Growth and Development

Preliminary Examination

Fall 2006

Instructions: Answer two (2) questions from Section I and two (2) questions from Section II. Make whatever additional assumptions you think are necessary.

Section I: Short Questions

Answer *two (2)* questions from this section.

Question 1

Consider a two country world in which there is mass one of identical households in each country. Index the two countries by F and H. Suppose that there are productivity differences between the two countries so that output in country i in period t is given by:

$$y_{it} = A_i k_{it}^{\alpha} n_{it}^{1-\alpha}$$

Households maximize utility according to the utility function:

$$\sum_t \beta^t u(c_t);$$

Note that labor is inelastically supplied. Assume that its value is $n_{it} = 1$ for all (i, t). Investment and consumption goods are completely mobile, but labor is fixed.
Thus, feasibility for this world economy is given by:

$$c_{Ft} + c_{Ht} + x_{Ht} + x_{Ft} \leq A_H k_{Ht}^{\alpha} n_{Ht}^{1-\alpha} + A_F k_{Ft}^{\alpha} n_{Ft}^{1-\alpha};$$

$$k_{Ht+1} \leq (1-\delta) k_{Ht} + x_{Ht};$$

$$k_{Ft+1} \leq (1-\delta) k_{Ft} + x_{Ft};$$

In everything below assume that all allocations are interior/ the non-negativity constraints on investments do not bind.

Assume that the representative agent in each country owns 50% of the initial stocks of capital in each country.

Assume that $A_H > A_F$.

(a) In which country is more capital employed in the competitive equilibrium?

(b) In which country is consumption higher in the competitive equilibrium?

Make whatever extra assumptions you think are necessary, but carefully spell them out. Also, fewer assumptions are better.

Question 2

"Does inventory investment amplify cyclical fluctuations in GDP?" Using your knowledge of a general equilibrium model where inventories arise endogenously, explain why the answer to this question may hinge critically on capital's share of output.

Question 3

Consider a model with an infinitely lived representative agent, with lifetime utility

$$\sum_{t=1}^{\infty} \delta^t \frac{1}{1-\sigma} c_t^{1-\sigma}$$

where $\delta \in (0,1)$ and $\sigma > 0$. Technology is given by

$$y_t = A k_t^{\alpha} h_t^{1-\alpha}$$

where k is physical, h is human capital and A is a positive parameter. Feasibility requires

$$c_t + k_{t+1} + h_{t+1} \leq y_t + (1-\mu)k_t + (1-\mu)h_t$$

and the non-negativity of c, k, h at all dates. The depreciation rate $\mu \in (0,1)$. At time zero the representative agent is endowed with k_0 and h_0 units of physical and human capital respectively. Consider the problem of maximizing the representative agent's utility subject to the resource constraints.

(a) What restrictions on σ, δ, A, α and μ, if any, ensure the existence of a positive balanced growth path for this economy?

(b) For what restrictions on σ, δ, A,α and μ is the growth rate equal to zero, asymptotically?

(c) Compute the k/h ratio along a balanced growth path with positive growth. Is it independent from initial conditions?

(d) Compute the growth rate of c, k and h along a balanced growth path.

Section II: Long Questions

Answer *two (2)* questions from this section.

Question 4

Consider the $A(k,h)$ model of endogenous growth with distortionary policies in place:

Consumer's problem:

$$Max \qquad \sum_t \beta^t c_t^{1-\sigma}/(1-\sigma)$$

$$\sum_t p_t((1+\tau_{ct})c_t + x_{kt} + (1+\tau_{ht})x_{ht}) \leq T + \sum_t [r_t k_t + w_t h_t n_t]$$

$$k_{t+1} \leq (1-\delta_k)k_t + x_{kt}$$

$$h_{t+1} \leq (1-\delta_h)h_t + (x_{ht} + g_t)$$

$$k_0 \quad \text{and} \quad h_0 \quad \text{given.}$$

Note that leisure does not enter the utility function and hence $n_t = 1$ for all t.

Assume that production and feasibility are given by:

$$c_t + x_{kt} + x_{ht} + g_t \leq y_t = Ak_t^\alpha (h_t n_t)^{1-\alpha}$$

Interpret g_t as government provided education.

Assume that $\delta_h = \delta_k$ in all questions below.

Make any additional assumptions you need, but clearly state them.

(a) What is the growth rate of output, etc., in the TDCE in the absence of any policies – i.e., $\tau_{ct} = \tau_{ht} = g_t = T = 0$?

(b) Suppose $\tau_{ct} = \tau > 0$ for all t and that the government balances its budget each period by subsidizing the formation of human capital – $\tau_{ht} < 0$ for all t. Assume that $g_t = T = 0$. What is the growth rate of output in this TDCE? Is it higher or lower than in a)?

(c) Consider the same tax policy as in b) but instead have the government balance its budget in each period by providing education, $g_t > 0$. Assume that $\tau_{ht} = T = 0$. What is the growth rate of output in this TDCE? Is it higher or lower than in a)? (It might be useful to assume that τ is 'small' here.)

(d) Under which of the three policies is welfare higher?

Question 5

A unit measure of perfectly competitive firms can each produce either of two goods. Within any period, a firm selects which good it will produce by locating in either sector 1 or sector 2. Total factor productivity in sector 1 is constant and normalized to 1. Total factor productivity in sector 2 is random across firms and takes on a value of either $A > 0$ or 0 . At the start of any period, each firm individually draws a value of $p \in [0, 1]$, from the time-invariant distribution F. A firm's p draw is itself a probability; specifically, p is the probability that, if it chooses to operate in sector 2, the firm will realize productivity A there (rather than 0).

Timing: After the probability draws (p) are realized at the start of the period (and before production), each firm is identified by (p, s), where s is the sector it was in at production-time last period (and thus where it begins the current period). At this time, the firm can either remain in its current sector s, or it can pay a fixed cost, Φ, to relocate to the other sector, $\widetilde{s} \neq s$. Next, *after the relocation decision has been made*, firms in sector 2 find out their actual current productivity. At that point, all firms select how much capital they will rent at rental rate $r + \delta$, where $\delta > 0$ is the rate of depreciation and $r > 0$ is the interest rate, and production takes place.

Production: The output of any firm choosing to produce in sector 1 is $y_1 = k_1^\alpha$ units of good 1, where k_1 is its chosen capital input. The output of any firm producing in sector 2 is $y_2 = Ak_2^\alpha$ units of good 2 if its realized TFP draw is A, where $\alpha \in (0, 1)$ is common to both sectors. Any firm located in sector 2 at production time that realizes a TFP of 0 will set $k = 0$ and produce nothing. There is no aggregate uncertainty or technological progress.

Households: Let good 1 be the numeraire, and let q denote the relative price of good 2. Good 1 is used for consumption, to pay the fixed moving cost, and for investment in capital. Good 2 is used only for consumption. A large number of identical households own both the economy's firms and the aggregate capital stock. Each household has preferences described by $\sum \beta^t u(C_t)$, where $\beta \in (0, 1)$ and the period utility function u is strictly increasing and concave in the composite consumption good, C. This composite good comes from a simple linear aggregator: $C = c_1 + \theta c_2$, where c_1 is the household's consumption of good 1, c_2 is its consumption of good 2, and $\theta > 0$.

Firms: Assume that the economy is in its steady state, so that r and q are constant. Define $z_1 \equiv 1$ and $z_2 \equiv qA$. At production-time, any firm that rents $k > 0$ to produce in sector $s = 1, 2$ will solve $\pi_s = \max_k [z_s k^\alpha - (r + \delta) k]$. Let $W(p, s)$ represent the expected, discounted value of a firm that begins the period in sector s and draws p. Suppose a firm begins the period in sector 1. If it chooses to remain there, its expected lifetime value will be $\pi_1 + \beta \int_0^1 W(p', 1) F(dp')$. If it instead pays the fixed cost to relocate to sector 2, it will be $-\Phi + p\pi_2 + \beta \int_0^1 W(p', 2) F(dp')$. Similarly, a firm beginning the current period with $(p, 2)$ will have expected lifetime value $p\pi_2 + \beta \int_0^1 W(p', 2) F(dp')$ if it remains in sector 2; otherwise, it will have $-\Phi + \pi_1 + \beta \int_0^1 W(p', 1) F(dp')$.

(a) Write down a pair of functional equations determining $W(p, s)$, $s = 1, 2$. Next, derive expressions defining $\overline{p}_s$, $s = 1, 2$, the threshold values of p at which a firm beginning the period in sector s will move to the other sector. Using these two threshold equations, show that

$$(\overline{p}_1 - \overline{p}_2) \pi_2 = 2\Phi. \tag{1}$$

(b) Using the thresholds defined above, take expectations of the functional equations you derived in (a) to obtain a second pair of functional equations determining $V(s)$, where $V(s) \equiv \int_0^1 W(p, s) F(dp)$, $s = 1, 2$. Next, solve for $V(1)$ and $V(2)$ as functions of π_1, π_2, $\overline{p}_1$, and $\overline{p}_2$. (Remember that the economy is in its steady state.)

(c) Use your solution for $V(1)$ and $V(2)$ and either threshold equation to derive an implicit equation in $\overline{p}_1$ and $\overline{p}_2$. Label this as $D(\overline{p}_1, \overline{p}_2) = 0$.

Throughout the remainder of this question, assume that Φ is a lump-sum tax that the government collects from each firm changing sectors, and that tax revenues are rebated to households. The government sets $\Phi = \varphi\pi_2$, where $\varphi > 0$ is a parameter and π_2 continues to represent the profit of any firm producing in sector 2 with productivity A. *All firms take Φ as given.*

(d) Use $\Phi = \varphi\pi_2$ to simplify equations (1) and $D(\overline{p}_1, \overline{p}_2)$. Assume $q = \theta$, so that firms operate in both sectors. Derive sufficient conditions for the existence of a unique stationary equilibrium (a unique pair $(\overline{p}_1, \overline{p}_2)$ that solves the two equations). What fraction of firms operate in sector 1? What is the gross reallocation of firms (the total number of firms changing sectors in any period)?

(e) Consider an economy where the government does not charge a relocation tax, $\varphi = 0$. What are the equilibrium values of $\overline{p}_1$ and $\overline{p}_2$? How are these values different in an otherwise identical economy where $\varphi > 0$? Explain why a firm's p is not sufficient to determine the good it will produce without knowing its start-of-period location.

(f) Continuing to assume $\varphi > 0$, how do the threshold probabilities change if there is an increase in θ? Why? Making any necessary assumptions about the form of the distribution F and the initial value of φ, can you determine how a rise in φ affects the number of firms in sector 1 and gross reallocation? Discuss the channels through which φ affects each.

Question 6

Consider the following model of competitive innovation with externalities. Innovation consists in the introduction of two goods: the new productive input - k, think of a new pharmaceutical plant and its specialized workers - and the new consumption good - c: the new drug that obtains from k. The latter requires only labor to be produced, as in $L_t = g(x_t)$, with $g(x_t)$ a monotone increasing and convex cost function, and x_t the amount of additional capacity. Productive capacity alone produces the consumption good (the pills) as in $c_t = f(k_t)$, with $f(k)$ a standard neoclassical production function.

The representative and competitive innovator comes into the market at time $t = 0$ with k_0 - the initial pharmaceutical plant and the knowledge to produce the new drug. During the first period, $c_0 = f(k_0)$ new pills are therefore available to consumers. As soon as this occurs, the formula for the new drug becomes public knowledge and anyone can build additional productive capacity x_0 using just $L_0 = g(x_0)$ units of labor. In other words, there is a complete spillover, as competitors can install productive capacity to produce the new drug without purchasing anything from the innovator: after $t = 0$ all they need is to purchase the labor input in the competitive market for labor. This is true for all $t = 0, 1, 2, ...$, hence the law of motion of aggregate capacity is $k_{t+1} = \zeta k_t + x_t$, with $\zeta > 0$.

There is a representative consumer, with utility function

$$\sum_{t=0}^{\infty} \delta^t [u(c_t) - wL_t]$$

and everyone behaves competitively. Given k_0, the inter-temporal competitive equilibrium is summarized by sequences of quantities $\{k_t, L_t\}_{t=0}^{\infty}$, solving ...

(a) Work out the competitive equilibrium for this economy.

(b) Find and characterize the equilibrium prices for the pills, the productive capacity and labor.

(c) If inventing/producing one unit of initial productive capacity costs $C > 0$, and the representative innovator is assumed to behave competitively since the very first period ($t = 0$), under which conditions will he/she finds profitable to innovate?

(d) Same question as in the item above, but assuming the innovator can behave as a monopolist in the very first period ($t = 0$). What this means is that the initial quantity k_0 is not taken as given, but it is chosen to affect the initial price of capacity (call this q_0) in order to maximize total profits, i.e. $[q_0 - C]k_0$.