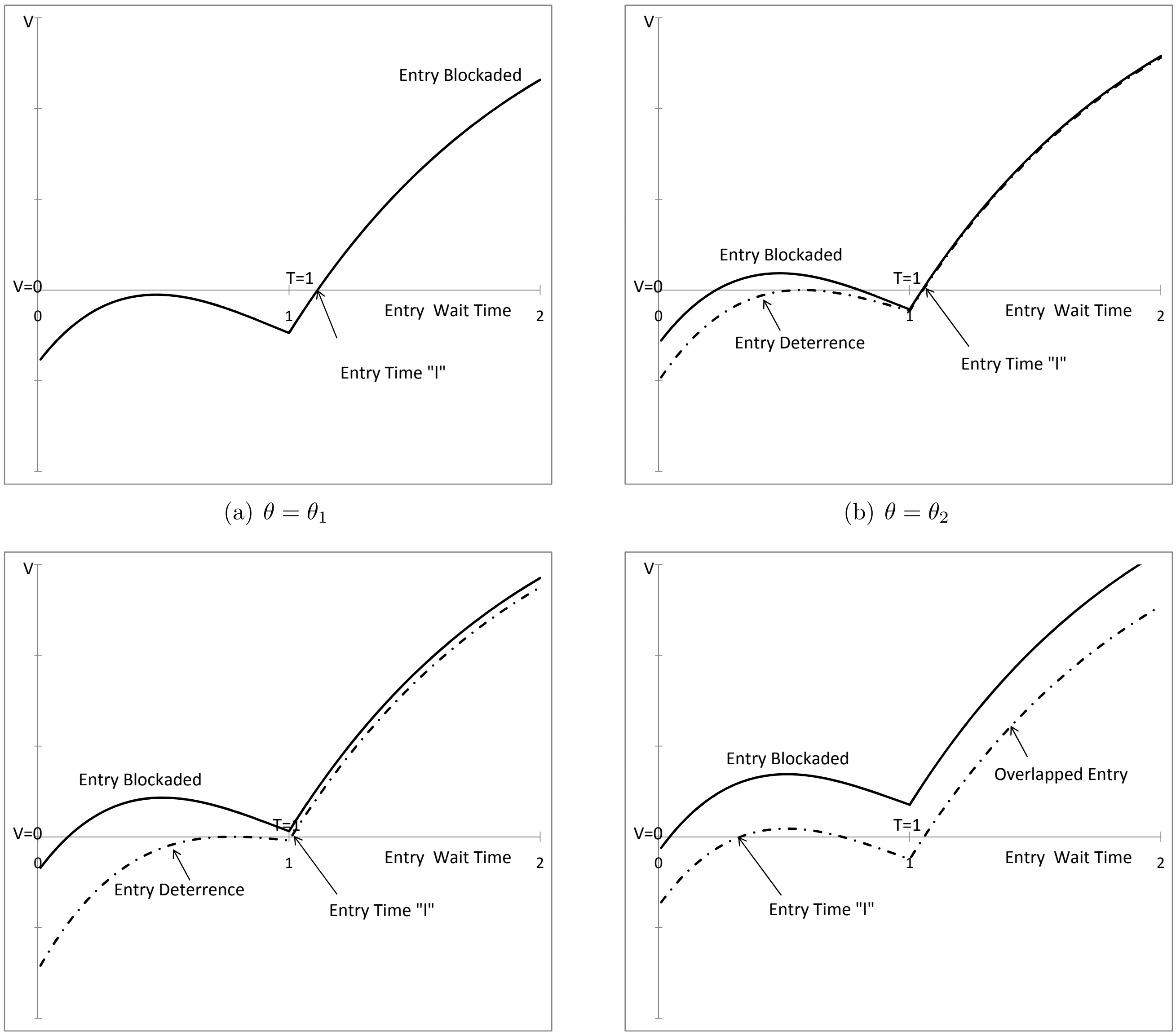


Figure 2: Value Function with Blockaded Entry: V^b



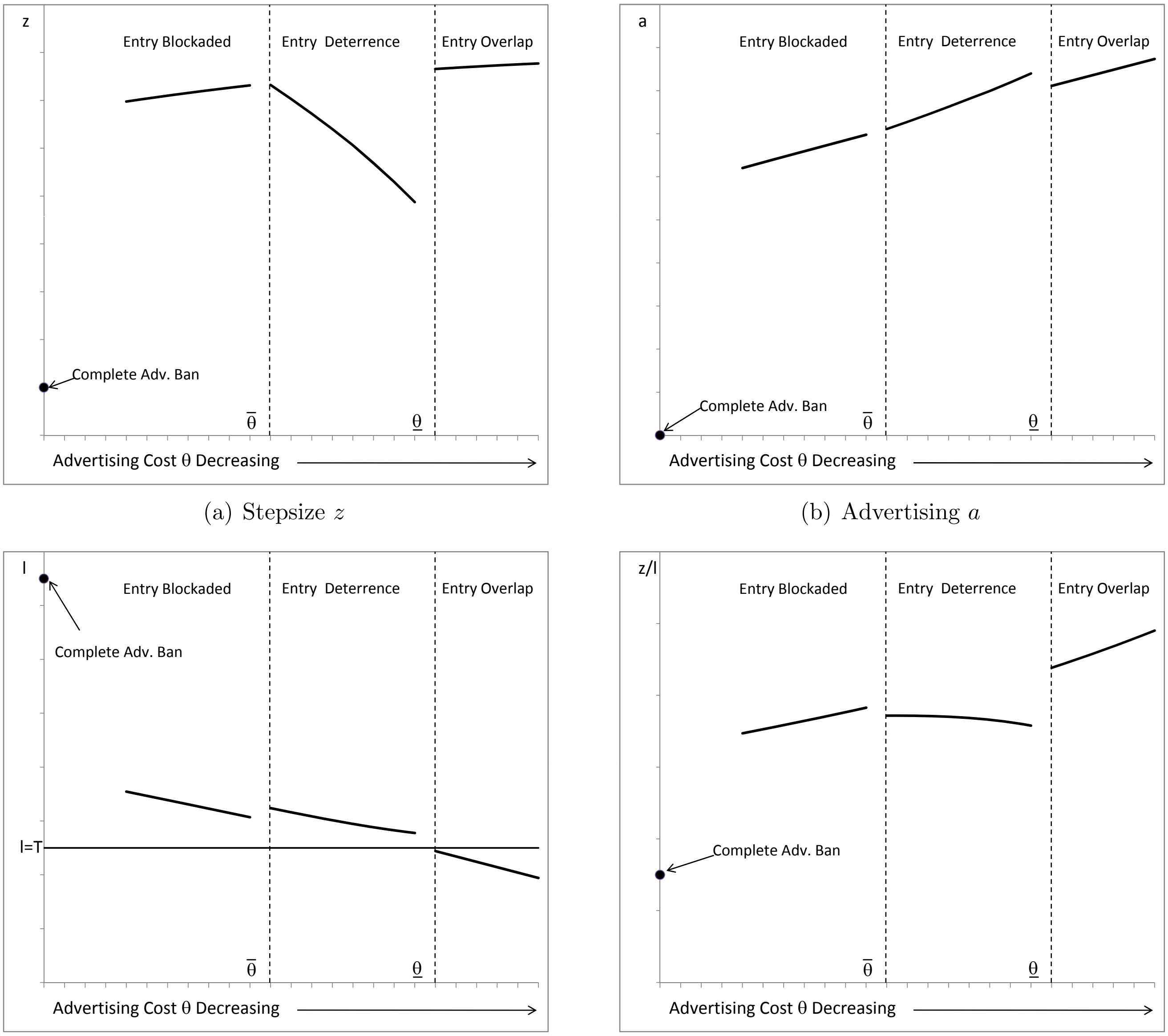
(a) $\theta = \theta_1$

(b) $\theta = \theta_2$

(c) $\theta = \theta_3$

(d) $\theta = \theta_4$

Figure 3: Entry Value as a Function of Entry Time l given ($\theta_1 > \theta_2 > \theta_3 > \theta_4$)



(a) Stepsize z

(b) Advertising a

(c) Entry l

(d) Innovation Growth z/l

Figure 4: Optimal Policy Function With Decreasing Advertising Cost

Blockaded Entry				
θ	z	a	l	z/l
0.0115	0.2136	0.31016	0.38826	0.55015
0.011	0.21404	0.3228	0.31806	0.67297
0.0105	0.21447	0.33542	0.25128	0.85352
0.01	0.21487	0.34801	0.21996	0.97687
0.0095	0.21525	0.36057	0.19148	1.1241
0.009	0.2156	0.37312	0.16541	1.3035
0.0085	0.21594	0.38565	0.14141	1.527
0.008	0.21625	0.39817	0.11919	1.8143

Entry Deterrence				
θ	z	a	l	z/l
0.0115	0.21324	0.31069	1.1478	0.18579
0.011	0.21029	0.32735	1.1321	0.18575
0.0105	0.20721	0.34456	1.117	0.18551
0.01	0.20399	0.36237	1.1026	0.18501
0.0095	0.20063	0.38087	1.0889	0.18424
0.009	0.19688	0.39897	1.0763	0.18292
0.0085	0.19298	0.41876	1.0649	0.18121
0.008	0.18874	0.43957	1.0551	0.17889

Table 1: Contrasting Entry Deterrence with Blockaded Entry

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Technical Appendix

Accumulation of Product Awareness Follows Poisson Process

Assume that the number of times a particular product's advertising information reaches a consumer in a fixed period of time follows a Poisson distribution. For a particular product entered the market in time r, for all $t \in [r, r+T]$, let $\lambda(t) = \int_{j=r}^{t} A(j)\phi \, dj$, then $prob(H(t) = x) = \frac{e^{-\lambda(t)}\lambda(t)^x}{x!}$. $A(j)$ is an indicator function of advertising. $A(j) = 1$ indicates that the firm advertises in period j, $A(j) = 0$ if otherwise. $\phi > 0$ is the exogenous diffusion rate of per unit time advertising. $H(t)$ is the number of times a consumer was exposed to the advertising up until period t. Therefore the total fraction of consumers see the advertisement at least once at t is the firm's "consumer awareness" at t.

$$s(t) \quad = \quad \sigma + (1 - e^{-\lambda(t)})(1 - \sigma) = 1 - (1 - \sigma) \cdot e^{-\lambda(t)}$$

In general, the above equation can be written for any give $t' > t$. Where

$$s(t') = 1 - (1 - s(t)) \cdot e^{-\int_t^{t'} A(j)\phi \, dj}$$

The above equation also provides the law of motion for the accumulation of market knowledge.

$$\dot{s}(t) = (1 - s(t)) \cdot A(t)\phi$$

This equation indicates that the return to advertising is marginally decreasing.

Proof: Advertising function $A(\cdot)$ can be replaced by a

Proof. I prove this proposition by constructing a step function with $A(t) = 1 \ \forall t < a$ and $A(t) = 0$ otherwise. I show that this function attains weakly higher value than any advertising function for a given pair of z and l. Since z and l are both fixed for the following analysis, I only consider the value function without the innovation cost $I(z) + ke^{-\eta l}$. In **Blockaded Entry Case**, first of all, consider at any $t \in [r, r+T)$. Given the value function of the firm after t is:

$$\int_{i=0}^{T-t} e^{-\rho i} \left[z \left(1 - (1 - s(t)) \cdot e^{-\int_{j=0}^{i} A(j+t)\phi \, dj} \right) - A(i+t)\theta \right] \, di$$

Claim 1: Firm would never advertise for just a instant of length zero, because by not advertising at all, firm would be strictly better off by saving $\theta > 0$ in that period and keep the same market size.

Claim 2: Now suppose firm advertises $\varepsilon > 0$ continuously, where $\varepsilon < T + r - t$. Firm is always better off by advertising in the periods $[t, t+\varepsilon]$, or by not advertising at all. Define the value gain/loss from delaying the advertising $\Delta \geq 0$ by (advertising during periods $[t+\Delta, t+\varepsilon+\Delta]$):

$$\begin{aligned} V^d(\Delta) &= \int_0^{\varepsilon} e^{-\rho i} \left(z(1 - (1 - s(t)) \cdot e^{-i\phi}) - \theta \right) \, di \\ &- \int_{\Delta}^{\Delta+\varepsilon} e^{-\rho s} \left(z(1 - (1 - s(t)) \cdot e^{-(i-\Delta)\phi}) - \theta \right) \, di \end{aligned}$$

After period $t + \varepsilon + \Delta$, $s(\cdot)$ and $A(\cdot)$ becomes the same for both schemes, so the values cancel out.

Next, I investigate the curvature of $V^d(\Delta)$ by taking first order derivative:

$$V^{d\prime}(\Delta) = e^{-\rho\Delta} \cdot \left[\frac{\phi z(1 - s(t))}{\rho + \phi} \cdot (1 - e^{-\varepsilon(\rho+\phi)}) - (1 - e^{-\rho\varepsilon}) \cdot \theta \right]$$

So if $\frac{\phi z(1-s(t))}{\rho+\phi} \cdot (1 - e^{-\varepsilon(\rho+\phi)}) - (1 - e^{-\rho\varepsilon}) \cdot \theta \geq 0$, $V^d(\Delta)$ is increasing and concave, otherwise, it is decreasing and convex. So the minimum is either $V^d(0)$ or $V^d(\infty)$, this is equivalent as saying comparing to any other schemes, firm is always better off by advertising in the periods $[t, t+\varepsilon]$ if $\frac{\phi z(1-s(t))}{\rho+\phi} \cdot (1 - e^{-\varepsilon(\rho+\phi)}) - (1 - e^{-\rho\varepsilon}) \cdot \theta \geq 0$; otherwise firm is better off by indefinitely delaying advertising thus by not advertising at all.

Now, given any $A(\cdot)$ function, we can construct a step function that is superior in the following way: First, by *Claim 1* I can take out all the instants of advertising. Then starting backwards from $t = r + T$ and going towards r, if continuous periods of advertising occur after t, use the criteria established in *Claim 2*, I can either delete advertising or make firm advertise starting at t. Continue this process until I hit $t = r$, the resulting function $A'(\cdot)$ must be at least weakly superior to $A(\cdot)$. And by construction, $\exists a \in [r, r+T]$ that $A'(t) = 1$, $\forall t \le a$ and $A'(t) = 0$, $\forall t > a$.

In **Overlapping Entry Case**, consider at any $t \in [r, T + r^1)$. The cumulative value of a firm in $[t, T + r^1)$ is:

$$\int_{i=0}^{T+r^1-t} e^{-\rho i} \left[\left(z + z^1 \cdot s^1(i+t)\right) \cdot \left(1 - (1 - h(t)) \cdot e^{-\int_{j=0}^{i} A(j+t)\phi \, dj}\right) - A(i+t)\theta \right] \, di$$

where $s^1(i+t) = 1-(1-s^1(t))\cdot e^{-\int_{j=0}^{i} A^1(j+t)\phi \, dj}$. Since both z^1 and A^1 are exogenous to the firm's decision, the same argument for **Blockaded Entry Case** (with z replaced by $(z+z^1\cdot s^1(i+t))$) can be used to show: *Claim 1*: If the firm advertise a cumulatively positive amount $\varepsilon > 0$ during $[r, T + r^1)$, it's always weakly better to advertise in $[r, r + \varepsilon]$.

Now suppose that $z^1 \cdot s^1(i + t) = 0$. Consider at any $t \in [r, r^2)$. The cumulative value of a firm in $[t, r^2)$ is:

$$\int_{i=0}^{r^2-t} e^{-\rho i} \left[z \cdot \left(1 - (1 - s(t)) \cdot e^{-\int_{j=0}^{i} A(j+t)\phi \, dj}\right) - A(i+t)\theta \right] \, di$$

then with exactly the same argument as above, if firm advertise at all during this period of time, say $\varepsilon > 0$, firm is always weakly better to advertise in $[r, r+\varepsilon]$. Now since $z^1\cdot s^1(i+t) > 0$, there is additional incentive for the firm to advertise early, so the following is true:

Claim 2: If the firm advertise a cumulatively positive amount $\varepsilon > 0$ during $[r, r^2)$, it's always weakly better to advertise in $[r, r + \varepsilon]$.

For advertising decisions in period $[r^2, T + r]$, the cumulative value of the firm in $[r^2, T + r]$

can be written as:

$$\int_{i=r^2(z,A)}^{T+r} e^{-\rho i - r} \left[\left(z \cdot (1 - s^2(i + r^2)) \right) \cdot \left(1 - (1 - s(r^2)) \cdot e^{-\int_{j=r}^{i} A(j)\phi \, dj} \right) - A(i + r^2)\theta \right] \, di$$

where $1 - s^2(i + r^2) = (1 - \sigma) \cdot e^{-\int_{j=r^2}^{i} A^2(n;z,A)\phi \, dj}$. Since I have shown in *Claim 1* that the successor firm would advertise in the beginning of its product's life, I can use $a^2(z, A)$ to represent $A^2(n; z, A)$ and write:

$$1 - s^2(i + r^2) = (1 - \sigma) \cdot \begin{cases} e^{-(i - r^2)\phi}, \text{ if } i - r^2 \leq a^2(z, A) \\ e^{-a^2(z,A)\phi}, \text{ if otherwise} \end{cases}$$

Suppose now $1 - s^2(i + r^2) = 1$ for all $i > r^2$, and suppose that θ is large enough that the firm advertises less than $T/2$ (in essence, no accommodated entry of advertising), then regardless of what r^2 is, I can use the argument in **Blockaded Entry Case** to show that if firm advertise at all during $[r, T + r]$, say $\varepsilon > 0$, firm is always weakly better to advertise in $[r, r + \varepsilon]$. Now, I know that $1 - s^2(i + r^2) \leq 1, = 1$ for all $i \in [L + r^1, r^2)$ and < 1 for all $i \in [r^2, r + L]$, so actually regardless of where r^2 is, the firm would actually have at least weakly less incentive to advertise. So *Claim 3* holds true:

Claim 3: Let θ be large enough, so that the firm advertises less than $T/2$ (in essence, no accommodated entry of advertising). Then regardless of what r^2 is, if the firm advertise a cumulatively positive amount $\varepsilon > 0$ during $[r, r + T]$, it's always weakly better to advertise in $[r, r + \varepsilon]$.

Now it only remains to see that if θ small and accommodated entry of advertising occurs, whether the firm would choose to advertise in the beginning of its successor's lifetime. Since $1s^2(i + r^2)$ is weakly decreasing, it has to be true that:

Claim 4: If the firm advertise a cumulatively positive amount $\varepsilon > 0$ during $[r^2, r+T]$, it's always weakly better to advertise in $[r^2, r^2+\varepsilon]$. Combine *Claim 3* and *Claim 4*, use the same backward construction method described in **Blockaded Entry Case**, the statement of **Overlapping**

Entry Case is proven. □

Proof of Lemma 1

Proof. Define the slope of V^b with respect to l to be:

$$V_l(l) = W'(l)\bar{z}(1-\sigma)e^{-\phi\bar{a}} + k\eta e^{-\eta l}$$

Where $W'(l) = -e^{-\rho(T-l)}(1-(1-\sigma)e^{-\phi(T-l)})$. Then:

$$V_l(0) = -e^{-\rho T}(1-(1-\sigma)e^{-\phi T})(1-\sigma)\bar{z}e^{-\phi\bar{a}} + k\eta e^{-\eta l}$$
$$V_l(T) = -\sigma(1-\sigma)\bar{z}e^{-\phi\bar{a}} + k\eta e^{-\eta l}$$

To prove the "Inverted U" Shape of V^b hold for any $\bar{z}$ and any $\bar{a}$, I assume the corner solutions $a = T$ and $a = 0$ hold for the above two conditions respectively. Given $\widetilde{z}(a) = \left[\frac{\int_{t=0}^{T} e^{-\rho t} s(t)\, dt}{I\gamma}\right]^{\frac{1}{\gamma-1}}$, then:

$$V_l(0) = -e^{-\rho T}(1-(1-\sigma)e^{-\phi T})(1-\sigma)\widetilde{z}(T) + k\eta e^{-\eta l}$$
$$V_l(T) = -\sigma(1-\sigma)\widetilde{z}(0) + k\eta e^{-\eta l}$$

Then $V_l(0) > -\sigma\widetilde{z}(T) + k\eta e^{-\eta l} > 0$. This is true because the assumption that no firm can make a positive profit entering at $l = 0$. This shows that the slope of V^b is positive at $l = 0$.

Also if it is possible for a firm to enter at a $l < T$ at all, it must be true that the entry fixed cost is decreasing fast enough so that at the moment of predecessor firm's expiration, firm 2 can earn strictly positive profit, at least momentarily. This implies that

$$\frac{(1-e^{-\rho t})\sigma}{\rho I\gamma}^{1/(\gamma-1)} \sigma(1-\sigma)e^{-\phi T} > k\eta e^{-\eta T}$$

which in turn implies that $V_l(T) < 0$. Since V_l is continuous, then $V_l = 0$ for some $l \in (0, T)$.

Furthermore, since both $W(l)$ and V^s are monotone in l, there must be a unique l at which $V_l = 0$, and V^b attains a local maximum. $\square$

Proof: Proposition 5

Proof. Define:

$$R(a) = \int_{t=0}^{T} e^{-\rho t} s(t) dt$$

Also define $\hat{q}^d = z^d(1-\sigma)e^{-\phi a^d}$. Further define:

$$\chi^d(a|\hat{q},\theta) = R(a) \cdot \frac{\hat{q}}{1-\sigma} e^{\phi a} - \int_{t=0}^{a} e^{-\rho t}\theta \, dt - I \cdot (\frac{\hat{q}}{1-\sigma} e^{\phi a})^\gamma$$

Claim 1: $\hat{q}^d$ is decreasing as θ is lowered. Suppose $\exists \theta_1 > \theta_2$, where both θ's are in $(\underline{\theta}, \overline{\theta}]$, and $\hat{q}_1 < \hat{q}_2$. By definition of z_2 and a_2, I know that $V^b(z_2, a_2, l|\theta_2) < 0$ for any $l \leq T$, so a firm would not enter before T. But by deviating to z_1 and a_1, it's clear that at $l_1^{\max} < T$, $\widehat{V}(z_1, a_1, l_1^{\max}|z_2, a_2, \theta_2) > V^b(z_1, a_1, l_1^{\max}|\theta_1) = 0$, so given $\hat{q}_1 < \hat{q}_2$, a successor firm can find a profitable deviation to enter before T. This is a contradiction to the fact that z_2 and a_2 are optimal under entry deterrence, hence $\hat{q}_1 \geq \hat{q}_2$. Now suppose $\hat{q}_1 = \hat{q}_2$, then z and a would remain the same. Since θ is lower, at l_1, $\widehat{V}(z_1, a_1, l_1^{\max}|z_1, a_1, \theta_2) > 0$, this violates the entry deterrence conditions. So $\hat{q}_1 > \hat{q}_2$.

Claim 2: z is decreasing and a is increasing as θ is lowered. From the first order condition of the enter deterrence problem, I have:

$$\phi z^d = -\frac{\partial V/\partial a}{\partial V/\partial z} = -\frac{R'(a^d)z^d - e^{-\rho a^d}\theta}{R(a^d) - I\gamma(z^d)^{\gamma-1}}$$

Assume z is increasing instead. At $\overline{\theta}$, it's true that $\partial V^b/\partial a = \partial V^b/\partial z = 0$. Then since z is increasing, $\partial V/\partial a > 0$ and $\partial V/\partial z < 0$. Also because $\hat{q}$ is decreasing by *Claim 1*, then a is increasing. Notice that $z^d = \frac{\hat{q}^d}{1-\sigma} e^{\phi a}$, I have:

$$\frac{\partial V^b}{\partial a} = e^{-\rho a^{det}} \cdot \left[\int_0^{T-a^{det}} e^{-\rho t} dt \cdot \phi \hat{q}^{det} - \theta \right]$$

Therefore, $\frac{\partial V^b}{\partial a}$ is decreasing in magnitude and $\frac{\partial V^b}{\partial z}$ is increasing in magnitude due to increase in z^d, hence z^d decreases, this gives a contradiction. So z^d is decreasing. Given z^d is decreasing, then $\partial V^b / \partial a < 0$ and $\partial V^b / \partial z > 0$. Suppose that a^d is decreasing, then by the similar reasons as above, I have a contradiction. Hence a^d is increasing.

Claim 3: l is decreasing as θ is lowered. Since $\hat{q}$ is decreasing, $\chi^d(a|\hat{q}, \theta)$ must be increasing as θ is lowered. Or else, for any l, $V^b(z_1, a_1, l|\theta_1) > V^b(z_2, a_2, l|\theta_2)$, violating equation (6) and the *Zero Profit Condition*. Since $\chi^d(a|\hat{q}, \theta)$ increases, it must be true $l > L$ is decreasing, since $W(l)\hat{q} = 0$. As depicted in Figure 3., the entry time under Entry Deterrence decreases from t_1 to t_2, as advertising cost is reduced from θ_1 to θ_2.

Claim 4: z/l is decreasing when θ is decreasing. First of all, by *Claim 3*, l decreases as $\hat{q}$ decreases, it's easy to show that $\hat{q}/l$ is decreasing as θ is lowered. On the other hand, $e^{\phi a} \cdot l$ is decreasing. So $e^{-\phi a}/l$ is increasing. Hence, combining the two facts z/l is decreasing. □

Proof: Proposition 6

Proof. Define:

$$R(a) = \int_{t=0}^{T} e^{-\rho t} s(t) dt$$

Since $\theta \in (\underline{\theta}, \overline{\theta}]$, I know that $l^d > T > l^{max}(z^b, a^b) \geq l^b$.

Next I focus on z and a. Because z^b and a^b maximize after entry payoff, so for any l, $V^b(z^b, a^b, l) \geq V^b(z^d, a^d, l)$.

First of all, $z^d \cdot (1-\sigma) e^{-\phi a^d} < z^b \cdot (1-\sigma) e^{-\phi a^b}$. This is immediate, because an incumbent firm can only reduce new entrant's value by decreasing $z \cdot (1-\sigma) e^{-\phi a}$. If on the contrary $z^d \cdot (1-\sigma) e^{-\phi a^d} \geq z^b \cdot (1-\sigma) e^{-\phi a^b}$, a new entrant's value to enter early is weakly increased, rendering entry deterrence unsuccessful. This eliminate the case where $z^d > z^b$ and $a^d < a^b$.

Then notice that $R'(a)z = z \cdot (1-\sigma)e^{-\phi a}\phi\frac{e^{-\rho a}-e^{-\rho T}}{\rho}$. Since (z^b, a^b) satisfies equation (4), it must be true that $z^b \cdot (1-\sigma)e^{-\phi a^b} = \frac{\theta\rho}{\phi(1-e^{-\rho(T-a^b)})} > z^d \cdot (1-\sigma)e^{-\phi a^d}$. Now suppose $z^d > z^b$, then since $z^d \cdot (1-\sigma)e^{-\phi a^d} < z^b \cdot (1-\sigma)e^{-\phi a^b}$, it must be true that $a^d > a^b$. In addition, by *Assumption 1*, I have $R(a^d) - I\gamma(z^d)^{\gamma-1} < R(a^*) - I\gamma(z^b)^{\gamma-1} = 0$. Then by first order necessary condition of the entry deterrence problem, this would imply that $\frac{\theta\rho}{1-e^{-\rho(T-a^b)}} > z^d \cdot (1-\sigma)e^{-\phi a^d} > \frac{\theta\rho}{1-e^{-\rho(T-a^d)}}$ hence $a^d < a^b$, a contradiction. So $z^b > z^d$.

Now suppose that $a^d < a^s$. From the first order necessary condition of the entry deterrence problem, I have $e^{-\rho a}[(1-\sigma)e^{-\phi a}(1-e^{-\rho(T-1)})/\rho - \theta] = -(R(a) - I\gamma(z)^{\gamma-1})$. LHS is a decreasing function of a. Since this function evaluated at a^b is zero by equation (4), I know that this function evaluated at a^d is greater than zero. Hence $R(a^d) - I\gamma(z^d)^{\gamma-1} < 0$. Then by *Assumption 2* and the fact that $z^d \cdot (1-\sigma)e^{-\phi a^d} < z^b \cdot (1-\sigma)e^{-\phi a^b}$, I have $z^d > z^b$, a contradiction. So $a^d > a^b$.

Since $z^d < z^b$ and $l^d > l^b$, it's clear that z/l is smaller in entry deterrence setting.

□