Ph.D. Preliminary Examination

FINANCIAL ECONOMICS

Fall 2007

This exam has two sections. Answer two questions from Section I and three from Section II. Define boldfaced terms. The time limit is four hours.

This exam has seven pages.

Section I

I.1. Consider a risk-free security with return $\bar{r}$ and a risky security with state-dependent return r. Assume that the set of states is a finite set S. An investor has deterministic initial wealth and a **multiple-prior (max-min) expected utility** function over state-contingent consumption plans. Assume that her von Neumann-Morgenstern utility is linear, and the set $\mathcal{P}$ of multiple probability measures (priors) on S is closed and convex.

Show that, if

$$\min_{P\in\mathcal{P}} E_P[r] \leq \bar{r} \leq \max_{P\in\mathcal{P}} E_P[r],$$

then the investor's optimal investment in the risky security is zero. Here, $E_P[r]$ denotes the expected return on the risky security under probability measure P.

I.2. Consider two-date security markets with N securities and S states. Suppose that there are short-sales restrictions on some but not all securities. That is, for a proper subset of N securities, holdings of these securities cannot be lower than some negative lower bounds.

State a necessary and sufficient condition for security prices to exclude (unlimited) **arbitrage under short-sales restrictions**. Your condition should be stated in terms of state prices. Prove that the condition is sufficient.

I.3. Consumers have preferences over consumption sequences $\{c_{t+n}\}_{n=0}^{\infty}$ determined by $\mathrm{E}_t\left[\sum_{n=0}^{\infty}\beta^n\left(c_{t+n}^{1-\gamma}-1\right)/(1-\gamma)\right]$ for some positive β and γ, with β small enough. Let e_t denote the aggregate endowment at time t. Consumers know e_t in period t and do not forget. The distribution of $\ln(e_{t+1}/e_t)$ given consumer information at time t is the same for all t. Markets are competitive and complete. Show that the yield curve for real discount bonds is flat.

I.4. The state price process for consumption payoffs is π_t, where $\mathrm{d}\pi_t = \pi_t[-r_t\mathrm{d}t + \sigma'_{\pi,t}\mathrm{d}W_t]$. Let p_t be the price level, measured in dollars per unit of consumption. Suppose that $\mathrm{d}p_t = p_t[\mu_{p,t}\mathrm{d}t + \sigma'_{p,t}\mathrm{d}W_t]$. Give an expression for the nominal interest rate.

Section II

II.1. Suppose that there is a single good available for consumption at a single date under uncertainty. Uncertainty is described be $S > 1$ states of nature. There are I agents whose preferences over state contingent consumption plans are specified by expected utility functions

$$\sum_{s=1}^{S} \pi_s v^i(c_s^i),$$

where c_s^i denotes consumption in state s, π_s is the strictly positive probability of state s (common to all agents), and v^i is the von Neumann-Morgenstern utility. Suppose that all utility functions v^i have **linear risk tolerance** (or, equivalently, hyperbolic risk aversion) with the same slope across all agents.

The total endowment is $\bar{\omega} = (\bar{\omega}_1, \ldots, \bar{\omega}_S)$ with $\bar{\omega}_s > 0$ for every state s and $\bar{\omega}_s \neq \bar{\omega}_{s'}$ for at least one pair of states s, s'.

(i) State a theorem asserting that agents' consumption plans at every Pareto optimal allocation lie in some two-dimensional subspace of the consumption space $\Re^S$. Be as general as you can. (No proof is required.)

(ii) Prove your theorem of (i) under additional assumption that utility functions v^i are quadratic.

(iii) Suppose that there are security markets and that they are complete. Let the market portfolio be a portfolio whose payoff is equal to the total endowment $\bar{\omega}$. Prove that the expected return on the market portfolio is strictly higher than the risk-free return in an equilibrium in security markets. If you find it convenient, you may impose the additional assumption of part (ii)

II.2. Consider a market for a single risky asset whose future payoff, denoted by $\tilde{v}$,is normally distributed with mean $\bar{v}$ and variance $\sigma_v^2 > 0$. There is one informed trader ("insider"), a market maker, and liquidity traders. There is a single round of trading. The demand of liquidity traders, denoted by $\tilde{z}$, is normally distributed with zero mean and variance $\sigma_z^2 > 0$, independent of $\tilde{v}$. Neither the insider nor the market maker can observe the liquidity demand.

The private information signal received by the insider prior to trading is a realization of the random variable

$$\tilde{s} = \tilde{v} + \tilde{\epsilon}$$

where $\tilde{\epsilon}$ is normally distributed with zero mean and variance $\sigma_\epsilon^2 > 0$, independent of $(\tilde{v}, \tilde{z})$. The insider is risk averse and has CARA utility with risk aversion $\rho > 0$. The market maker is risk neutral.

The insider submits a market order acting as a monopolist. She maximizes expected utility of profit from her market order. The market maker observes only the total market order of the insider and liquidity traders, sets price so that her expected profit be zero, and trades to clear the market. Traders have access to risk-free borrowing and lending with return equal to one.

(i) State a definition of an equilibrium in this model.

(ii) State a definition a linear equilibrium. Provide a sketch of a derivation of a linear equilibrium.. A "closed-form" solution is not required.

(iii) Define a full-information equilibrium as an equilibrium when the market maker observes the insider's signal. Find a full-information equilibrium.

II.3. Consider an economy with I consumers and one consumption good. There is an ex ante trading date and a consumption date. Consumers have identical preferences over stochastic consumption prospects c represented by

$$\mathrm{E}\left[\frac{c^{1-\gamma}}{1-\gamma}\right].$$

The coefficient of relative risk aversion γ is positive and different from one. Consumer $i \in \{1, 2, \ldots, I\}$ has endowments of consumption equal to $(y_i + z_i)e$ where y_i, z_i and e are all strictly positive random variables. The values of

$$\sum_{i=1}^{I} y_i \text{ and } \sum_{i=1}^{I} z_i$$

are non-random, and $(z_1, \ldots, z_I)$ is independent of $(e, y_1, \ldots, y_I)$. At the ex ante trading date, there is a complete set of markets for payoffs that depend only on $(e, y_1, \ldots, y_I)$. Consumers are price takers. In the ex ante market, the value of a payoff d that depends only on $(e, y_1, \ldots, y_I)$ is given by $\mathrm{E}[\pi d]$, for some strictly positive random variable π.

Determine the equilibrium π, up to an irrelevant normalization, and carefully explain your answer.

II.4. Consider an economy with a representative consumer whose preferences over consumption rates c_t are given by

$$\mathrm{E}_0\left[\int_0^\infty e^{-\rho t}\ln(c_t)\mathrm{d}t\right],$$

for some positive ρ. The consumer is endowed at date 0 with a positive stock of k_0 units of consumption. The stock of consumption goods must remain non-negative and can be accumulated according to

$$\mathrm{d}k_t = k_t R_t^{-1}\mathrm{d}R_t - c_t\mathrm{d}t,$$

where

$$\mathrm{d}R_t = R_t\left[\mu(x_t)\mathrm{d}t + \sigma(x_t)\mathrm{d}W_t\right]$$

and

$$\mathrm{d}x_t = \nu(x_t)\mathrm{d}t + \eta(x_t)\mathrm{d}W_t,$$

and W_t is a standard Brownian motion.

(i) Determine the optimal consumption function.

(ii) Determine the risk-free interest rate, assuming a standard decentralization.

Suppose that $\mu(x) = \alpha + \beta x$ and $\sigma(x) = \delta\sqrt{x}$. In addition, $\nu(x) = \kappa(\mu - x)$ and $\eta(x) = \theta\sqrt{x}$ for some positive κ, μ and θ.

(iii) Show that the risk-free rate follows a first-order autoregression.

II.5. Assume that the Internal Revenue Service's (IRS) depreciation allowances for computing corporate taxable income imply that depreciation for taxes is different than economic depreciation. Assume also that companies use their book capital to compute the depreciation allowance with inflation in the economy equal to 2 percent in all periods.

(i) Write out the maximization problem for a typical corporation. (Hint: Be careful to distinguish the reproducible cost of capital and the book value of capital.)

(ii) Derive the first-order necessary conditions for the corporation.

(iii) Explain intuitively how changing IRS depreciation allowances and inflation will affect the corporation's decisions and stock price.

II.6. Derive the market value of corporate equity in an economy with an accrual-based capital gains tax, implying that capital gains are taxed when they accrue rather than when they are realized. (Hint: relate the problem to one with a tax on retained earnings.)