

Consistent versus Inconsistent Systems

Recall that a system of linear equations comprises of a number of linear equations involving the same variables. For example

$$\begin{aligned}2x_1 - 2x_2 + 8x_3 &= 10 \\4x_1 - 6x_2 + 2x_3 &= 4 \\2x_2 + x_3 &= 6\end{aligned}$$

is a system of three linear equations.

A **solution** to this system of linear equations would be a list of numbers in the place of the variables that would make all three linear equations true statements. The *unique solution* to this linear system requires $x_1 = 2$ and $x_2 = x_3 = 1$. To verify that this is indeed a solution to the system just plug into the three linear equations the recommended numbers, and make sure that the equations become true statements.

Intuitively, finding a solution to the system of equations amounts to locating the intersection of the three lines. Think of each linear equation as a line in the $x_1x_2x_3$ -plane. The solution to the system is simply the (x_1, x_2, x_3) point that is common to all three lines.

However, not always there will be a unique solution to the system. Sometimes there can be a *set of solutions* or even *no solution* at all as the following examples indicate.

Example with a solution set

$$\begin{aligned}2x_1 - 3x_2 &= 0 \\4x_1 - 6x_2 &= 0\end{aligned}$$

Verify that the solution set is $\{(x_1, x_2) : x_1 = \frac{3x_2}{2}\}$. Notice that the solution set has infinite possible solutions since geometrically, the two lines coincide. Hence every point on the overlapping lines is a potential sound solution to the system of equations.

A linear system with either a *unique solution* or a *solution set* is called a **consistent** system.

Example with no solution

$$\begin{aligned}2x_1 - 2x_2 + 8x_3 &= 10 \\6x_1 - 6x_2 + 24x_3 &= 4 \\2x_2 + 4x_3 &= 6\end{aligned}$$

In this case the system of equations has no solution. Notice that if you multiply the first equation by 3 you will get the second expression but with a different (contradictory) result.

A linear system with *no solution* is often referred to as an **inconsistent** system.