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Ph.D. Preliminary Examination
MACROECONOMIC THEORY
Spring 2009

Majors and Minors : Answer ALL FOUR parts.

PLEASE MAKE YOUR ANSWERS NEAT AND CONCISE

Make whatever assumptions you need to answer the questions. BE SURE TO STATE THEM CLEARLY.

Part I

In the following there are 9 questions for 60 points. Answer questions for a total value of 50 points. Be as BRIEF as you can and good luck.

Industry and Search

Imagine a firm got a worker and it can produce one unit of output per period for 10 periods (there is a zero interest rate). If the firm does not operate today, it loses its license and is out. The worker is risk neutral, can either take the job or run a hot dog operation what yields .25 units per period for five years and sick for another 5 years during which she will be on disability insurance collecting .1 units of output.

- (1) (5 points) What is the minimum wage that the worker would accept.
- (2) (5 points) If the firm had to pay an entrance fee to open, what would be the maximum fee under which the firm would enter.
- (3) (5 points) Pose a wage that is the bargaining solution with the firm having twice the weight than the worker.
- (4) (5 points) Briefly describe what could happen if there was a possibility of multiple entry of firms. How could the free entry condition be applied?

Recursive Competitive Equilibrium

There is an economy with many identical agents with preferences given by

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \alpha(1-n_t)^{\frac{1}{2}} + \gamma P_t^{\frac{1}{2}} \right] \right\}$$

where c_t is their own consumption at time t , n_t is the fraction of their own time worked at time t , and P_t are public parks. Their initial wealth is A .

The technology to produce output uses capital (that depreciates at rate δ) and labor:

$$Y_t = F(K_t, N_t)$$

- (5) (5 points) What conditions would be satisfied in a Pareto Optimum in steady state?

Imagine now that the government levies income taxes and issues debt to pay for the parks. Its initial debt is B .

- (6) (15 points) Define (recursively) the set of government policies that constitute an equilibrium together with all the necessary elements.
- (7) (5 points) Can you define an equilibrium with a policy such that debt is kept forever at its initial level? Be as precise as possible about the conditions that such a policy satisfies.

Imagine now that this is a small open economy and borrowing and lending can occur and sell at the international rate \bar{r} .

- (8) (10 points) Define Recursive competitive equilibrium for this case and for the appropriate policies.
- (9) (5 points) Give an expression for the wage, and for the stock of capital.

Part II

Consumption/Savings with Production Risk

Consider an economy with a continuum of consumers/entrepreneurs of measure 1, indexed by i .

Each consumer has access to a production technology that uses capital and labor to produce a homogeneous good. This production technology is specific to the entrepreneur who owns the technology. Letting k_{it} denote the amount of capital entrepreneur i has at date t , the production function is

$$y_{it} = z_{it} f(k_{it}, n_{it})$$

where z_{it} is a random variable that is *iid* over time and across consumers, f is a constant returns to scale function and n_{it} denotes the amount of labor hired by entrepreneur i . The economy has no aggregate uncertainty. Capital fully depreciates after one period.

Entrepreneurs can invest their capital only in their own production technology and cannot rent/sell it to other consumers. In addition to capital, entrepreneurs trade among themselves one-period risk-free bonds. Let r_t denote the interest rate on bonds purchased at date t . The profits generated by entrepreneur i are given by

$$\pi_i(k_i, z_i; w) = \max_{n_i} z_i f(k_i, n_i) - w n_i$$

The consumer/entrepreneur's problem is to maximize life-time utility

$$\max_{c_{jt}, k_{jt+1}, b_{jt+1}} E \sum_{t=0}^{\infty} \beta^t u(c_{jt})$$

subject to

$$c_{jt} + k_{jt+1} + b_{jt+1} = \pi_i(k_{jt}, z_{jt}; w_t) + w_t + (1 + r_{t-1}) b_{jt}$$

where, note that the endowment of labor of each entrepreneur is normalized to be 1.

Assume $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma > 1$. Let λ^* denote the initial distribution of assets (k_{i0}, b_{i0}) across consumers. The market clearing condition for bonds is $\int b_{it} di = 0$ for all t . There is no constraint (other than the natural debt limit) on how much the consumer can borrow using the risk-free bond. We require however $k_{jt+1} \geq 0$.

- (1) Write the consumer's problem recursively. Define a recursive competitive equilibrium.
- (2) Characterize the consumer's decision rules in the stationary steady-state: derive expressions relating the consumption and portfolio decisions to primitives of the model. Show that these consumption rules aggregate into an expression for aggregate consumption as a function of the aggregate capital stock. Use this aggregation result to characterize the steady state level of the capital stock. (Hint: You may find it useful to let human wealth be defined as the present discounted value of wages.)
- (3) Does $\beta(1+r) = 1$ in this economy?

Part III

Please answer as much as you can of the following question

Consider a closed economy inhabited by a continuum of infinitely lived agents (each agent is indexed by i) whose preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left(\log(c_{it}) - \frac{l_{it}^{\phi}}{\phi} \right)$$

$1 > \beta > 0, \phi > 1$

where c_{it} is consumption (of a single good) and l_{it} is labor. Agents have no initial wealth, and in each period they are endowed with labor productivity e_{it} given by

$$e_{it} = \exp(\varepsilon_{it} + A_i)$$

where

$$A_i \rightarrow N\left(\frac{-\sigma_A^2}{2}, \sigma_A^2\right) \text{ i.i.d. across agents, constant through time}$$

$$\varepsilon_{it} \rightarrow N\left(\frac{-\sigma_{\varepsilon}^2}{2}, \sigma_{\varepsilon}^2\right) \text{ i.i.d. across agents and time}$$

Agents can trade a full set of Arrow securities contingent on the realization of their ε and can sell their effective labor $e_{it}l_{it}$ to competitive firms who use a constant returns to scale technology that transforms one unit of effective labor into one unit of consumption good. Let w be the price of one unit of effective labor in terms of the consumption good. There is no capital nor aggregate uncertainty in the economy. Assume throughout that appropriate law of large numbers apply, so that for each period and for each possible value of the permanent shock A , the measure of agents experiencing temporary shocks in a given set is constant.

- (1) Denote with $p_{\varepsilon'}(A, \varepsilon)$ and $b_{\varepsilon'}(A, \varepsilon)$ the price and the quantity purchased (by an agent experiencing shocks A, ε) of an Arrow security paying off 1 unit of consumption tomorrow if the agent tomorrow experiences shock ε' , and with $c(A, \varepsilon), l(A, \varepsilon)$ consumption and labor supply of the same agent. Write down its budget constraint

(2) Define a competitive equilibrium for this economy

(3) Show that in equilibrium

$$p_{\varepsilon'}(A, \varepsilon) = \beta\pi(\varepsilon')$$

where $\pi(\varepsilon')$ is the pdf of ε' ,

$$c(A, \varepsilon) = c(A)$$

i.e. consumption is independent from the temporary shock ε , and

$$b_{\varepsilon'}(A, \varepsilon) = c(A) - wl(A, \varepsilon') \exp(\varepsilon' + A)$$

(4) Solve for the risk free rate and for the wage per unit of effective labor in this economy

(5) Solve for $c(A)$ and for $l(A, \varepsilon)$ (remember that if $x \rightarrow N(\mu, \sigma^2)$ then

$$E(\exp(x)) = \exp(\mu + \frac{1}{2}\sigma^2)$$

(6) Show that GDP and ex-ante welfare (i.e. welfare before you know your A and the sequence of your ε s) in this economy are increasing in σ_ε^2 . Explain why

(7) Show that GDP is constant in σ_A^2 and ex ante welfare is decreasing in σ_A^2 . Explain why

(8) Solve for the difference between labor income inequality and consumption inequality (measure inequality as the standard deviations of the log). What are the key parameters that determine this difference? Explain why

Part IV

This problem asks you to work with the version of the neoclassical growth model with human capital and exogenous labor supply.

Consider the following Planner's Problem:

$$\max \quad \sum \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to:

$$(i) \quad c_t + x_{kt} + x_{ht} \leq y_t = Ak_t^\alpha z_t^{1-\alpha},$$

$$(ii) \quad k_{t+1} \leq (1 - \delta_k)k_t + x_{kt},$$

$$(iii) \quad h_{t+1} \leq (1 - \delta_h)h_t + x_{ht},$$

$$(iv) \quad z_t \leq n_t h_t$$

$$(iv) \quad 0 \leq \ell_t + n_t \leq 1,$$

$$(v) \quad h_0, k_0 \text{ given.}$$

(1) Show that the value function for this problem is homogeneous of degree $(1 - \sigma)$ in the initial capital stocks, (h_0, k_0) .

(2) Characterize the homogeneity properties of the optimal decision rules in initial capital stocks, (h_0, k_0) . Specifically if the initial conditions on the capital stocks are $(\eta h_0, \eta k_0)$ instead of (h_0, k_0) where $\eta > 0$, how are the optimal time paths for consumption, labor supply, investment, etc., affected? Prove your claims.

Suppose now that the government has a fixed sequence of expenditures that it must finance, g_t , and that it can use taxes on capital and labor income, τ_{kt} and τ_{zt} .

- (3) Define a competitive equilibrium for this environment.
- (4) What is the Ramsey problem here for a benevolent government? In particular, carefully derive and explain the implementability constraint for this environment.
- (5) Assume that $\delta_k = \delta_h$. What is $\frac{\tau_{kt}}{\tau_{zt}}$ in this case?
- (6) Assume that $\delta_k = \delta_h$. What can you say about $\lim_t \tau_{kt}$? About $\lim_t \tau_{zt}$?