Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MAJORS

Fall 2007

The time limit for this exam is $3\frac{1}{4}$ hours.

Notation:

$I\!R$ is the set of real numbers

$I\!R^n_+ = \{x \in I\!R^n : x_1 \geqq 0 \,\&\ldots\&\, x_n \geqq 0\}$

$I\!R^n_{++} = \{x \in I\!R^n : x_1 > 0 \,\&\ldots\&\, x_n > 0\}$.

For vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ in $I\!R^n$:

$x \gg y$ means $x_1 > y_1 \,\&\, \cdots \,\&\, x_n > y_n$

$x \geqq y$ means $x_1 \geqq y_1 \,\&\, \cdots \,\&\, x_n \geqq y_n$.

BE SURE you clearly define all **boldfaced/underlined** terms. Also, please be sure to define precisely any notation that you introduce.

NOTE: This examination should have 13 pages including this one. (Check to make sure!)

This exam has four (4) parts. Answer a total of FOUR (4) questions: ONE question from EACH part.

Part I

Answer *one (1)* question from Part I.

Question I.1

Let $\succeq$ be a reflexive, transitive, complete and strictly increasing (i.e., strongly monotone) preference relation on the consumption set $X = I\!R^L_+$. Preference relation $\succeq$ is said to be *homothetic* if the following holds for every $x, x' \in I\!R^L_+$ and every $\lambda > 0$:

$$\text{if } x \sim x' \quad \text{then} \quad \lambda x \sim \lambda x',$$

where $\sim$ is the indifference relation of $\succeq$.

Prove that $\succeq$ is homothetic if and only if it there exists a **utility representation** u of $\succeq$ such that u is homogeneous of degree 1.

Question I.2

Consider two real-valued random variables Y and Z on a probability space. You may think about Y and Z as two contingent claims on a state space. You may assume that the state space is finite. Suppose that Y can take only one of two possible values y_1, y_2 with respective probabilities $\pi_1 > 0$ and $\pi_2 > 0$ such that $\pi_1 + \pi_2 = 1$. Suppose further that the expectations of Z conditional on $\{Y = y_1\}$ and $\{Y = y_2\}$ are zero, that is $E[Z|Y = y_1] = 0$ and $E[Z|Y = y_2] = 0$.

(a) Prove that $Y + Z$ is **more risky** (in the sense of second-order stochastic dominance) than Y.

(b) Prove that $Y + 2Z$ is more risky than $Y + Z$.

Part II

Answer *one (1)* question from Part II.

Question II.1

This question applies to pure exchange economies with ℓ commodities and n traders, $i = 1, \ldots, n$, each having initial endowment vector $e_i \in I\!R^\ell$ and preferences $\preceq_i$ which are assumed throughout to be continuous complete preorders on the consumption set $X_i \subseteqq I\!R^\ell$.

(a) State the first welfare theorem.

(b) Prove the first welfare theorem.

(c) Does the conclusion of the first welfare theorem hold when the following complications separately (one at a time) are present?

Justify each answer by one of the following methods: pointing out that it doesn't affect your proof, explaining how your proof can be modified to encompass the complication, providing a counterexample to the conclusion of the first welfare theorem when this one complication is present, or explaining precisely how the complication prevents your proof from being modified to demonstrate that the first welfare theorem holds despite the complication. The complications are as follows:

(i) preferences that are convex but not strictly/strongly convex

(ii) preferences that are weakly convex but not convex

(iii) preferences that are nonsatiated but not locally nonsatiated

(iv) preferences that are strictly monotone but consumption sets X_i are not necessarily convex

(d) Briefly discuss (i.e., in an essay of 50–300 words) the economic significance of the first welfare theorem.

Question II.2

Consider a pure exchange economy with ℓ commodities and n consumers, $i = 1, \ldots, n$, each having initial endowment vector $e_i \in \mathbb{R}^\ell$ and preferences $\preceq_i$ defined on the consumption set $\mathbb{R}^\ell_+$. Each $\preceq_i$ is assumed to be a continuous complete preorder which satisfies strict convexity and strict monotonicity.

(a) What can be said about the aggregate excess demand Z of this economy? (I.e., state the theorem of Sonnenschein *et al* and be sure to specify the domain and range of the mapping Z.)

(b) For each property of Z you state in part (a), identify which assumption(s) on preferences are needed for the property and then prove that the assumption(s) you identify imply the property.

(c) Briefly discuss (i.e., in an essay of 50–300 words) the economic significance of the characterization of aggregate excess demand.

Part III

Answer *one (1)* question from Part III.

Question III.1

(a) Show that every finite game possesses a Nash equilibrium in which no player places a strictly positive probability on a weakly dominated strategy.

(b) Improve this result from (a) by showing that every finite game possesses a Nash equilibrium σ in which for every player i, σ_i is not dominated.

(c) Show by an example that the result in the previous point (b) requires finiteness.

Question III.2

(a) Define the set of correlated strategies and correlated equilibria for a finite game.

(b) Define an augmented game, and show how the Bayesian-Nash equilibrium of the augmented game and the correlated equilibrium are related.

(c) Prove that the set of correlated equilibrium payoffs is a closed, convex, non-empty set.

Part IV

Answer *one (1)* question from Part IV

Question IV.1

(a) Define the Nash equilibrium operator NE^δ and the Sub-game perfect Equilibrium operator SP^δ from subsets of the set of feasible and incentive compatible payoffs to subsets of the same set of payoffs.

(b) Define a self-generated set for each of the two operators in part (a).

(c) Show an example where the two operators are different.

Question IV.2

(a) Define a simple strategy profile.

(b) State the necessary and sufficient condition insuring that a simple strategy profile is a perfect equilibrium.

(c) Prove your statement from (b).