Column Space and Null Space of a Matrix

The **column space** of a matrix A is the set $Col(A)$ of all linear combinations of the columns of A. Basically, determining whether a vector **b** is in $Col(A)$ amounts to showing that the **b** vector is in the span of A .

In set notation, $Col(A) = \{\mathbf{b} : \mathbf{b} = A\mathbf{x} \text{ for some } \mathbf{x} \text{ in } \Re^n\}$.

The **null space** of a matrix A is the set $Nul(A)$ of all solutions to the homogeneous equation $A\mathbf{x} = 0$.

In set notation, $Nul(A) = \{\mathbf{x} : \mathbf{x} \text{ is in } \Re^n \text{ and } A\mathbf{x} = \mathbf{0}\}$

Example

Let $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and let $\mathbf{v} = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$.

Determine if **v** belongs to the null space of A.

To test if **v** satisfies $A\mathbf{v} = \mathbf{0}$, simply compute

$$A\mathbf{v} = \begin{bmatrix} 2 & 4 & -4 \\ -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Thus **v** belongs to the null space of A.