

Basic Variables and Free Variables

A **pivot position** in a matrix A is a location that corresponds to a leading entry in an echelon form of A . The column that contains the entry in the pivot position is called the **pivot column**.

A **basic variable** is that specific variable that corresponds to the pivot column. Variables that are not basic are called **free variables**.

For example, take the reduced echelon matrix $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

Recall that this signifies that $x_1 = 2$, $x_2 = 1$, $x_3 = 1$ since all variables x_1 , x_2 , x_3 are basic.

On certain occasions though, there can be a solution set or even no solution. In the system of linear equations

$$2x_1 - 3x_2 = 0$$

$$4x_1 - 6x_2 = 0$$

the reduced echelon form is $\begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

In this case $x_1 - \frac{3}{2}x_2 = 0$. Therefore the only true statement we can deduce, is that the solution set is $\{(x_1, x_2) : x_1 = \frac{3x_2}{2}\}$. In this system, x_1 is a basic variable whereas x_2 is a free variable.