Characterization of Invertible Matrices

A square matrix does *not* have to be invertible. So when is, a square matrix invertible? The Invertible Matrix Theorem provides conditions that the square matrix needs to fulfill, to guarantee the existence of an inverse.

The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given A, the statements are either *all true* or *all false*.

a. A is an invertible matrix.
b. A is row equivalent (the coefficient matrix) to the $n \times n$ identity matrix.
c. A has n pivot positions.
d. The equation $A\mathbf{x} = 0$ has only the trivial solution.
e. The columns of A form a linearly independent set.
f. The number 0 is *not* an eigenvalue of A (eigenvalues are examined later).